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Stellar Evolution and Convection in 3D Hydrodynamic Simulations of a Complete Burning Phase

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Abstract: Our understanding of stellar evolution and nucleosynthesis is limited by the uncertainties coming from the complex multi-dimensional processes in stellar interiors, such as convection and nuclear burning. Three-dimensional stellar models can improve this knowledge by studying multi-D processes, but only for a short time range (minutes or hours). Recent advances in computing resources have enabled 3D stellar models to reproduce longer time scales and include nuclear reactions, making the simulations more accurate and allowing to study explicit nucleosynthesis. Here, we present results from 3D stellar simulations of a convective neon-burning shell from a 20 $M_\odot$ star, run with an explicit nuclear network from its early development to complete fuel exhaustion. We show that convection halts when fuel is exhausted, stopping its further growth after the entrainment of fresh material. These results, which highlight the differences and similarities between 1D and multi-D stellar models, have important implications for the evolution of convective regions in stars and their nucleosynthesis.

Keywords: convection; hydrodynamics; nuclear reactions, nucleosynthesis, abundances; stars: evolution; stars: interiors; stars: massive

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1. Introduction

Today, the structure and evolution of a star are normally investigated through one-dimensional (1D) stellar models. Thanks to their simplified prescriptions for reproducing the stellar physical processes, 1D models are the most used tools for reproducing the life of a star, being able to effectively produce entire grids of stellar models, useful for different fields. However, the prescriptions adopted need to be carefully calibrated, either through observations or multi-D stellar modeling; however, a calibration performed with short-time-range simulations may not be correct. Classic examples of such prescriptions are the mixing length theory (MLT) [1] and convective boundary mixing (CBM, also known as 'overshoot') [2], employed to determine the extent of convective regions in stars; these two prescriptions can lead to degeneracy and sometimes yield significantly different results. Multi-dimensional hydrodynamic simulations of specific portions of stars are able to couple the fluid motions with other physical processes such as nuclear burning, rotation, and magnetic fields. This approach is disfavored by the great computing resources required for these simulations. For these reasons, only small sections of a star can be modeled, and for a time scale compatible with hours of stellar evolution. Therefore, it is possible to improve our understanding of these processes only with a combined effort from both 1D and multi-D stellar simulations. Having a correct knowledge of convection and CBM is of great relevance for asymptotic-giant-branch (AGB) stars, for their impact on the formation of the ^{13}C-pocket and its consequent nucleosynthesis. In fact, CBM under the convective envelope during the third dredge-up is responsible for the formation of the ^{13}C-pocket; therefore, its specific implementation in the models strongly affects the size and chemical composition of the ^{13}C-pocket, and consequently the final nucleosynthesis of the star [3–6].

In order to better understand the behavior of CBM over the long time scale, and particularly its relation to nuclear burning and fuel exhaustion, we have produced a new set of hydrodynamic simulations with the `PROMPI` code [7], following the evolution of a neon-burning shell in a 20 $M_\odot$ star continuously from its early development to fuel exhaustion, assuming initial conditions from a state-of-the-art 1D stellar model [8]. Although this is a different environment than what found in AGB stars, we believe that significant information about CBM can be extracted that has a more general validity. We present here the evolution of this shell and its entrainment over time, and we draw a comparison with similar studies available in the literature.

2. Methods

The four simulations presented in this work were run with the stellar hydrodynamical code `PROMPI` [7], starting from the same initial conditions assumed from a 1D stellar evolution model of a 20 $M_\odot$ star run with the `MESA` code [9]. The 1D model assumes solar metallicity, Schwarzschild criterion and exponential decaying overshoot for all convective boundaries (see [8] for more details). The structure of the neon-burning shell, namely its density, temperature, and composition, was remapped onto a wedge in spherical geometry, of radius 3.6–8.5 $\times$ 10^8 cm and angular size 26°. The 3D simulations were then run with a numerical resolution of 256 $\times$ 128^2 cells.

In the 3D hydrodynamic simulations, an explicit nuclear network is used for reproducing nuclear energy generation, including the isotopes n, p, ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, ^{23}Na, ^{24}Mg, ^{28}Si, ^{31}P, ^{32}S, and ^{56}Ni. This extended network is used here only for reproducing neon burning, but it can potentially cover all the energy-generating nuclear reactions from helium to oxygen burning. The four 3D simulations differ by the nuclear reaction rates, which were multiplied by a 'boosting factor', a constant value that increases energy generation. Boosting the stellar luminosity is commonly carried out in hydrodynamic codes in order to accelerate the simulations by increasing the convective speed and reducing the computation cost (e.g., [10]).

Here, we present four hydrodynamic simulations run with boosting factors 1 (nominal luminosity), 5, 10, and 50 (see [8] for more details). It is important that we include a simulation that was not boosted (boosting factor equals 1) in order to have consistent results comparable with the 1D model.

3. Results

In order to visualize the output of the simulations, we show a vertical cross-section from one of the simulations in Figure 1, run at nominal luminosity with a resolution of 2048 $\times$ 1024^2. The figure shows the neon mass fraction in color scale, and includes the convective region in the center of the domain, characterized by plumes and eddies, surrounded by stable radiative regions. In addition, it is possible to see the entrainment of neon-rich material (in red) in the top-right corner.

We also show the time evolution of the four simulations in Figure 2, which represents the angularly averaged kinetic energy (in color scale) along the radius as a function of time. Each panel shows a simulation with a different boosting factor, from 1 in the top-left panel to 50 in the bottom-right one. The boosting has an effect on both the time scale of the simulations (x-axis), and the amount of kinetic energy produced (color scale). In all simulations, the convective zone in yellow gradually grows due to entrainment, until it stops growing when fuel is exhausted and convection halts. In these final phases, the gradual end of convection displays fluctuations between higher and lower levels of energy, as visible in the figure, before it finally stops. This is the first time in the literature that convection in hydrodynamic simulations stops due to fuel exhaustion, and these simulations show that entrainment does not proceed indefinitely, engulfing the stars, but it naturally halts when also convection dies.

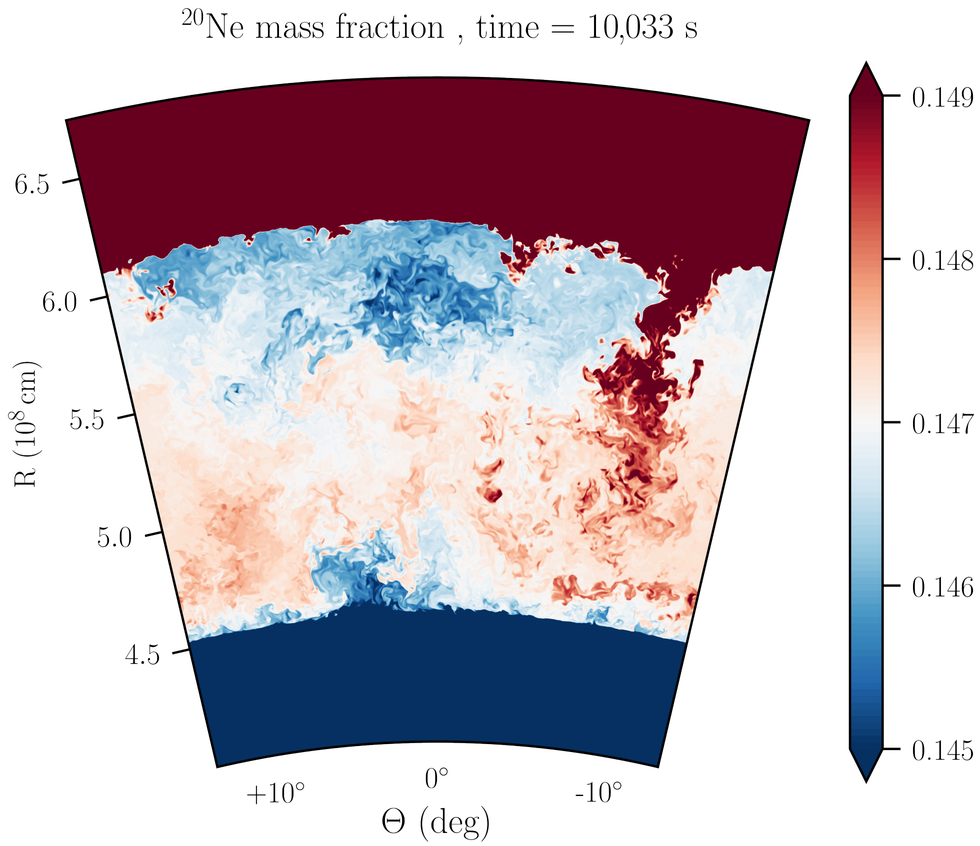

Figure 1. Vertical cross-section of the neon mass fraction (in color scale), taken from the high-resolution nominal luminosity simulation after 10,033 s. The plot shows entrainment of neon-rich material from the upper stable region into the convective zone.

Studying the evolution of the four simulations, we can parameterize turbulent entrainment with a simple law that relates the entrainment rate E, defined as the entrainment over the convective velocity v_e/v_{rms}, to the bulk Richardson number Ri_B, which represents a measure of the convective boundary stiffness. The law was defined by [7,11] through the free parameters A, n as $E = A\ Ri_B^{-n}$. We can use the new hydrodynamic simulations we presented here to estimate the free parameters A, n, by measuring E from Figure 2, and computing Ri_B by integrating the Brunt-Väisälä frequency across the convective boundary (see [12] for definitions and implementation).

In Figure 3, we show this new estimate of the entrainment rate (the line of best fit in log scale, blue) compared to another neon-shell from [12] (red), run for a shorter time scale, and entrainment estimated in the 1D models of [13]. In the legend, we give the estimated A and n for each work.

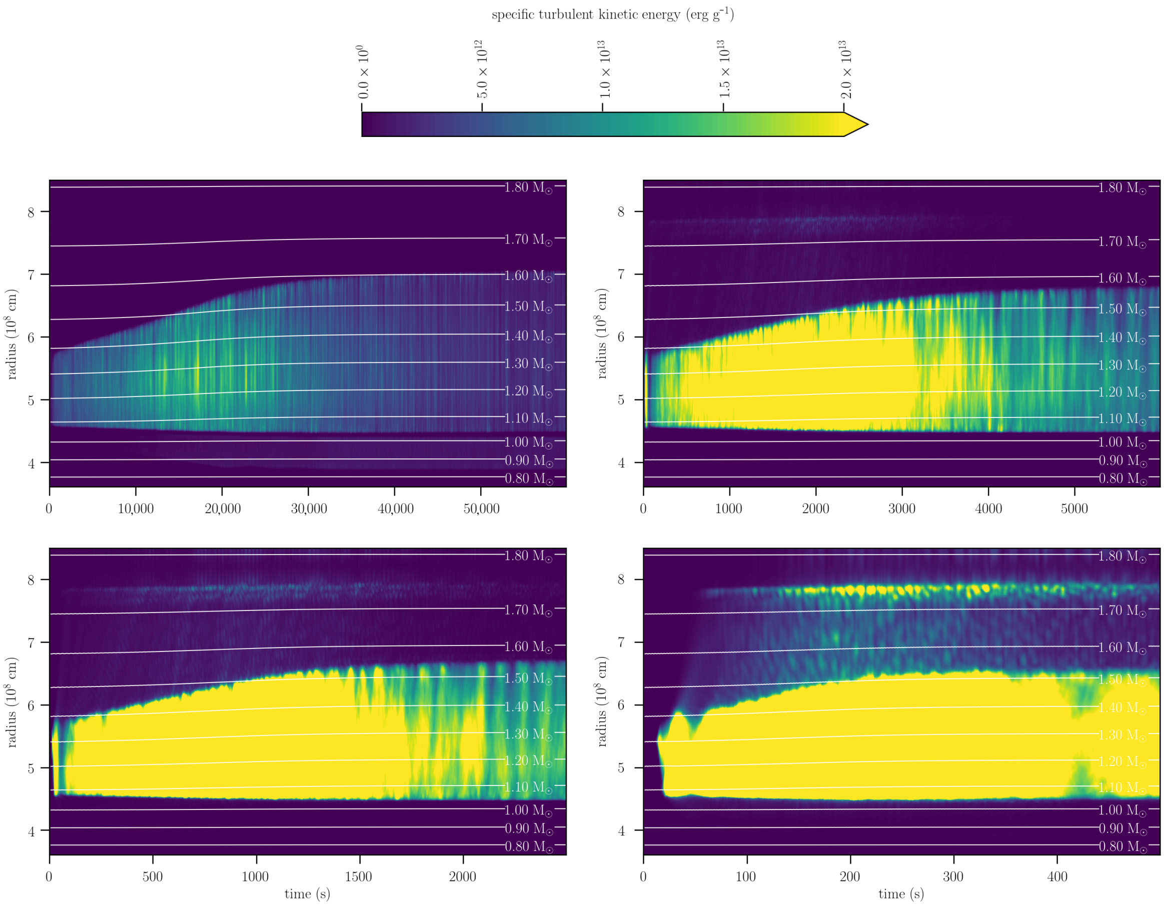

Figure 2. Time evolution of the angularly averaged kinetic energy (in color scale) for four simulations with different luminosity (from top left to bottom right, boosting factor 1, 5, 10, 50). In yellow, the convective zone grows with time due to entrainment. In white, the isomass contours.

We can see from the plot that the three studies predict different slopes for the entrainment law. In particular, overshoot in 1D models is always found to be much slower that in hydrodynamic simulations. On the other hand, 3D models of stars have such short time scales that it is not guaranteed that entrainment will reach an equilibrium status. The difference between the two 3D models in Figure 3 is that the one from [12] (red) was run for a fraction of the shell evolution, while the one presented here (blue) was run for the entire shell lifetime. Indeed, the steeper slope found here makes entrainment compatible with both 1D models at large Ri_B and 3D models at small Ri_B. This is because hydrodynamic models are the best for simulating late phases of massive stars, where Ri_B is small, while 1D models are better constrained during the main sequence, when Ri_B is large. In this sense, the new entrainment law found here could explain CBM in both 1D and 3D models.

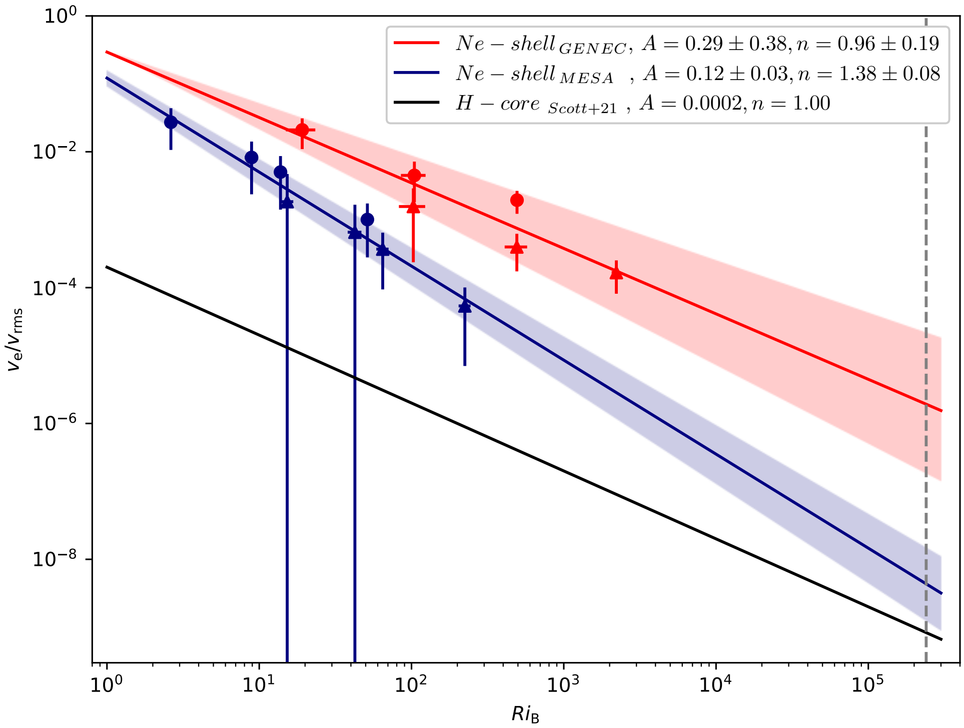

Figure 3. Entrainment rate vs. bulk Richardson number and linear regressions: `MESA` Ne-shell from this study (blue), `GENEC` Ne-shell from [12] (red), and H-core from 1D [13] (black). Triangles are lower convective boundaries; circles are upper boundaries. The dashed vertical line is the convective H-core. Error bars are standard deviations, and the shaded regions are parameter uncertainties (only slope uncertainty for `GENEC` Ne-shell, since amplitude error is too large in log scale).

4. Discussion

We presented here a new set of 3D hydrodynamic simulations of a neon-burning shell from a 20 $M_\odot$ star, run with the `PROMPI` code. We have reproduced the entire evolution of the shell, from its early phases until fuel exhaustion, studying the interplay between nuclear reactions and turbulent convection, and constraining the convective boundary mixing (or overshoot) through the entrainment law.

The main conclusion of this work, as also described in [8], is that entrainment does not proceed indefinitely as a physical process, but it naturally halts when fuel is exhausted in the convective region and convection stops. This result is derived by 3D simulations based on a state-of-the-art 1D stellar model and validated by the fact that the nuclear energy generation rate was not modified in the nominal luminosity simulation. The entrainment law that is parameterized from the data presented here has a steeper slope than what was found in previous multi-D simulations of advanced phases, which did not cover the entire nuclear burning evolution.

These results are of great interest for their application to different fields in astrophysics, including AGB stars. Indeed, the exact amount of overshoot is expected to have a strong impact on the structure and evolution of AGB stars, considering that CBM under the convective envelope during the third dredge-up is shown to be essential for producing the ^{13}C-pocket, while the importance of CBM under the He-intershell is still debated. Recent hydrodynamic studies (e.g., [8,12,14,15]), including the one presented here, seem to suggest that entrainment could be more vigorous compared to traditional prescriptions; therefore, its impact on the physics of AGB stars could be remarkable.

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F.R., R.H. and V.V.; investigation, F.R., R.H. and V.V.; resources, R.H., W.D.A., C.M. and M.M.; data curation, F.R., R.H. and C.G.; writing—original draft preparation, F.R., R.H., V.V., W.D.A., C.G., C.M., M.M., A.S.M. and T.R.; writing—review and editing, F.R., R.H., V.V., W.D.A., C.G., C.M., M.M., A.S.M. and T.R.; visualization, F.R. and R.H.; V.V.; supervision, F.R., R.H., W.D.A., C.G., A.S.M. and T.R.; project administration, F.R. and R.H.; funding acquisition, R.H. All authors have read and agreed to the published version of the manuscript.

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Data Availability Statement: The data underlying this article will be shared upon reasonable request to the corresponding author.

Conflicts of Interest: Dr. Casey A. Meakin is employed by Pasadena Consulting Group. The remaining authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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