The effect of starspots on the ages of low-mass stars determined from the lithium depletion boundary

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ABSTRACT
In a coeval group of low-mass stars, the luminosity of the sharp transition between stars that retain their initial lithium and those at slightly higher masses in which Li has been depleted by nuclear reactions, the lithium depletion boundary (LDB), has been advanced as an almost model-independent means of establishing an age scale for young stars. Here we construct polytropic models of contracting pre-main sequence stars (PMS) that have cool, magnetic starspots blocking a fraction β of their photospheric flux. Starspots slow the descent along Hayashi tracks, leading to lower core temperatures and less Li destruction at a given mass and age. The age, $\tau_{\rm LDB}$, determined from the luminosity of the LDB, $L_{\rm LDB}$, is increased by a factor $(1 - \beta)^{-E}$ compared to that inferred from unspotted models, where $E \simeq 1 + d\log \tau_{\rm LDB}/d\log L_{\rm LDB}$ and has a value ~ 0.5 at ages $< 80\,{\rm Myr}$, decreasing to ~ 0.3 for older stars. Spotted stars have virtually the same relationship between K-band bolometric correction and colour as unspotted stars, so this relationship applies equally to ages inferred from the absolute K magnitude of the LDB. Low-mass PMS stars do have starspots, but the appropriate value of β is highly uncertain with a probable range of $0.1 < \beta < 0.4$. For the smaller β values our result suggests a modest systematic increase in LDB ages that is comparable with the maximum levels of theoretical uncertainty previously claimed for the technique. The largest β values would however increase LDB ages by 20–30 per cent and demand a re-evaluation of other age estimation techniques calibrated using LDB ages.

Key words: stars: magnetic activity; stars: low-mass – stars: evolution – stars: pre-main-sequence – clusters and associations: general – starspots

1 INTRODUCTION

During their approach to the main sequence, the cores of contracting low-mass ($\leqslant 0.5\,M_\odot$) pre-main sequence (PMS) stars reach temperatures sufficient to ignite lithium. Rapid mixing in these fully-convective stars then leads to complete Li destruction throughout the star on a time scale that is short compared with their contraction time scales (e.g. Basri, Marcy & Graham 1996; Rebolo et al. 1996; Bildsten et al. 1997). The time it takes to reach this Li-burning threshold increases with decreasing stellar mass. If a set of coeval low-mass stars in a young cluster are spectroscopically examined for the presence of photospheric Li, there is an abrupt transition from stars that have depleted all their initial Li to stars of only slightly lower luminosity (and mass) that retain their initial Li. The luminosity, $L_{\rm LDB}$, at this "Lithium Depletion Boundary" (LDB) can therefore be used to estimate the age of the cluster, $\tau_{\rm LDB}$ (Stauffer, Schultz & Kirkpatrick 1998; Stauffer et al. 1999; Jeffries & Oliveira 2005).

$\tau_{\rm LDB}$ is approximately proportional to $L_{\rm LDB}^{-1/2}$ (see section 2.3), so with typical uncertainties in distances and bolometric corrections, LDB age estimates can have precisions of 5–10 per cent.

Of greater importance is that relatively few assumptions need be made in order to calculate $L_{\rm LDB}$ as a function of age; analytical calculations and evolutionary models incorporating differing treatments of opacities, equations of state and convection produce age estimates that differ by less than 15 per cent (Ushomirsky et al. 1998; Jeffries & Naylor 2001). Burke, Pinsonneault & Sills (2004) conducted a series of calculations, altering the physical inputs of their evolutionary models within plausible bounds, finding "theoretical uncertainties" in LDB ages that were just 3–8 per cent depending on the stellar mass at the LDB.

The theoretical robustness of LDB age estimates compares favourably with competing methods of cluster age determination (e.g. Soderblom 2010; Soderblom et al. 2013). Ages determined from isochrone-fitting to high-mass stars, as they leave the ZAMS and turn off the main-sequence, depend sensitively on the effects of rotation and the assumed levels of convective overshoot in the core (e.g. Maeder & Meynet 1989; Meynet & Maeder 2000; Ekström et al. 2012). Isochrone fitting to low-mass PMS stars is subject to significant uncertainties in the efficiency of convection and the adopted outer atmospheres (e.g. D'Antona & Mazzitelli 1994; Baraffe et al. 1998), and there are further uncertainties introduced

in transforming observables such as spectral type into effective temperatures. Resultant age uncertainties can be factors of two, even given a large set of coeval stars at a range of masses (e.g. Hillenbrand, Bauermeister & White 2008; Hillenbrand 2009). For these reasons, a review of techniques for estimating young stellar ages by Soderblom et al. (2013) concluded that the LDB method provides the most reliable means of setting the absolute age scale of PMS stars.

LDB age determinations now exist for 9 clusters and associations (see Soderblom et al. 2013 and references therein), ranging in age from 20–130 Myr. At ages < 20 Myr, the extent of the superadiabatic envelope leads to rapidly growing model dependencies on the adopted opacities and convection treatment (Burke et al. 2004). At ages $\geqslant 150$ Myr, the sensitivity of the technique declines, but a greater limitation is that L_{LDB} becomes so small that spectroscopy in even nearby clusters becomes impractical.

Despite the optimism that LDB ages are robust to theoretical uncertainties, the assumptions made and the effects neglected by *all* evolutionary models could still lead to systematic errors. An important example is the neglect of the effects of rotation and magnetic activity – both of which are manifestly present in low-mass PMS stars. Whilst the effects of rotation on the hydrostatic structure of low-mass PMS stars and their Li depletion are likely to be small (e.g. Mendes, D'Antona & Mazzitelli 1999; Burke et al. 2004), the effects of the consequent dynamo-generated magnetic fields may be more significant (e.g. Ventura et al. 1998; D'Antona, Ventura & Mazzitelli 2000). There is growing observational evidence from fast-rotating, magnetically active stars in tidally-locked eclipsing binaries, young clusters and the field, that magnetic activity may increase the radii of low-mass stars (Lopez-Morales 2007; Morales, Ribas & Jordi 2008; Jackson, Jeffries & Maxted 2009; Stassun et al. 2012). The mechanism by which it does so is still unclear; but could include the magnetic stabilisation of the star against convection (Gough & Tayler 1966; Moss 1968; Mullan & Macdonald 2001; Feiden & Chaboyer 2013), a reduction in convective efficiency due to a turbulent dynamo (Feiden & Chaboyer 2014) or the blocking of emergent flux by dark, magnetic starspots (Spruit 1982; Spruit & Weiss 1986; Jackson & Jeffries 2014). If PMS stellar radii are increased by magnetic activity, they could have lower central temperatures and hence less Li depletion at a given age. Using an arbitrary lowering of the mixing length parameter to simulate a reduction in convective efficiency, Somers & Pinsonneault (2014) show that spreads in Li abundance seen in the G/K stars of young clusters can be explained by varying levels of magnetic activity connected with a spread in rotation rates. They also claim that a similar increase in radius associated with magnetic fields in lower mass stars would delay the onset of Li burning and that current LDB ages may be underestimates.

In Jackson & Jeffries (2014) we investigated how the presence of dark, magnetically induced starspots influences the evolution and radii of low-mass PMS stars. Using a polytropic model, we showed that the radii of spotted, fully convective PMS stars are increased by a factor $\simeq (1 - \beta)^{0.45}$ compared with unspotted stars of the same luminosity, where β is the fraction of photospheric flux blocked by spots. In this contribution we use a similar approach to explore how starspots will affect ages determined from the LDB. We find that for plausible levels of spot coverage on active, low-mass PMS stars, that current LDB ages are *systematically* underestimated by a factor comparable to, or larger than, the maximum levels of theoretical uncertainty previously claimed for the technique.

The plan of the paper is as follows: section 2 describes how we adapt and calibrate a simple polytropic model to predict the

time at which Li is depleted by a given fraction in fully convective low-mass stars. Provided that the critical temperature at which Li burning commences and the properties of the atmospheric opacities do not change rapidly with mass, this leads to a relatively simple expression for the increase in age implied for spotted stars at a given L_{LDB}. In section 3 we then consider the properties of the starspots and how these alter bolometric corrections and the relationship between absolute magnitude of the LDB and age. In section 4 we discuss the implications of the results and in particular, the consequences of older LDB ages for evolutionary models and the calibration of isochronal ages in the HR diagram.

2 EFFECT OF STARSPOTS ON THE AGE OF LITHIUM DEPLETION

The presence of dark starspots on the photosphere will slow the contraction of PMS stars, leading to larger radii at a given luminosity (Jackson & Jeffries 2014). This will delay the onset of lithium depletion and hence in a cluster of coeval stars, alter the relationship between L_{LDB} and τ_{LDB}. We evaluate this as follows:

A set of model evolutionary tracks are analysed to determine both the age, t_{Li} and radius, R_{Li} as a function of mass, M, and luminosity, L, at which low mass stars achieve a given level of lithium depletion (typically, 95 or 99 percent thresholds are used by observers, e.g. Jeffries & Oliveira 2005).

A polytropic model is introduced to represent fully convective low-mass stars as they descend Hayashi tracks towards the LDB. Constants defining the surface opacity of the polytropic model are chosen to fit the results of published (unspotted) evolutionary tracks as a function of M in the Hertzsprung-Russell (HR) diagram.

The polytropic model is used to determine the effect of starspots on R and L as a function of M, age and starspot coverage. Under the assumption that similar constants characterise the polytropic models for spotted and unspotted stars at the LDB, and that Li depletion occurs at the same value of R/M in spotted and unspotted stars, we derive a simple analytic expression for the relative change in the LDB age of spotted stars as a function of spot coverage.

2.1   Masses and radii of stars at the LDB

Fig. 1 shows evolutionary tracks and associated isochrones for low mass stars ($0.07 < M/M_\odot < 0.5$). These are outputs plotted from the BCAH98 theoretical isochrones (Baraffe et al. 1998) for metallicity [M/H]=0.0, helium mass fraction Y=0.275 and a convective mixing length of one pressure scale height ($L_{\mathrm{mix}} = H_p$). The tabulated model data include a lithium depletion factor, X_{Li}, as a function of mass and age, representing the fraction of the initial photospheric Li remaining. These data are interpolated by fitting $X_{\mathrm{Li}} = \frac{1}{2}[1 - \tanh((t - t_{50})/\tau_{\mathrm{Li}})]$ to the tabulated isochrones, where the constant t_{50} represents the (mass-dependent) epoch of 50 per cent Li depletion and τ_{Li} is a time constant for Li destruction (see Fig. 2a). These results are used to calculate the age at which 50, 95 and 99 per cent lithium depletion occurs as a function of mass. These points are marked on the individual evolutionary tracks in Fig. 1 and are plotted as a function of luminosity in Fig.2b. The time scale over which Li is burned is indicated by the gap between the curves in Fig. 2a. It is quite uniform, although becomes slightly shorter (in terms of a logarithmic age increment) at lower masses and luminosities.

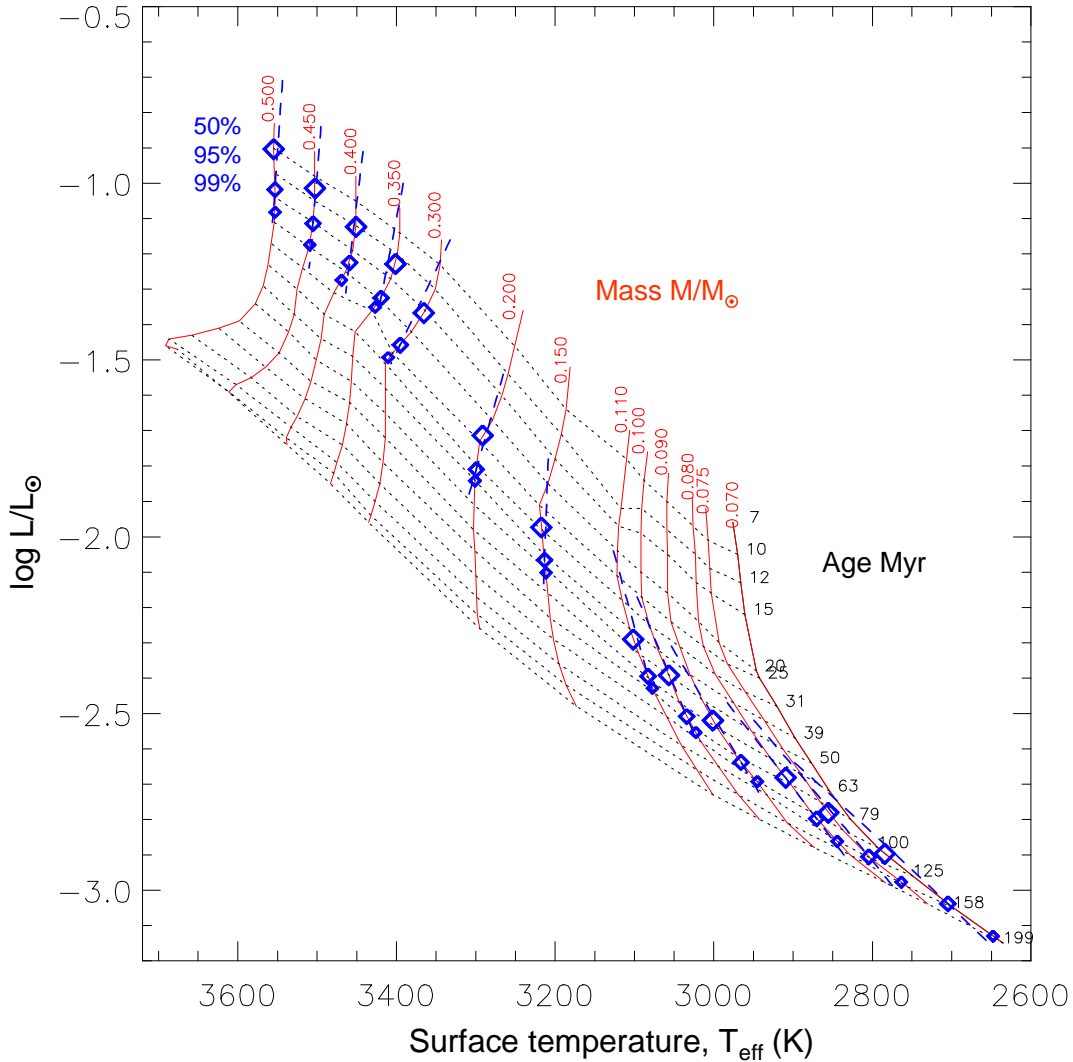Extrapolating these results to stars with varying levels of spot

Figure 1. Model isochrones and evolutionary tracks for young low mass stars from the Baraffe et al. (1998) models of stellar evolution ([M/H]=0.0, Y=0.275, $L_{\mathrm{mix}} = H_p$). Diamonds of decreasing size along each evolutionary track indicate the surface temperature and luminosity at the age of 50, 95 and 99 percent lithium depletion. Blue dashed lines show evolutionary tracks predicted by a polytropic model of a fully convective unspotted star in the Hayashi zone with parameters fitted to match the published numerical models as they approach the 95 percent Li depletion age (see section 2.2).

coverage requires an additional relation representing the dependence of the rate of lithium depletion on the temperature, T_c, and density, ρ_c, at the stellar core. The reaction rate for lithium depletion is extremely temperature sensitive ($\propto T_c^{20}$) with a much weaker dependence on density. The resultant narrow range of burning temperatures allows T_c and R at the time of Li depletion to be approximated by power laws, at least for a non-degenerate star (e.g. Bildsten et al. 1997). For fully convective, non degenerate, PMS stars the core properties scale approximately as $T_c \propto M/R$ and $\rho_c \propto M/R^3$. Hence the ratio of radius to mass for a given level of lithium depletion is expected to be nearly invariant, or at least only a slowly varying function of mass.

Fig. 3 shows the critical radius to mass ratio, $[R/M]_{\mathrm{Li}}$ at which 50, 95 and 99 per cent of Li is depleted as a function of surface temperature and mass, determined from the BCAH98 models. For $M > 0.11\,M_\odot$, $[R/M]_{\mathrm{Li}}$ only changes as $\sim M^{-0.1}$, which is consistent with the analytic prediction of Bildsten et al.

(1997) that the radius at the LDB for non-degenerate stars varies as $M^{7/8}\mu^{3/4}T_{\mathrm{eff}}^{-1/6}$, where μ is the mean atomic mass and the effective temperature of Hayashi tracks varies little with mass. For stars (and brown dwarfs) with $M \leqslant 0.11\,M_\odot$, $[R/M]_{\mathrm{Li}}$ peaks and decreases slowly towards lower masses. This is presumably due to the onset of electron degeneracy and the consequent increase in the effective atomic mass, μ_{eff}, that is defined in terms of the ratio of gas temperature to pressure (see section 2.2 and Ushomirsky et al. 1998).

The fact that $[R/M]_{\mathrm{Li}}$ does not vary strongly with M or T_{eff} over the ranges relevant to low-mass PMS stars, in both the non-degenerate and partially degenerate regimes, is what allows us to obtain analytic expressions for the effects of starspots on LDB ages.
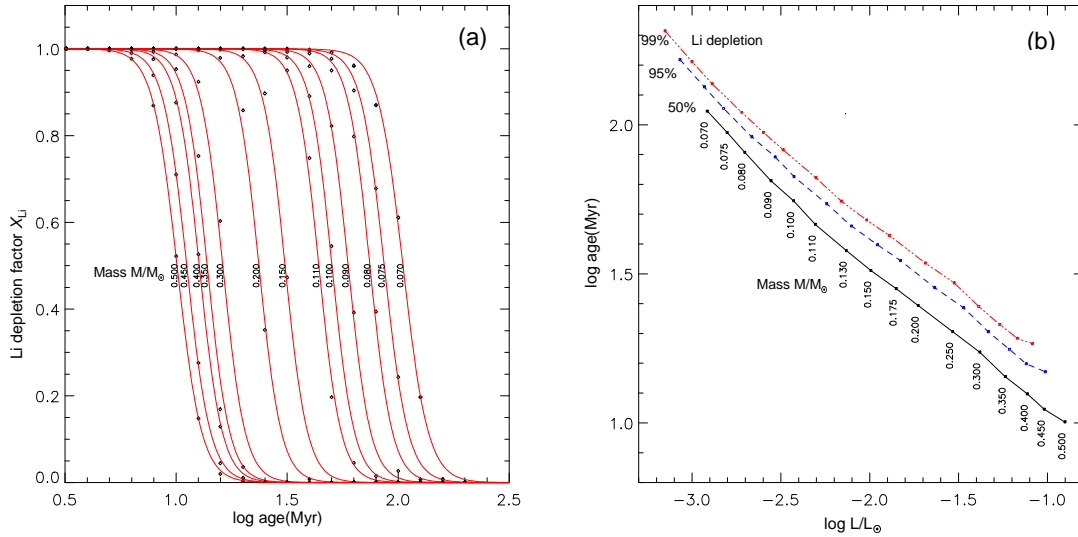
Figure 2. Variation of the lithium depletion factor, X_{Li} with age for low mass stars. Crosses in the left hand plot show tabulated values of X_{Li} for the BCAH98 model of stellar evolution ([M/H]=0.0, Y=0.275, $L_{mix} = H_p$). Curves show the function $X_{Li} = \frac{1}{2}[1 - \tanh((t - t_{50})/\tau_{Li})]$ fitted at each mass (where t_{50} is the 50 percent Li depletion and τ_{Li} the time constant for decay). The right hand plots show the 50, 95 and 99 per cent Li depletion age as a function of luminosity using these fitted parameters.

2.2 Polytropic models as function of mass

Low-mass stars over the mass range shown in Fig. 1 are predicted to remain fully convective at least until the end of any Li burning (D'Antona & Mazzitelli 1994). Fully and efficiently convective stars can be represented by a polytropic model with the equation of state of an ideal gas, $P = N_a k_b \rho T / \mu$, where N_a is Avagadro's number and k_b is Boltzmann's constant. For this case the adiabatic pressure relation is $P \propto \rho^{5/3}$ and the polytropic index $n = 3/2$. This relation is valid even when the core of the star becomes partially degenerate at the lower mass and older extents of the parameter ranges considered in this paper. In an isentropic star the degree of electron-degeneracy is constant (Stevenson 1991). Hence the effect of degeneracy is simply to increase the effective mean molecular mass, μ_{eff}, thus reducing the temperature through the star by a constant factor μ/μ_{eff}. The adiabatic exponent remains unchanged and the star is described by a $n = 3/2$ polytrope, independent of the degree of degeneracy (Hayashi & Nakano 1963; Ushomirsky et al. 1998).

The first step is to fit a polytropic model to the evolutionary tracks of the *unspotted* PMS stars in Fig.1 as they approach the epoch of Li depletion. A fully convective, contracting polytropic PMS star follows a (Hayashi) track in the HR diagram given by

$$\frac{L}{L_\odot} = C^{-A} \left(\frac{M}{M_\odot}\right)^B \left(\frac{T}{T_\odot}\right)^A, \qquad (1)$$

where T is the photospheric temperature and A, B and C are constants calculated from the polytropic index, n and the indices a and b used to describe how the Rosseland mean opacity depends on the temperature and density at the photosphere ($\kappa = \kappa_0 \rho^a T^b$, see Prialnik 2000; Jackson & Jeffries 2014). In practice κ varies in a complex manner with temperature such that it cannot be described by single values of a and b over the full range of temperatures and masses shown in Fig. 1. Instead, local values of A and B are determined as a function of mass from the evolutionary tracks. With n fixed at $3/2$, the value of A is determined from the slope of the evolutionary tracks at constant mass ($1/A = \frac{\partial \log T}{\partial \log L}$). This
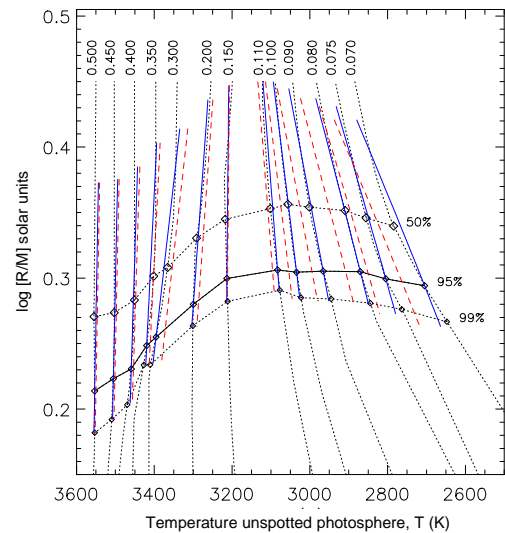

Figure 3. The critical radius/mass ratio, $[R/M]_{Li}$ at which 50, 95 and 99 per cent Li depletion occurs (marked with diamonds of decreasing size) shown on evolutionary tracks in the R/M versus temperature plane for a range of labeled stellar masses. Dotted lines show evolutionary tracks for (unspotted) BCAH98 models, where the x-axis shows the temperature of the unspotted photosphere (T_{eff} for an unspotted star). Blue lines show tracks predicted using a polytropic model of an unspotted star in the Hayashi zone with parameters fitted to match the evolutionary tracks in the HR diagram (see Fig. 1). Red dashed lines show the displacement of the tracks resulting from 30 percent spot coverage of dark starspots ($\beta = 0.3$, see section 2.4).

slope changes with time, so we restrict the fit to the 0.7 dex of time immediately preceding the epoch of 95 per cent lithium depletion, t_{95} (see Fig. 1). The rationale for this is that it will only be the presence of spots and the consequent structure of the star
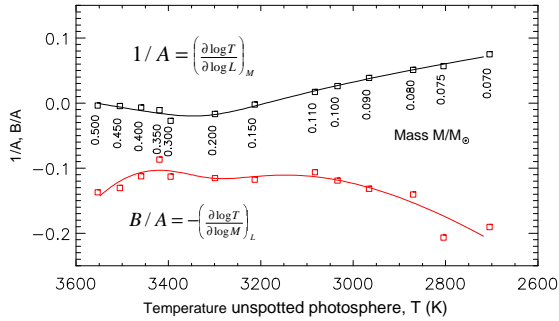
Figure 4. The variation of the exponents A and B (defined in equation 1, plotted as $1/A$ and B/A respectively) defining the polytropic model of a star in the Hayashi zone as a function of mass and photospheric temperature (at the epoch of 95 per cent Li depletion). Squares give values at individual masses. The curves are fifth order polynomial fits.
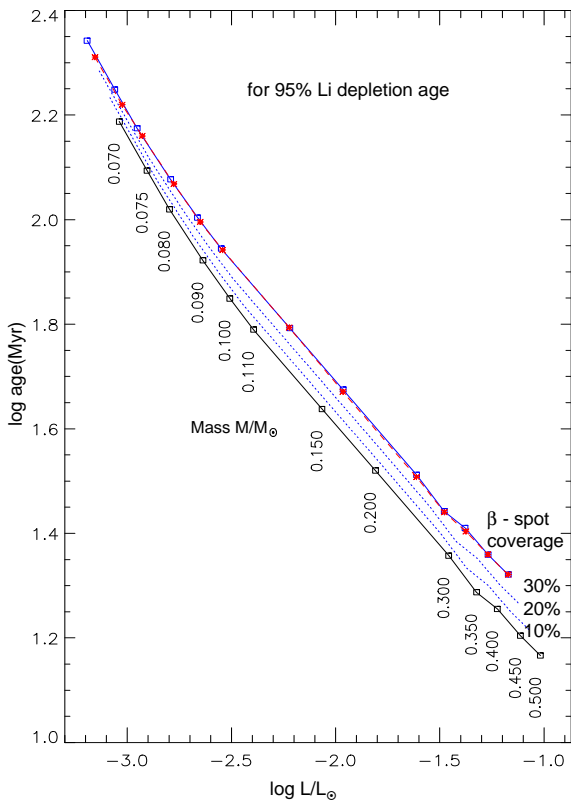

Figure 5. The effect of starspots on the Li depletion age as a function of luminosity. The lower solid line shows the 95 percent Li depletion age for an unspotted star. Blue dotted and solid lines show the effect of increasing levels of spot coverage (β=0.1, 0.2 and 0.3) where, at a given mass (marked with fiducial squares in the lower and upper curves), the Li depletion age is increased by a factor $(1-\beta)^{-1}$ and the luminosity is reduced by a factor $(1-\beta)$. The red line (and mass points marked as red crosses) show results for β=0.3 taking account of the small changes in parameters defining the polytropic model caused by starspot coverage (see section 2.4).

during this period that affects Li depletion. The value of B is determined in a similar way from the spacing between the evolutionary tracks ($B/A = -\frac{\partial \log T}{\partial \log M}$) as a function of M and T_{eff} (at t_{95}). The constant C is determined from the luminosity at t_{95}. The resulting values of $1/A$ and B/A are shown as a function of temperature and mass in Fig. 4. Tracks based on these simple polytropic models (dashed lines in Fig. 1) are able to reproduce the results of the numerical calculations for stars approaching the Li depletion epoch.

2.3 The effect of starspots on R and L

Jackson & Jeffries (2014) used a similar polytropic model to investigate the effect of dark photospheric spots on the evolution of low mass stars as they descend Hayashi tracks, where the luminosity of the star is taken to result solely from the release of gravitational energy. When spots form on the stellar surface their immediate effect is to reduce L by a factor $(1-\beta)$, with no change in radius or temperature of the unspotted photosphere, T_u. Here β is the equivalent filling factor of completely dark starspots that would produce the same reduction in L as the actual coverage of dark starspots at the actual (non zero) spot temperature. The subsequent effect of starspots is to reduce the rate of descent along the Hayashi track by a factor $(1-\beta)$. In the long-term there is an increase in R at a given age t, relative to the radius of unspotted stars of similar mass. Jackson & Jeffries showed that provided starspots were formed more than about 1 dex previously in $\log t$, then[1]

$$\frac{R}{R_\odot} = [Zt(1-\beta)]^{\frac{A-4}{4-3A}} \left(\frac{M}{M_\odot}\right)^{\frac{8-2A-4B}{4-3A}}, \qquad (2)$$

$$\frac{L}{L_\odot} = (1-\beta)C^{\frac{-4A}{4-A}}[Zt(1-\beta)]^{\frac{2A}{4-3A}} \left(\frac{M}{M_\odot}\right)^{\frac{4B-4A}{4-3A}}, \qquad (3)$$

where $Z = C^{\frac{-4A}{4-A}} \frac{(10-2n)(4-3A)}{3(4-A)} \left(\frac{L_\odot R_\odot}{GM_\odot^2}\right)$.

These expressions can be used as follows to determine the change in the Li depletion age of a spotted star, $t_{\text{Li,s}}$, relative to that of an unspotted star, $t_{\text{Li,u}}$.

(i) Consider an unspotted star of mass M requiring a critical value of $[R/M]_{\text{Li}}$ to achieve a specified level of Li depletion, which is reached at an age $t_{\text{Li,u}}$ (see Fig.3). From equation 2 the same value of $[R/M]_{\text{Li}}$ is reached for a spotted star at an age of $(1-\beta)^{-1}t_{\text{Li,u}}$. Hence *at a fixed mass*, $[t_{\text{Li,s}}/t_{\text{Li,u}}]_M = (1-\beta)^{-1}$, corresponding to a vertical shift of a mass point upwards in Fig 2b. This assumes that the photospheric temperature and density of the spotted star are not changed enough to alter the values of A and B sufficiently to affect the comparison, and that $[R/M]_{\text{Li}}$ is the same for spotted and unspotted stars.

(ii) However, the corresponding change in luminosity at fixed mass between an unspotted star, L_u at age, $t_{\text{Li,u}}$ and a spotted star, L_s at age $t_{\text{Li,u}}(1-\beta)^{-1}$ is given by equation 3 as $[L_s/L_u]_M = (1-\beta)$. This corresponds to a horizontal shift to the left in Fig. 2b, assuming again that the indices in equation 3 that depend on A and B do not change significantly for the spotted star.

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[1] Given that evidence for spots is plentiful on stars as young as ~ 1 Myr (e.g. Herbst et al. 2002), this assumption seems reasonable at ages $>$ 10 Myr.
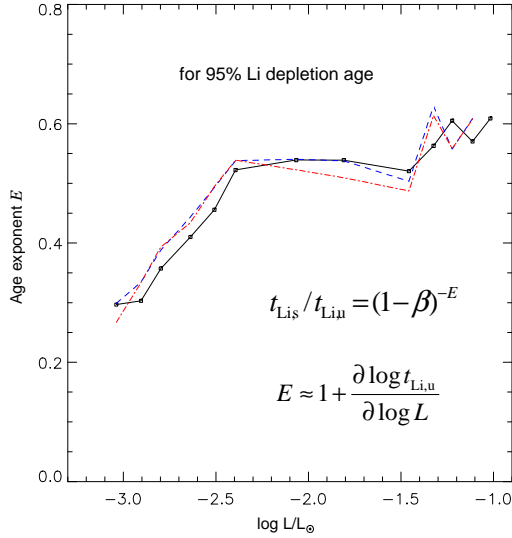
Figure 6. Variation of the exponent, E defining the increase in Li depletion age of a spotted star, $t_{\mathrm{Li,s}}$ relative to that of an unspotted star, $t_{\mathrm{Li,u}}$ of similar luminosity. The solid black line shows E calculated from the slope of the 95 percent Li depletion age curve (Fig. 2). The blue dashed line shows the exponent calculated from the curves in Fig. 5 where, at a given mass, the Li depletion age is increased by a factor $(1-\beta)^{-1}$ and the luminosity is reduced by a factor $(1-\beta)$. The red dot-dashed line shows results for β=0.3 taking account of the small changes in parameters defining the polytropic model caused by starspot coverage (see section 2.4).

Fig. 5 shows a logarithmic plot of t_{Li} versus luminosity using a 95 per cent Li depletion criterion. The lower solid line shows the relation for an unspotted star interpolated from the BCAH98 models shown in Fig. 2. The blue dashed and solid lines show the effect of 10, 20 and 30 per cent spot coverage where, for each mass point, the age has been increased by a factor $(1-\beta)^{-1}$ and the luminosity reduced by a factor $(1-\beta)$.

These two effects can be combined into one simple approximation that $t_{\mathrm{Li,s}} \simeq t_{\mathrm{Li,u}}(1-\beta)^{-E}$, where $E-1$ is the gradient of the original (unspotted) t_{Li} versus L relationship. i.e. $E \simeq 1 + \partial \log t_{\mathrm{Li,u}}/\partial \log L$. This approximation is true for small β and remains accurate for larger β provided $\log t_{\mathrm{Li,u}}$ varies linearly with $\log L$. Fig. 6 compares the value of E determined from the slope of the unspotted Li depletion age curve in Fig. 5 with the effective values of E found from the positions of the age curves for spotted stars shown in Fig. 5 that were shifted according to the two-step process described above. Results differ by only 0.03 rms over the mass range 0.07< $M/M_\odot$ <0.4. $E \simeq 0.5$ over most of this mass range, so to first order we can say that *the LDB ages inferred from models of spotted stars are older than those inferred from standard models by a factor of* $\sim (1-\beta)^{-1/2}$, but by a slightly smaller factor for older (> 80 Myr) clusters with $L_{\mathrm{LDB}} < 10^{-2.5} L_\odot$, where $E \sim 0.3$ (see Figs. 5 and 6).

A further effect of starspots is to change the inferred mass of stars at the LDB. From the mass points in Fig. 5 it can be seen that starspots both increase the Li depletion age at a given luminosity and increase the mass of the star that reaches its Li depletion age at this luminosity. From equations 2 and 3 the stellar mass at the LDB scales with spot coverage as $(M_s/M_u)_{\mathrm{LDB}} = (1-\beta)^{-(A-4)/(2A-4B)}$. As A is large compared with B (see Fig. 4), $(M_s/M_u)_{\mathrm{LDB}} \simeq (1-\beta)^{-1/2}$.

2.4   The effect of changes in polytropic constants

In calculating the change in Li depletion age with β it is implicitly assumed that the constants defining the polytropic model of a spotted star are the same as those of an unspotted star. The validity of this approximation can be tested by evaluating $t_{\mathrm{Li,s}}$ taking account of the changes in critical parameters with spot coverage. In order to do this it is convenient to define the change in Li depletion age and luminosity in terms of the temperature T (defined as the temperature of the spot-free region of the photosphere), the critical radius for Li depletion, $[R/M]_{\mathrm{Li}}$ and the exponent A. From equations 2 and 3 the change in Li depletion age at fixed mass due to starspot coverage is actually

$$\left(\frac{t_{\mathrm{Li,s}}}{t_{\mathrm{Li,u}}}\right)_M = \frac{1}{(1-\beta)} \frac{\frac{4-A_s}{4-3A_s}[R/M]_{\mathrm{Li,s}}^{-3} T_s^{-4}}{\frac{4-A_u}{4-3A_u}[R/M]_{\mathrm{Li,u}}^{-3} T_u^{-4}} , \qquad (4)$$

where suffix s denotes values of A, $[R/M]_{\mathrm{Li}}$ and T for a spotted star and suffix u denotes values for an unspotted star. Using the relation $L = (1-\beta)R^2 T^4$, the change in luminosity at the Li depletion age is

$$\left(\frac{L_{\mathrm{Li,s}}}{L_{\mathrm{Li,u}}}\right)_M = (1-\beta)\frac{[R/M]_{\mathrm{Li,s}}^2 T_s^4}{[R/M]_{\mathrm{Li,u}}^2 T_u^4} \qquad (5)$$

It was shown in Jackson & Jeffries (2014) that the effect of starspots is to offset the Hayashi track of the spotted star relative to that of an unspotted star of the same mass in the HR diagram. The offset at fixed mass and age depends on the slope of the Hayashi track, as $(T_s/T_u)_{M,t} = (1-\beta)^{-2/(3A-4)}$ (see equation 9 of Jackson and Jeffries). The change in T produces small changes in $[R/M]_{\mathrm{Li}}$ and A. Fig. 4 shows the displacement of the polytropic tracks of $[R/M]_{\mathrm{Li}}$ versus T due to spot coverage which gives an upper bound for the change in $[R/M]_{\mathrm{Li}}$ for each mass point. Similarly the change in A for a spotted star can be estimated by interpolating the polynomial approximation of $1/A$ versus $\log T$ in Fig. 4.

Fig. 5 also shows an LDB-age relation for a heavily spotted star (β=0.3) taking account of this change in T due to spot coverage and the resulting changes in $[R/M]_{\mathrm{Li}}$ and A. The curve is almost identical to the curve calculated assuming that $[R/M]_{\mathrm{Li}}$ and A are fixed, although at low masses the equivalent mass points are displaced parallel to the curve so the *mass* of a star at L_{LDB} is slightly smaller than would otherwise be estimated. In effect, the change of surface temperature at low masses reduces the age of the spotted star (at fixed mass) but increases its luminosity, such that the change of age at fixed luminosity is only slightly modified. Fig. 6 compares the age exponent E taking account of changes in T, ($[R/M]_{\mathrm{Li}}$ and A with the results assuming these parameters are fixed. For a high level of spot coverage (β=0.3) the age exponent differs by < 0.02 rms over the mass range 0.07< $M/M_\odot$ <0.4 and therefore we judge this a negligible source of uncertainty when estimating LDB ages.

3   THE EFFECTS OF STAR SPOTS ON THE BOLOMETRIC CORRECTION

As luminosities and temperatures cannot be measured directly, the observational route to inferring an LDB age involves estimating the bolometric magnitude of the LDB using its apparent magnitude, along with distance, extinction and either empirical or theoretical relationships between bolometric correction and the colour or spectral type of stars at the LDB (e.g. Jeffries & Oliveira 2005; Binks
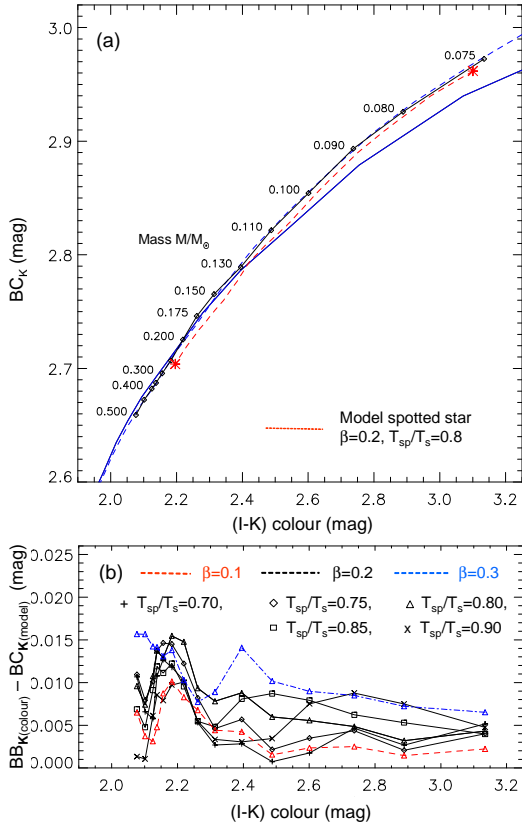
Figure 7. The effect of starspots on the BC_K bolometric correction inferred from an $I - K$ colour. The upper plot shows BC_K versus colour using BT-Settl model atmospheres (Allard et al. 2011). The blue solid and dashed lines show curves for unspotted stars at fixed ages of 1 Gyr and 150 Myr respectively. The solid black line shows BC_K values at the epoch of 95 per cent Li depletion. The red dashed line shows the predicted values of BC_K for spotted stars with β=0.2 and a spot temperature contrast of 0.8. The lower plot show the difference in the value BC_K if this evaluated from its colour (assuming there are no star spots present) compared to its value for a spotted star predicted using a two-temperature model. Results are shown for a range of spot temperature contrast ratios, and spot coverages β.

& Jeffries 2014). In this section we consider how the presence of starspots affects the relationship between bolometric correction and colour for spotted stars. We focus on K magnitudes and bolometric corrections versus $I - K$, as these are most frequently used data in the LDB literature.

3.1 Colour transformations for unspotted stars

Fig. 7a shows curves of the K-band bolometric correction, BC_K, as a function of $I - K$ colour interpolated from the "BT-Settl/AGSS2009" model which use synthetic colours and magnitudes derived from the BT-Settl model atmospheres (Allard, Homeier & Freytag 2011) and the solar abundances of Asplund el al. (2009). Consider first the case of an unspotted star; the appropriate curve of BC_K versus colour is calculated with temperatures and gravities corresponding to t_{95} for each mass track. This is shown as a solid black line in Fig. 7a.

Also shown in Fig, 7a is a BC_K versus colour curve at a fixed age of 1 Gyr which approximates to the ZAMS. As expected this shows similar values of BC_K at bluer colours and higher mass stars

but is discrepant at lower masses. The reason is presumably that t_{95} for the higher mass stars is closer to their time of arrival on the ZAMS than for lower mass stars, and hence their gravities are more similar for a given $I - K$. The maximum discrepancy though is only 0.03 mag at $0.07 M/M_\odot$ which would introduce systematic errors of only ~ 1.5 percent in the LDB age estimates for older (>100 Myr) clusters. This systematic error can be minimised by using a bolometric correction curve at an age more appropriate to the expected LDB age of the population. For example the curve for a fixed age of 150 Myr (see Fig. 7a) is virtually indistinguishable from one in which BC_K is calculated at t_{95} at each mass[2] .

3.2 The effect of starspots on colour magnitude curves

Starspots affect the magnitude and colour of stars on the Hayashi track in two ways;

• There is a small increase in the temperature *of the unspotted photosphere* of the spotted star, T_s (< 4 percent) relative to that of an unspotted star, T_u of the same age and luminosity. The magnitude of the change is found from the polytropic model as $(T_s/T_u)_{L,t} = (1 - \beta)^{-(1-B/4)/(A-B)}$ (see equation 10 of Jackson & Jeffries 2014). This makes the unspotted surface of a spotted star slightly hotter (bluer) than a totally unspotted star.

• There is also a contribution to the observed stellar flux from the spotted area of the photosphere that always makes the star appear redder. The relative magnitude of this contribution depends on the spot temperature T_{sp} and the fraction of the surface covered by starspots, $\gamma = \beta/(1 - T_{sp}^4/T_s^4)$

A simple two-temperature model is used to calculate the effect of starspots on the bolometric correction at t_{95}, where the K-band flux is the sum of the flux from the unspotted surface of area $(1 - \gamma)4\pi R^2$ and temperature T_s, and the flux from the spotted surface of area $\gamma 4\pi R^2$ and temperature T_{sp}. This gives a combined K-band bolometric correction of

$$BC_{K,\text{star}} = 2.5 \log\left(\frac{1 - \gamma}{1 - \beta} 10^{\frac{BC_{K,T_s}}{2.5}} + \frac{\gamma(\frac{T_{sp}}{T_s})^4}{1 - \beta} 10^{\frac{BC_{K,T_{sp}}}{2.5}} \right),$$
(6)

where $BC_{K,T}$ is the bolometric correction at temperature T for an unspotted star at the Li depletion age. A similar expression is used to evaluate the I-band bolometric correction in order to determine BC_K as a function of colour.

The red dashed line in Fig. 7a shows BC_K as a function of $I - K$ for a spotted star with β=0.2 and T_{sp}/T_s=0.8. At higher masses the K-band flux from the spotted area reddens the star by ~ 0.1 mag, but at the same time BC_K is increased by ~ 0.05 mag such that the spotted star still lies on the original BC_K versus $I - K$ curve for unspotted stars. At lower masses the K-band flux from the spotted areas is much smaller relative to flux from the unspotted area, such that the presence of starspots induces a slight blue shift due to the increase in temperature of the unspotted photosphere, but again a compensatory change in BC_K means the relationship between BC_K and $I - K$ is hardly changed.

Fig. 7b shows the difference between the bolometric correction estimated from stellar colour (neglecting spot coverage) and

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[2] Or, if using an empirical bolometric correction relation derived from main sequence stars, a small theoretical correction, given by the gap between the 150 Myr and 1 Gyr curves in Fig. 7a, could be applied.

the model value from equation 6 for varying levels β and T_{sp}/T_s. In general the difference in values of BC_K increases with β but peaks for values of T_{sp}/T_s between 0.8 and 0.85 when the flux from the spotted area is significantly reddened but still makes a significant contribution to the stellar flux. The results show that bolometric corrections inferred from $I-K$ colour neglecting the effects of starspots are systematically overestimated by just ~ 0.01 mag for spot coverage $\beta = 0.3$. This corresponds to a systematic error of only ~ -0.7 per cent in Li depletion age. A similar analysis of BC_K versus $V - K$ colour shows an expected systematic error of ~ -0.03 in bolometric correction which corresponds to just $\sim +1.4$ per cent error in LDB age. We regard these systematic uncertainties as negligible given that $\beta = 0.3$ will lead to changes in LDB age of $\sim +20$ per cent according to the results of section 2.3.

A more general treatment of this effect to include the many ways that bolometric corrections could be estimated using different photometric systems, or by using relationships between spectral type and bolometric correction, is beyond the scope of this paper. The details will depend exactly on which bands are used, how spectral types are determined and how the weaker flux from spotted regions contributes to the considered indicator.

4   DISCUSSION

4.1   The effects of starspots on LDB determinations and LDB ages

The main result of this paper is that ages estimated by the LDB method for spotted stars are systematically older by a factor of $(1 - \beta)^{-E}$ compared with estimates using theoretical models for unspotted stars. The size of the effect depends on spot coverage β and the slope of the relationship between τ_{LDB} and L_{LDB}, characterised by the index E and illustrated in Fig. 6. This *relative* age increase should apply whatever theoretical models of unspotted stars are used to estimate the LDB age and the values of E will be similar too, because the relationship between τ_{LDB} and L_{LDB} is almost model independent (e.g. Jeffries & Naylor 2001; Burke 2004). It should also apply for any choice of Li depletion factor to define the LDB. We have shown that the value of E is robust to the details of the polytropic models and that working in terms of observed absolute magnitudes and bolometric corrections does not change the basic result. Hence the main uncertainty in evaluating the significance of the effect is the value of β.

PMS stars at the LDB have a narrow range of spectral types from about M4 at 20 Myr (e.g. Binks & Jeffries 2014) to M7 at 130 Myr (e.g. Cargile, James & Jeffries 2010). Levels of magnetic activity in such stars appear to be governed by a rotation-activity connection, and as the spin-down timescales for stars at this mass are longer than their LDB ages, it is likely that most stars at or around the LDB will be rapidly rotating and highly magnetically active (e.g. Reiners & Basri 2008). This is confirmed from the short rotation periods inferred from light curve modulation seen in a large fraction of M-type PMS stars at ages of 10–150 Myr (e.g. Irwin et al. 2007, 2008; Messina et al. 2010) and from their "saturated" levels of chromospheric and coronal activity (Messina et al. 2003; Jackson & Jeffries 2010; Jeffries et al. 2011). The same rotational modulation provides indirect evidence for spot coverage on PMS M-stars. We are not aware of Doppler imaging results for the mid-M or cooler objects at the LDB in this age range, but results for young, early M-dwarfs also provide ample evidence for extensive spot coverage (e.g. Barnes & Collier Cameron 2001). For older

(150 Myr) clusters and the latest spectral types of stars at the LDB, it is possible that levels of magnetic activity and consequent spot coverage begin to decline (Reiners & Basri 2008, 2010), although stars at the LDB in the Pleiades (125 Myr) and Blanco 1 (132 Myr) are still chromospherically active (Stauffer et al. 1998; Cargile et al. 2010).

The appropriate value of β for active main sequence stars, let alone PMS stars, is a matter of debate. The evidence is reviewed extensively by Jackson & Jeffries (2013, 2014) and Feiden & Chaboyer (2014). Rotational modulation of light curves gives only a lower limit to β – any axisymmetric spot coverage will not contribute – but a β of at least 0.1 is probable in the most active stars based on the upper envelope of their photometric amplitudes (e.g. Messina, Rodonò & Guinan 2001; Messina et al. 2003). Doppler imaging is also likely to yield lower limits to spot coverage due to insensitivity to axial symmetry and limited angular resolution (Solanki & Unruh 2004). Spectroscopic constraints on spot coverage from modelling the TiO bandheads and spectral energy distributions of active main sequence G- and K-dwarfs suggest spot coverage in the range 20–50 per cent and spotted to unspotted photospheric temperature ratios of 0.65-0.80, leading to $0.1 < \beta < 0.4$ (O'Neal, Neff & Saar 1998; Stauffer et al. 2003; O'Neal et al. 2006). Whether such values can be extrapolated to younger and lower mass PMS stars is uncertain.

Indirect estimates of β for low-mass PMS stars can come from assuming starspots are responsible for their larger radii compared with models for inactive PMS stars of similar mass and age. This was the approach adopted by Jackson & Jeffries (2014) who showed that the radii of fully convective PMS stars are increased by a factor $(1 - \beta)^{-0.45}$ at a given luminosity. From radius estimates for PMS M-dwarfs in NGC 2516, at an age of 150 Myr, they inferred $\beta \sim 0.5$. This value should probably be regarded as an upper limit; Macdonald & Mullan (2013) have shown that the inclusion of magnetic inhibition of convection in addition to dark starspots reduces the required values of β.

In summary, the value of β is quite uncertain (with an uncertainty that grows with decreasing mass and hence older LDB ages) but probably lies in the range 0.1–0.4 for the types of star considered here. At the lower end of this range the presence of starspots represents a small, but systematic increase of ~ 5 per cent to LDB ages (slightly less for clusters older than 80 Myr, because E is smaller – see Fig.6). To put this in context, even this correction is similar in size to making factors of two adjustment to the mixing length adopted by evolutionary models or making quite drastic changes to the radiative opacities or assumed boundary conditions of the outer atmosphere (see figs. 3–5 in Burke et. al. 2004). Obviously if the spot coverage were at the upper end of the range suggested above, then LDB ages could be systematically increased by as much as 30 per cent; a far larger correction than those due to any of the factors considered by Burke et al. (2004).

Any systematic increase in LDB ages would alter the conclusions of the comparison with ages determined from fitting upper main sequence stars and the nuclear turn-off in the HR diagram. The introduction of further convective core overshooting or extra mixing due to rotation would be required in order to those ages into concordance with the revised LDB ages (see the discussions in Stauffer et al. 1998; Jeffries et al. 2013; Soderblom et al. 2013). A further effect of spots might be to explain some of the "blurring" of the LDB that has been noted in some clusters (e.g. Jeffries & Oliveira 2005; Jeffries et al. 2013). In principle, the LDB should be sharp, with little difference in luminosity between those stars exhibiting no lithium and those of slightly lower mass with their

undepleted initial abundance. In practice, there are stars with Li intermingled with Li-depleted stars along the cluster sequence in a colour-magnitude diagram. In section 3.2 we showed that spotted stars have virtually the same relationship between bolometric correction and colour as unspotted stars. Thus a coeval sequence of stars in the colour-magnitude diagram is still a sequence of stellar luminosities even if some stars are more spotted than others. However a star with larger β will reach the LDB (and hence be Li-depleted) at higher luminosity and hence brighter absolute magnitude. Conversely, we expect Li-rich spotted stars to coexist in the colour-magnitude plane with Li-poor stars with lower spot coverage that have already reached the LDB. The size of any intermingling region will be $\sim 2.5 \log(1-\Delta\beta)$ mag, where $\Delta\beta$ is the range of β values.

4.2 LDB ages and other magnetically caused structural changes

The analysis in Sections 2 and 3 considers only the effect of starspots on the LDB age. However, other authors have considered a number of ways that the presence of dynamo-generated magnetic fields might alter the structure of low-mass stars. The degree to which magnetic fields affect LDB age estimates depends primarily on how they effect the rate of heat loss from the star and hence the rate of change of radius with time. Broadly speaking there are two ways in which this can happen (e.g. Chabrier, Gallardo & Baraffe 2007). The first is a reduction in the efficiency of radiative heat transfer from the surface of the star due to the presence of starspots, which we have discussed here and which can be characterised with β, the fraction of flux blocked from leaving the star (Spruit 1982). The second is a reduction in the efficiency of convective heat transfer in the interior of the star, which increases the radius of the star at a uniform surface temperature. This may be characterised as a reduction in mixing length (Somers and Pinsonneault & 2014), or by changes to the Schwarzschild stability criterion (Mullan & Macdonald 2001, Feiden & Chaboyer 2014).

Somers & Pinsonneault (2014) claimed that a reduction in mixing length would increase LDB ages and this was numerically established by Burke et al. (2004). Here we can briefly see how this result fits into our polytropic framework and show that the increase in LDB age can be more generally expressed in terms of an increase in radius at a given age rather than in terms of β. By equating the rate of change of gravitational potential energy to its luminosity, for a polytropic star (of index n) contracting along a vertical Hayashi track, we can write down the time taken to reach a particular radius (in this case the radius at which Li is depleted)

$$t_{\text{Li}} = \frac{1}{10-2n} \frac{GM}{4\pi\sigma T_{\text{eff}}^4} \frac{1}{R_{\text{Li}}^3} \qquad (7)$$

We now say that the radius of a star at a given age is magnetically inflated by a factor $(R_i/R_u)_t$ where suffix i refers to the inflated star and suffix u the reference star. If we further assume that the surface temperature is reduced by a factor $(T_i/T_u)_t = (R_i/R_u)_t^{-1/2}$, such that M/L is unchanged, then the Li depletion age is increased by a factor $t_{i,\text{Li}}/t_{u,\text{Li}} = (R_i/R_u)_t^2$ and the luminosity decreased by a factor $L_{i,\text{Li}}/L_{u,\text{Li}} = (R_i/R_u)_t^{-2}$. These two factors are equivalent to the shifts of $(1-\beta)^{-1}$ and $(1-\beta)$ established for spotted stars in section 2.3 provided that $(R_i/R_u)_t \simeq (1-\beta)^{-0.5}$. But that is precisely what was found in Jackson & Jeffries (2014), where we showed that $(R_i/R_u)_{L,t} = (1-\beta)^{-0.5}$ if the Hayashi tracks are vertical. Hence, although this is a very simple model (the Hayashi tracks are not vertical and the polytropic index might be

changed depending on the radial profile of magnetic fields – e.g. Feiden & Chaboyer 2014), it shows that if radii are inflated by a given factor, the onset of Li-burning will be delayed and LDB ages increased by a similar amount irrespective of the exact mechanism(s) causing the radii to be larger.

Whilst there might be little difference in L_{LDB} for stars of a given age that are made larger by either starspots or a reduction in convective efficiency, there *is* a key observational distinction. Starspots change the bolometric correction and colours of stars in compensatory ways so that the colour and magnitude of a star at the LDB closely mimics those of an unspotted star of slightly different mass. On the contrary, stars that are made larger but have a uniform surface temperature will be significantly redder than unspotted stars of similar luminosity or magnitude. The reddening with respect to non-magnetic stars grows (e.g. for radii increased by 11 per cent) from 0.12 mag in $I-K$ at $M=0.5\,M_\odot$ ($\tau_{\text{LDB}} \simeq 20$ Myr) to 0.35 mag at $M=0.075\,M_\odot$ ($\tau_{\text{LDB}} \simeq 150$ Myr). Thus the *colour* of the LDB for inflated stars with a uniform photospheric temperature would be redder than if the inflation were caused by dark photospheric inhomogeneities. The difference should be quite marked at the older end of the age range considered if radii were increased to this extent.

5 SUMMARY

Polytropic models of fully convective PMS stars are used to investigate the effects of cool starspots on ages determined from the Lithium Depletion Boundary (LDB). The effect of spots is to slow the rate of descent along Hayashi tracks leading to lower core temperatures at a given age or luminosity. This delays the onset of Li destruction and means that if the luminosity at which Li is depleted can be measured – the LDB – then the age inferred from this should be increased. Our specific findings are:

• The age, τ_{LDB}, inferred from the luminosity of the LDB, L_{LDB}, is *systematically* increased by a factor $(1-\beta)^{-E}$ compared with an age estimated from unspotted evolutionary models. Here, β is the equivalent coverage of dark starspots and the exponent E is related to the rate of change of τ_{LDB} with L_{LDB}; $E \simeq 1 + d\log\tau_{\text{LDB}}/d\log L_{\text{LDB}}$. For ages < 80 Myr, $E \sim 0.5$ and decreases towards ~ 0.3 at older ages as shown in Fig. 6.

• We also find that spotted stars show virtually the same relationship between K-band bolometric correction and colour as unspotted stars, so that estimates of age based on the absolute K magnitude will be affected in exactly the same way.

• The appropriate value(s) of β to adopt are highly uncertain, but may lie in the range 0.1–0.4. Even at the lower end of this range, the systematic shift in LDB ages is comparable with the largest theoretical uncertainties previously claimed for the technique (e.g. Burke et al. 2004). For the largest values of β considered, LDB ages would be increased by 30 per cent (16 per cent at older ages), a significant change that would require re-evaluation of the physics in higher mass stellar models if ages inferred from these were to agree with LDB ages.

• We note that any magnetic effect that causes PMS stellar radii to be larger at a given age, whether that be starspots or a reduced convective efficiency, will produce similar increases in inferred LDB ages. However, increased radii with uniform photospheric temperatures due to suppressed convection in the interior will cause the LDB to be observed at significantly redder colours than for spotted stars of equivalent radii, or for non-magnetic stars.

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