Chapter 8
Dispersion of the Bending Wave in a Fluid-loaded Elastic Layer

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Abstract A plane strain problem is considered for an elastic layer immersed into a compressible fluid. The dispersion relation for anti-symmetric waves is studied. The associated three-term long-wave low-frequency expansion for a fluid-borne bending wave is derived, along with similar expansions corresponding to Kirchhoff and Timoshenko-Reissner type fluid-loaded plates. The results of comparative asymptotic analysis are presented. The role of plate inertia and fluid compressibility are discussed.

Key words: Fluid-loaded elastic layer, Plate theories, Dispersion, Asymptotic

8.1 Introduction

Fluid-structure interaction problems for elastic plates have been investigated since long ago. However, asymptotic considerations in this area were usually restricted to the classical Kirchhoff theory, e.g. see [2, 3]. Only a few publications has approached the subject using original equations in dynamic elasticity, e.g. see [5, 6, 9]. Until now to the best of authors' knowledge there is no direct comparisons of the asymptotic (not just numerical) results, obtained from linear elasticity and approximate plate models. At the same time, nowadays there is a significant demand of more rigorous and accurate predictions inspired by advanced industrial applications, including soft robotics, e.g. see [8].

In this paper we study a plane strain problem for an elastic layer, governed by 2D equations in elasticity, in contact with a compressible non-viscous fluid. The related dispersion equation for anti-symmetric waves is analysed at the long-wave low-frequency limit. The ratio between dimensionless wavelength and frequency is

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not taken to be the same as for a bending wave on a free plate and corresponds to a specific fluid-borne bending wave, e.g. see [10]. A three-term asymptotic expansion of the aforementioned dispersion relation is derived and implemented for testing the approximate dispersion relations for two simplified fluid-structure interaction models based on thin plate asymptotic theories including the classical Kirchhoff theory as well as Timoshenko-Reissner type theory, e.g. see [1, 7] and references therein. The roles of fluid compressibility and plate inertia are also addressed.

It is shown in particular that the leading order term in the derived expansion of the ”exact” dispersion relation also follows from the dispersion relation for a Kirchhoff plate immersed into incompressible fluid, neglecting the plate inertia. Moreover, fluid compressibility appears to be outside the range of validity of both classical and refined plate based formulations studied in the paper. It is also established that the adapted refined theory has a higher asymptotic accuracy than the classical one. This observation is far from being obvious, since consideration starts from the assumption that fluid loading may be considered as a prescribed external stress field. The latter assumption has been proved to be justified even at a higher order, although it formally supports the asymptotic scaling characteristic of a bending wave on a plate with traction free faces not incorporating accurately enough the effect of the fluid.

The paper is organised as follows. The linear equations in plane elasticity and fluid dynamics are presented in Sect. 8.2, along with the approximate formulations based on the Kirchhoff and Timoshenko-Reissner types plate theories. All associated dispersion relations are derived in Sect. 8.3. The Sect. 8.4 is concerned with a comparative analysis of asymptotic expansions.

8.2 Basic Equations

Consider free in-plane vibrations of an elastic layer of thickness $2h$ immersed in a non-viscous compressible fluid. Let the mid-line of the layer be the $x_2 = 0$ axis of the Cartesian coordinate system $(-\infty < x_1, x_2 < \infty)$, see Fig. 8.1. Throughout the paper we use the following notation: ρ and ρ_0 are solid and fluid densities, respectively; E is Young's modulus, ν is the Poisson ratio, λ and μ are Lamé elastic constants, c_1 and c_2 are the longitudinal and transverse wave speeds in solid, c_0 is the wave speed in fluid.

The equations of motion in terms of the elastic potentials ϕ and ψ and the fluid potential φ can be written as

$$\Delta\phi - \frac{1}{c_1^2}\frac{\partial^2\phi}{\partial t^2} = 0, \qquad \Delta\psi - \frac{1}{c_2^2}\frac{\partial^2\psi}{\partial t^2} = 0, \tag{8.1}$$

and

$$\Delta\varphi - \frac{1}{c_0^2}\frac{\partial^2\varphi}{\partial t^2} = 0, \tag{8.2}$$

where