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Analysis of G-type Seismic Waves in Dry Sandy Layer overlying an Inhomogeneous Half-space

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Abstract

The paper investigates the dispersion of G-type seismic waves along the interface of dry sandy and inhomogeneous isotropic elastic media. The maximum energy flow which projects the fact that a large amount of energy can flow along the interface for horizontally polarized shear wave with group velocity lesser than the shear wave velocity in the upper mantle has been shown in closed form by means of dispersion equation. Effect of sandiness parameter and inhomogeneities on phase velocity has been shown in graphs. A case study has also been made for isotropic homogeneous over an inhomogeneous half-space.

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Keywords: G-type wave; Dry sandy layer; Phase velocity; Dispersion equation; Transform technique.

1. Introduction

The study of surface waves remains a constant field of interest to the seismologists and geophysicists to estimate the damage caused by its devastating nature. The existence of the low-velocity layer in the earth mantle was first established by Gutenberg [1, 2]. G-type seismic waves are horizontally polarized seismic waves of shear type that can propagate with a group velocity of about 4.4 km/s. G-type waves exhibit a transient pulse-like character in records that are followed by a train of dispersed Love waves especially for continental path. The group velocities of Love waves are of period range from 40-300s but in case of Love waves with long periods (60-300*s*) are known as G-type waves. It has been observed that G-type waves take 2.5h to make a round trip of the earth. A sequence of G-type waves may be observed after a long earthquake.

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Many researchers have already made some contribution on G-type waves. Gutenberg [1], Lehmann [3], Mal [4] have discussed the generation of G-type seismic waves. Aki [5] has also shown the generation and propagation Niigata earthquake on June 16, 1964. Chattopadhyay and Keshri [6] pointed out the generation of G-type seismic waves under the influence of initial stress. Chattopadhyay et al. [7, 8] also studied the G-type waves in viscoelastic and fiber-reinforced media. Other extensive works have been done by many researchers like Jeffreys [9], Bhattacharya [10], Haskell [11] etc. Recently Kundu et al. [12] have also established the effect of G-type waves inhomogeneous layer over pre-stressed inhomogeneous substratum.

In this paper we represented the low velocity layer by assuming the variation in rigidity and density. Finally we have obtained the dispersion equation of G-type wave in dry sandy layer overlying an inhomogeneous isotropic half-space. Assuming the variations in rigidity and density in the half-space which reduces the equation of motion into Hill's equation with periodic coefficients is later solved by method introduced by Valeev [13]. It was also examined that a class of system of linear differential equations with periodic coefficients which may be converted to linear difference equations using Laplace transformation which in turn may be solved by the method infinite determinants. Keeping the terms upto first order, the Laplace transform of the displacements is obtained.

2. Formulation of the Problem

We have considered a dry sandy medium of thickness H overlying semi-infinite inhomogeneous isotropic elastic half-space. The x-axis is taken horizontally and the z-axis as vertically downwards where as the origin has been taken at the interface of these two media.

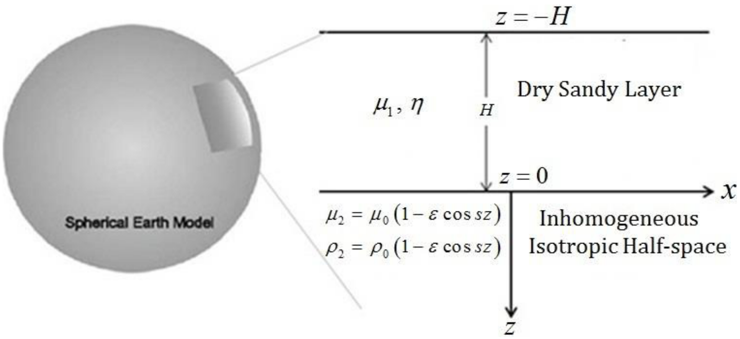

Fig. 1. Formulation of the Problem

The variations in rigidity and density of half-space are assumed as

$$\mu_2 = \mu_0\left(1-\varepsilon\cos sz\right)$$
$$\rho_2 = \rho_0\left(1-\varepsilon\cos sz\right)$$

where ε & s represents the small positive constant and real depth parameter respectively.

3. Solution of the Problem

We consider the propagation of horizontally polarized surface waves of shear type propagating along the x-axis. So the displacement components are $u=0, w=0$ and $v=v\left(x,z,t\right)$. Therefore the equation of those waves in dry sandy elastic medium is given by

$$\frac{\mu_1}{\eta}\left[\frac{\partial^2 v_1}{\partial x^2}+\frac{\partial^2 v_1}{\partial z^2}\right]=\rho_1\frac{\partial^2 v_1}{\partial t^2} \tag{1}$$

where v_1 denotes the y-component of the displacement vector.

μ_1, ρ_1, η denote the rigidity, density and the sandiness parameter of the dry sandy medium respectively. When $\eta > 1$ the layer corresponds to the sandy materials and $\eta = 1$ corresponds to the isotropic elastic solid.

Following Tomar and Kaur [14] and considering $\beta_1 = \sqrt{\dfrac{\mu_1}{\eta\rho_1}}$ as the shear wave velocity the above equation (1) takes the form
above equation (1) takes the form

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial z^2} = \frac{1}{\beta_1^2}\frac{\partial^2 v_1}{\partial t^2} \tag{2}$$

Using the method of separation of variables and taking $v_1 = V_1(z)e^{ik(x-ct)}$ the equation (2) becomes

$$\frac{d^2V_1}{dz^2} + s_1^2 V_1 = 0 \tag{3}$$

where $s_1 = k\sqrt{\dfrac{c^2}{\beta_1^2} - 1}$ and $\beta_1 = \sqrt{\dfrac{\mu_1}{\eta\rho_1}}$

Hence
$$v_1(x,z,t) = (c_1 \cos s_1 z + c_2 \sin s_1 z)e^{ik(x-ct)} \tag{4}$$

For the lower inhomogeneous medium the displacement $v_2(x,z,t)$ satisfies the following differential equation

$$\frac{\partial}{\partial x}\left[\mu_2(1-\varepsilon\cos sz)\frac{\partial v_1}{\partial x}\right] + \frac{\partial}{\partial z}\left[\mu_2(1-\varepsilon\cos sz)\frac{\partial v_2}{\partial z}\right] = \rho_2(1-\varepsilon\cos sz)\frac{\partial^2 v_2}{\partial t^2} \tag{5}$$

Since stresses and displacements are continuous at the interface and the upper layer is stress free hence the boundary conditions are

$$\begin{aligned} &\text{(i)}\ v_1 = v_2 \ \text{at}\ z = 0 \\ &\text{(ii)}\ \mu_1\eta\frac{\partial v_1}{\partial z} = \mu_2(1-\varepsilon\cos sz)\frac{\partial v_2}{\partial z}\ \text{at}\ z=0 \\ &\text{(iii)}\ \frac{\partial v_1}{\partial z} = 0\ \text{at}\ z = -H \end{aligned} \tag{6}$$

Since the upper surface is stress free, hence using the boundary condition (iii) of eq. (6) we get

$$\frac{C_1}{\cos s_1 z} = \frac{C_2}{\sin s_1 z} = R_0 \ \text{(say)} \tag{7}$$

Therefore eq. (6) becomes

$$v_1(x,z,t) = R_0 \cos s_1(z+H)e^{ik(x-ct)} \tag{8}$$

For the lower inhomogeneous half-space the displacement $v_2(x,z,t)$ satisfies the following differential equation

$$\frac{\partial}{\partial x}\left[\mu_2(1-\varepsilon\cos sz)\frac{\partial v_1}{\partial x}\right] + \frac{\partial}{\partial z}\left[\mu_2(1-\varepsilon\cos sz)\frac{\partial v_2}{\partial z}\right] = \rho_2(1-\varepsilon\cos sz)\frac{\partial^2 v_2}{\partial t^2} \tag{9}$$

where the density ρ_2 is assumed to be constant.

Now taking
$$v_2(x,z,t) = V_2(z)e^{ik(x-ct)} \tag{10}$$

Now using eq. (9), the eq. of motion for lower non-homogeneous medium may be written as

$$\begin{aligned} &\frac{d^2V}{dz^2} + e^{isz}\left[-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2V_2(z) + \frac{\varepsilon}{2}k^2V_2(z) - \frac{\varepsilon}{2}\frac{d^2V}{dz^2} - \frac{\varepsilon si}{2}\frac{dV_2}{dz}\right] + e^{-isz}\times\left[-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2} + \frac{i\varepsilon s}{2}\frac{dV_2}{dz} - \right. \\ &\qquad \left. \frac{\rho_2 k^2c^2\varepsilon}{2\mu_2}V_2(z) + \frac{k^2\varepsilon}{2}V_2(z)\right] + k^2\left(\frac{\rho_2 c^2}{\mu_2} - 1\right)V_2(z) = 0 \end{aligned} \tag{11}$$

This is the Hill's differential equation which can be solved by the method given by Valeev [13].

We apply the Laplace transformation w.r.to '*z*', that is we multiply eq. (11) by e^{-rz} and integrate w.r.to *z* from 0 to ∞ we get

$$\int_0^\infty e^{-(r+is)z}\left\{-\frac{\varepsilon}{2}\frac{d^2V_2}{dz^2}+\frac{i\varepsilon s}{2}\frac{dV_2}{dz}-\frac{\rho_2k^2c^2\varepsilon}{2\mu_2}V_2(z)+\frac{k^2\varepsilon}{2}V_2(z)\right\}dz+\int_0^\infty e^{-(r-is)z}\left[-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2V_2(z)+\frac{\varepsilon}{2}k^2V_2(z)-\frac{\varepsilon}{2}\frac{d^2V}{dz^2}-\frac{\varepsilon si}{2}\frac{dV_2}{dz}\right]dz+\int_0^\infty e^{-rz}\left[\frac{d^2V_2}{dz^2}+\left\{\frac{\rho_2c^2}{\mu_2}-1\right\}k^2V_2(z)\right]dz=0 \tag{12}$$

From the boundary condition (i) of (5) i.e., $v_1=v_2$ at $z=0$

therefore
$$V_2(0)=R_0\cos(s_1H) \tag{13}$$

Now we consider $q(0)=\left(\frac{dV_2}{dz}\right)_{z=0}$ and $\mu_2^{(0)}=\mu_2(1-\varepsilon)$, then from the boundary condition (ii) of (5) gives

$$q(0)=-\frac{\mu_1\eta R_0s_1\sin(s_1H)}{\mu_2^{(0)}} \tag{14}$$

Considering the Laplace transform of $V_2(z)$ as

$$F(r)=\int_0^\infty e^{-rz}V_2(z)\,dz \tag{15}$$

Using (14) and (15) in eq. (12) we obtained

$$\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2+\frac{\varepsilon si}{2}(r+is)-\frac{\varepsilon}{2}(r+is)^2\right\}F(r+is)+\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon}{2}(r-is)^2-\frac{\varepsilon si}{2}(r-is)\right\}F(r-is)+\left(r^2-\psi^2\right)F(r)=r\lambda_1+\lambda_2 \tag{16}$$

where
$$\lambda_1=(1-\varepsilon)V_2(0),\ \lambda_2=(1-\varepsilon)q(0)\text{ and }\psi^2=k^2\left(1-\frac{\rho_2c^2}{\mu_2}\right) \tag{17}$$

To find the $F(r)$ from eq. (16) we replace r by $(r+ism)$ and then divide throughout by $(ism)^n$ $(m\neq 0)$. We obtain the following infinite system of linear system of linear differential eqns. in the quantities $F(r+ism)$ $[m=0,\pm1,\pm2,\pm3,.....]$

$$(ism)^{-n}\left[-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2+\frac{\varepsilon si}{2}\{r+is(m+1)\}-\frac{\varepsilon}{2}\{r+is(m+1)\}^2\right]F\{r+is(m+1)\}+(ism)^{-n}\left[-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon si}{2}\{r+is(m-1)\}-\frac{\varepsilon}{2}\{r+is(m-1)\}^2\right]\times F\{r+is(m-1)\}+(ism)^{-n}\left\{(r+ism)^2-\psi^2\right\}F(r+ism)=(ism)^{-n}\left\{(r+ism)\lambda_1+\lambda_2\right\} \tag{18}$$

where r has been considered as a parameter in the coefficients. To consider the special case of separately for $m=0$ we include eq. (16) in (18) by considering $(ism)^{-n}=1$ when $m=0$.

Solving the system of difference eqns. (18) we obtain $F(r)$ as the ratio of infinite determinants

e.g.,
$$F(r)=\frac{\Delta_1}{\Delta_2} \tag{19}$$

where
$$\Delta_1=\begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (is)^{-n}\left\{(r+is)^2-\psi^2\right\} & (is)^{-n}\left\{(r+is)\lambda_1+\lambda_2\right\} & 0 & \cdots \\ \cdots & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.+\frac{\varepsilon si}{2}(r+is)-\frac{\varepsilon}{2}(r+is)^2\right\}\end{matrix} & (r\lambda_1+\lambda_2) & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right.\\ \left.-\frac{\varepsilon si}{2}(r-is)-\frac{\varepsilon}{2}(r-is)^2\right\}\end{matrix} & \cdots \\ \cdots & 0 & (-is)^{-n}\left\{(r-is)\lambda_1+\lambda_2\right\} & (-is)^{-n}\left\{(r-is)^2-\psi^2\right\} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

$$\Delta_2 = \begin{vmatrix} \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & (is)^{-n}\left\{(r+is)^2-\psi^2\right\} & \left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon si}{2}r-\frac{\varepsilon}{2}r^2\right\} & 0 & \cdots \\ \cdots & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right. \\ \left.+\frac{\varepsilon si}{2}(r+is)-\frac{\varepsilon}{2}(r+is)^2\right\}\end{matrix} & \left(r^2+\psi^2\right) & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right. \\ \left.-\frac{\varepsilon si}{2}(r-is)-\frac{\varepsilon}{2}(r-is)^2\right\}\end{matrix} & \cdots \\ \cdots & 0 & \begin{matrix}(-is)^{-n}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right\} \\ -(-is)^{-n}\left\{\frac{\varepsilon si}{2}r+\frac{\varepsilon}{2}r^2\right\}\end{matrix} & (-is)^{-n}\left\{(r-is)^2-\psi^2\right\} & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \end{vmatrix}$$

The first approximation of the eqn. (19) can be written as

$$F(r)=\frac{r\lambda_1+\lambda_2}{r^2-\psi^2}\approx\frac{r(1-\varepsilon)B_1}{r^2-\psi^2}-\frac{(1-\varepsilon)B_2}{\mu_2^{(0)}\left(r^2-\psi^2\right)} \tag{20}$$

where $B_1=R_0\cos(s_1H)$
$B_2=(-R_0)\mu_1\eta s_1\sin(s_1H)$

The second approximation of the eq. (19) can be written as $F(r)=\dfrac{\Delta_3}{\Delta_4}$ (21)

where

$$\Delta_3 = \begin{vmatrix} (is)^{-n}\left\{(r+is)^2-\psi^2\right\} & (is)^{-n}\left\{(r+is)\lambda_1+\lambda_2\right\} & 0 \\ \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right. \\ \left.+\frac{\varepsilon si}{2}(r+is)-\frac{\varepsilon}{2}(r+is)^2\right\}\end{matrix} & (r\lambda_1+\lambda_2) & \begin{matrix}\left\{-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2\right. \\ \left.-\frac{\varepsilon si}{2}(r-is)-\frac{\varepsilon}{2}(r-is)^2\right\}\end{matrix} \\ 0 & (-is)^{-n}\left\{(r-is)\lambda_1+\lambda_2\right\} & (-is)^{-n}\left\{(r-is)^2-\psi^2\right\} \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} (is)^{-n}\left\{(r+is)^2-\psi^2\right\} & -\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon si}{2}r-\frac{\varepsilon}{2}r^2 & 0 \\ \begin{matrix}-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2 \\ +\frac{\varepsilon si}{2}(r+is)-\frac{\varepsilon}{2}(r+is)^2\end{matrix} & \left(r^2+\psi^2\right) & \begin{matrix}-\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2+\frac{\varepsilon}{2}k^2 \\ -\frac{\varepsilon si}{2}(r-is)-\frac{\varepsilon}{2}(r-is)^2\end{matrix} \\ 0 & (is)^{-n}\left\{-\frac{\varepsilon}{2}(\rho_2/\mu_2)k^2c^2+\frac{\varepsilon}{2}k^2-\frac{\varepsilon si}{2}r-\frac{\varepsilon}{2}r^2\right\} & (-is)^{-n}\left\{(r-is)^2-\psi^2\right\} \end{vmatrix}$$

Neglecting the terms containing ε^2 and higher orders we get

$$s^{2n}\Delta_3=(\lambda_1r+\lambda_2)\left\{(r+is)^2-\psi^2\right\}\left\{(r-is)^2-\psi^2\right\}+\left\{(r-is)\lambda_1+\lambda_2\right\}\left\{(r-is)^2-\psi^2\right\}\times\left\{\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2-\frac{\varepsilon}{2}k^2+\frac{\varepsilon si}{2}(r-is)+\frac{\varepsilon}{2}(r-is)^2\right\}+$$
$$\left\{(r+is)\lambda_1+\lambda_2\right\}\left\{(r-is)^2-\psi^2\right\}\times\left\{\frac{\varepsilon}{2}\frac{\rho_2}{\mu_2}k^2c^2-\frac{\varepsilon}{2}k^2+\frac{\varepsilon si}{2}(r+is)+\frac{\varepsilon}{2}(r+is)^2\right\}$$

and $s^{2n}\Delta_4=\left(r^2-\psi^2\right)\left\{(r-is)^2-\psi^2\right\}\left\{(r+is)^2-\psi^2\right\}$

Then the eqn. reduces to

$$F(r)=\frac{\lambda_1 r+\lambda_2}{r^2-\psi^2}+\frac{\varepsilon}{2}\frac{V_2(0)(r+is)+q(0)}{(r^2-\psi^2)\{(r+is)^2-\psi^2\}}\left\{\frac{\rho_2}{\mu_2}k^2c^2-k^2-is(r+is)+(r+is)^2\right\}$$
$$+\frac{\varepsilon}{2}\frac{V_2(0)(r-is)+q(0)}{(r^2-\psi^2)\{(r-is)^2-\psi^2\}}\left\{\frac{\rho_2}{\mu_2}k^2c^2-k^2-is(r-is)+(r-is)^2\right\} \tag{22}$$

With the help of inversion formula $V_2(z)$ can be written as

$$V_2(z)=\frac{1}{2\pi}\int_{\gamma-i\infty}^{\gamma+i\infty}F(r)e^{rz}\,dr \tag{23}$$

The residues R_1, R_2, & R_3 at the poles $r=\psi$, $r=\psi+is$, $r=\psi-is$ are represented as

$$R_1=\left\{\frac{\psi V_2(0)+q(0)}{2\psi}\right\}\left\{(1-\varepsilon)+\frac{\varepsilon\psi^2}{s^2+4\psi^2}\right\}e^{\psi z}-\frac{\varepsilon A_1}{s^2+4\psi^2}\left\{\frac{q(0)-\psi V_2(0)}{2\psi}\right\}e^{\psi z}+\frac{\varepsilon V_2(0)}{2}\left\{\frac{s^2+2\psi^2}{s^2+4\psi^2}\right\}e^{\psi z} \tag{24}$$

$$R_2=-\frac{i\varepsilon}{4s}\frac{\{\psi V_2(0)+q(0)\}\{A_1+\psi^2+is\psi\}}{\psi(2\psi+is)}e^{(\psi+is)z} \tag{25}$$

$$R_3=\frac{\varepsilon}{4s}\frac{\{\psi V_2(0)+q(0)\}\{A_1+\psi^2+is\psi\}}{\psi(2\psi-is)}e^{(\psi-is)z} \tag{26}$$

where $A_1=\frac{\rho_2}{\mu_2}k^2c^2-k^2$

The eqns. (24), (25) & (26) reveals that the conditions for a large amount of energy to be confined near the surface are

$$q(0)+\psi V_2(0)=0 \tag{27}$$

$$q(0)-\psi V_2(0)=0 \tag{28}$$

and $$2\psi^2+s^2=0 \tag{29}$$

Now the eqns. (27) and (28) gives

$$q(0)=\pm\psi V_2(0)$$

Thus the dispersion relation obtained can be written as

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right)=\pm\frac{1-\varepsilon}{\eta}\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2}-1}} \tag{30}$$

where $\beta_1=\sqrt{\frac{\mu_1}{\eta\rho_1}}$ and $\beta_2=\sqrt{\frac{\mu_2}{\rho_2}}$

The eqn. (30) is the dispersion eqn. for propagation of G-type seismic waves in dry sandy layer over an inhomogeneous half-space.

We now consider only the positive sign for further consideration.

The eq. (29) provides $$kc=\sqrt{\frac{\mu_2}{2\rho_2}(2k^2+s^2)} \tag{31}$$

Thus the group velocity is given by

$$W=\frac{d}{dk}(kc)=\frac{\beta_2\sqrt{2}k}{\sqrt{2k^2+s^2}} \tag{32}$$

4. Particular Cases:

4.1 Case-I

When $\eta = 1$, i.e. the upper layer becomes a elastic solid material then eqn. (30) reduces to

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right) = (1-\varepsilon)\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2}-1}} \tag{33}$$

This is the dispersion eqn. of G-type seismic wave in an elastic solid material over an inhomogeneous half-space.

4.2 Case-II

When $\varepsilon = 0$, i.e. the lower half-space becomes homogeneous then eqn. (30) transforms into

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right) = \frac{1}{\eta}\frac{\mu_2}{\mu_1}\frac{\sqrt{1-\frac{c^2}{\beta_2^2}}}{\sqrt{\frac{c^2}{\beta_1^2}-1}} \tag{34}$$

which is the dispersion eqn. of G-type seismic wave in dry sandy layer over an homogeneous half-space.

4.3 Case-III

When both $\eta = 1$ and $\varepsilon = 0$, i.e. the upper layer is an elastic solid material and lower half-space becomes homogeneous the eqn. (30) converts into

$$\tan\left(kH\sqrt{\frac{c^2}{\beta_1^2}-1}\right) = \frac{\mu_2}{\mu_1}\frac{\sqrt{1-\left(c^2/\beta_2^2\right)}}{\sqrt{\left(c^2/\beta_1^2\right)-1}} \tag{35}$$

which is the dispersion eqn. of G-type seismic wave an elastic solid material over an homogeneous half-space and this eqn. coincides with the classical eqn. of Love type waves.

5. Numerical Calculation and Discussion

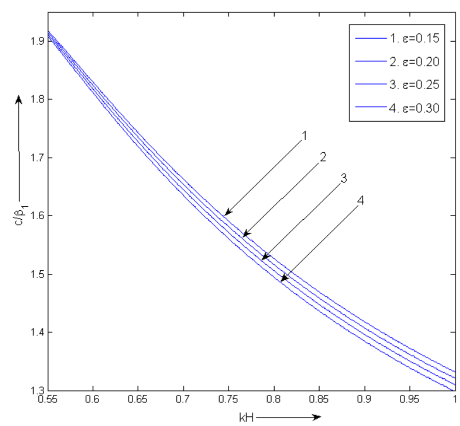

Fig. 2. Dimensionless phase velocity c/β_1 against dimensionless wave number kH with changing values of ε

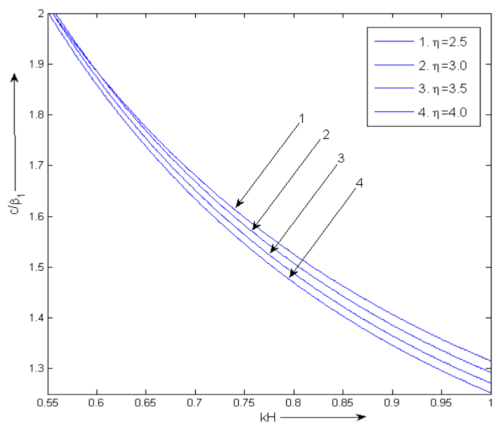

Fig. 3. Dimensionless phase velocity c/β_1 against dimensionless wave number kH with changing values of η

Referring Gubbins (1990) we have taken the values of $\mu_1 = 6.54\times10^{10}\ N/m^2$, $\rho_1 = 3409 Kg/m^3$, $\mu_2 = 6.34\times10^{10}\ N/m^2$, $\rho_2 = 3364 Kg/m^3$ for the graphical presentation of phase velocity and wave number of G-type wave in dry sandy layer over inhomogeneous half-space. Figure 2 clarifies that dimensionless phase velocity decreases with the increase of wave number and for different values of $\varepsilon = 0.15, 0.2, 0.25, 0.3$ with a fixed value of sandiness parameter $\eta = 2$.

Figure 3 demonstrates that dimensionless phase velocity also decreases with the increase of wave number and for different values of $\eta = 2.5, 3.0, 3.5, 4.0$ with a fixed value of $\varepsilon = 0.15$.

6. Conclusions

We have derived the dispersion equation of G-type seismic wave in a dry sandy layer over an inhomogeneous half-space with the help of transform technique and Valeev's method. It is clear that the phase velocity is influenced by sandiness parameter (η) and the inhomogeneity parameter (ε). We have obtained the condition for the large amount of energy that is confined near the surface and also derived the expression for the group velocity. Variations in phase velocity and wave number have been plotted graphically. Thus we can conclude that

- Dimensionless phase velocity (c/β_1) decreases gradually with the increase of dimensionless wave number (kH)
- Dimensionless phase velocity (c/β_1) decreases with the increase of sandiness parameter (η)
- Dimensionless phase velocity (c/β_1) decreases with the increase of inhomogeneity parameter (ε).

The study will not only enrich to determine the causes of earthquake but also may be useful to predict the nature of Love waves having long period.

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