Charge transport properties of bulk Ta₃N₅ from first principles

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Abstract

Tantalum nitride is considered a promising material for photoelectrochemical water splitting, however its charge transport properties remain poorly understood. We investigated polaronic and band transport in Ta₃N₅ using first-principles calculations. We first studied the formation of small polarons using density-functional theory (DFT) and DFT+U and hybrid functionals. We found that electron small polarons may occur but hole polarons are not energetically favorable. The estimated polaronic mobility for electrons is at least three orders of magnitude smaller than that measured in Ta₃N₅ films, suggesting that the main transport mechanism for both electrons and holes is band-like. Since band transport is strongly affected by the carrier effective masses, and Ta₃N₅ in known to have large electron and hole effective masses, we also investigated whether substitutional impurities or strain may help lower the effective masses. We found significant reduction in both electron and hole effective masses (up to 17% for electron and 39% for hole) under applied strain, which may lead to substantial improvement (up to 30% for electron and 15% for hole) in the carrier mobilities.

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I. INTRODUCTION

Tantalum nitride (Ta₃N₅) is considered a promising material for photoelectrochemical water splitting due to its suitable band gap (~ 2.1 eV)¹⁻³ for visible light absorption and favorable band edge alignments for both hydrogen and oxygen evolution reactions.³ However, recent experiments have reported low photocurrent and poor overall performance of Ta₃N₅ photoanodes for water oxidation,^{4,5} which is likely caused by poor charge transport properties^{6–8} as well as by rapid photodegradation of the samples in water.^{5,9}

Recently, using first-principles calculations we computed the effective masses of holes and electrons in bulk Ta_3N_5 ,⁸ which were found to be large along several directions, e.g. $2.70m_0$ for electrons and $3.56m_0$ for holes along the ΓY direction;⁸ these values are significantly larger, for example, than those computed for the hybrid organic-inorganic perovskites CH₃NH₃PbI₃, considered to be materials with high carrier mobilities^{10,11}: $m_e = 0.09-0.26m_0$ and $m_h = 0.08-0.31m_0$. Our results suggested that Ta₃N₅ may have low carrier mobilities and hence unfavourable bulk charge transport properties. We also found that the effective masses of holes are larger (up to ~ 3 times) than those of electrons in most directions, indicating that holes may have lower mobility; this is consistent with the experimental finding of Pinaud et al., 6 who reported that hole transport plays a major role in the poor photoactivity of Ta₃N₅ photoanodes. Carrier mobilities and lifetimes of electrons in Ta₃N₅ films were recently measured by Ziani et al.⁷ using ultra-fast transient absorption spectroscopy and Hall effect measurements; these authors obtained electron mobilities in the range of 1.3 to 4.4 cm²/V·s—as low as those reported for other photoanode materials, such as Fe₂O₃ $(1.53 \text{ cm}^2/\text{V}\cdot\text{s})^{12})$ and TiO₂ $(0.1-4 \text{ cm}^2/\text{V}\cdot\text{s})^{13}$ —and lifetimes of electrons in the range of 3.1 to 8.7 ps, a few orders of magnitude lower than that reported, e.g. for BiVO₄ (~ 40 ns).¹⁴

Hence, understanding the charge transport mechanisms in Ta_3N_5 is key to improve its performance for photoelectrochemical water splitting. In this work we used first-principles calculations based on density-functional theory to study the charge transport properties of bulk Ta_3N_5 . We first investigated the formation and transport of small polarons; our results indicated that electron small polarons may occur in bulk Ta_3N_5 , but hole polarons are not energetically favorable. However, the mobility estimated for electron small polaron hopping is orders of magnitude smaller than that reported experimentally, suggesting that the main charge transport mechanism for both electrons and holes is band like. Therefore we next investigated whether substitutional impurities (Nb, V, and P) or strain may lower the carrier effective masses of bulk Ta₃N₅. We found a substantial reduction in both electron and hole effective masses (up to 17% for electron and 39% for hole) under applied strain of 1–2%, which may lead, for example, to an improvement of 30% and 15% in electron and hole mobilities, respectively, for applied biaxial strain of 1.5% (estimated considering only scattering by acoustical phonons). On the other hand, the presence of substitutional V and P atoms did not improve the band transport properties of Ta₃N₅ while the incorporation of Nb at Ta sites led to a small increase (~ 6%) in the hole mobility.

The rest of the paper is organized as follows: in Section II we describe the details of our first-principles calculations and in Section III we present and discuss our results. Our conclusions are provided in Section IV.

II. COMPUTATIONAL DETAILS

We performed first-principles calculations based on density-functional theory $(DFT)^{15}$ using the generalized gradient approximation (GGA) proposed by Perdew, Burke and Ernzerhof (PBE)¹⁶ as well as $DFT+U^{17}$ and hybrid functionals. We employed the QUANTUM ESPRESSO (QE) package,¹⁸ plane-wave basis sets with energy cutoff of 160 Ry, and normconserving Hartwigsen-Goedecker-Hutter (HGH)¹⁹ pseudopotentials.

The Ta₃N₅ crystal has an orthorhombic structure (space group Cmcm) with 32 atoms in the conventional unit cell; our calculated lattice parameters⁸ obtained with the PBE functional, a = 3.87, b = 10.22, and c = 10.26 Å, are in excellent agreement with the experimental values: a = 3.89, b = 10.21, and c = 10.26 Å.²⁰ To study the effects of impurities and strain on the electronic and charge transport properties of the system (Sec. III B) we used a $1 \times 1 \times 1$ supercell with 32 atoms and a $12 \times 6 \times 6$ Monkhorst-Pack grid.²¹ The possible formation of small polarons in Ta₃N₅ (Sec. III A) was investigated using a $3 \times 1 \times 1$ supercell with 96 atoms (see Fig. 1) and a $3 \times 3 \times 3$ Monkhorst-Pack grid. A U parameter¹⁷ was applied to the tantalum 5d and nitrogen 2p orbitals and different values of U were used as discussed in Section III A. For calculations with hybrid functionals we used sc-hybrid,²² PBE0,²³ and HSE.²⁴ In this case, the k-point grid was reduced to the Γ point only and the energy cutoff was reduced to 120 Ry for computational convenience.

III. RESULTS AND DISCUSSION

A. Small polarons in Ta_3N_5

We first discuss polaronic transport. Electrons and holes moving in an ionic crystal, such as Ta_3N_5 , can be trapped by electron-phonon interaction and form polarons. When the radius of this quasi-particle is of the order of the inter-ionic distance, the trapped charge forms a so called *small polaron*,^{25,26} whose transport occurs via thermally-activated hopping. Small polarons are stable in various transition metal oxides considered potential photoanodes for water splitting, such as TiO_2 ,²⁷ $BiVO_4^{28,29}$ and Fe_2O_3 ,³⁰ and the transport properties of these materials are affected by their presence. In nitrides, hole small polarons were found to be metastable, e.g. in AlN (0.3 eV higher in energy than the nonpolaronic configuration) and energetically stable in Mg-doped AlN and GaN.³¹

As polarons originate from electron-phonon interactions, we first checked the accuracy of our calculations in reproducing the phonons of Ta_3N_5 (detailed results are presented in the Supplemental Material³²). Within the framework of density functional perturbation theory (DFPT),^{33,34} we computed the infrared and Raman frequencies of Ta_3N_5 at the Brillouin Zone center (see Table SI) using PBE, and we found excellent agreement with the theoretical and experimental data of Ref. 35. We also found that frequencies larger than 300 cm⁻¹ are dominated by N displacements. We obtained phonon dispersion curves along the ΓX , ΓY and ΓZ directions (Fig. S1) and from the slopes of linear fits to the acoustic branches near the Γ point we computed the group velocities and the elastic constants of bulk Ta_3N_5 ; our results are consistent with those obtained using the energy-strain method³⁶ (see Table SII). The good agreement between our results and previous theoretical and experimental data showed that our phonon calculations are accurate.

Electron and hole polarons were simulated in bulk Ta_3N_5 by adding or removing an electron, respectively, from a $3 \times 1 \times 1$ supercell with 96 atoms (see Fig. 1), and applying a small perturbation around a selected atom in order to break the symmetry of the crystal; in particular, we prepared the system into an initial distorted configuration by increasing the bond lengths by ~ 10% around the selected atom. We then minimized the system geometry in the presence of a uniform background insuring charge neutrality. For self-trapped electrons we considered inequivalent Ta atoms 1 and 2 and for hole small polarons we considered N

atoms 3 and 4 (see Fig. 1). Note that N3 is bonded to three Ta atoms while N4 is connected to four Ta atoms.



FIG. 1. Ball and stick representation of a $3 \times 1 \times 1$ supercell of Ta₃N₅ indicating the sites investigated for electron (Ta atoms 1 and 2) and hole (N atoms 3 and 4) localization. Ta and N atoms are shown as gray and blue spheres, respectively.

We first investigated the formation of small polarons in Ta₃N₅ as a function of the U parameter added to the Kohn-Sham Hamiltonian. Fig. 2 shows the polaron formation energy, E_p , obtained for different values of U(Ta) and U(N). $E_p = E_T^P - E_T^{NP}$ is the total-energy difference between the polaronic (E_T^P) and nonpolaronic (E_T^{NP}) configurations and negative values of E_p indicate that the polaronic configuration is energetically more stable. As expected, the larger the U, the stronger the charge localization and the larger E_p . Electron small polarons are stable for $U(\text{Ta}) \geq 3$ eV. Using U(Ta) = 7 eV—a value which reproduces the band gap obtained in our previous work⁸ using sc-hybrid functional²² and G_0W_0 (see Table SIII)—we found that the self-trapped electron at Ta1 and Ta2 are 0.74 and 0.56 eV more stable than the delocalized configuration, respectively. We note that a value of U = 7 eV is in reasonable agreement with that computed from first principles in Ref. 37 (U = 6.47 eV). On the other hand, as shown in Fig. 2(b), hole small polarons are only energetically favorable for $U(\text{N}) \geq 4$ eV, a value much bigger than the one computed in Ref. 37 (U = 0.06 eV).

These results point at stable electron polarons and unfavorable hole ones. However DFT+U is known to overestimate the charge localization in some cases^{38,39} and, in order to check the robustness of our results, we performed calculations using hybrid functionals, namely sc-hybrid,²² PBE0,²³ and HSE.²⁴ We started from the optimized geometry for



FIG. 2. Formation energy, E_p , of (a) electron and (b) hole small polarons in Ta₃N₅ as a function of the parameter U(Ta) for electron and U(N) for hole (see text).

both polaronic and nonpolaronic configurations obtained with DFT+U (U(Ta) = 7 eV and U(N) = 5 eV and we performed self-consistent calculations using hybrid functionals without geometry relaxation. For electron small polarons we found $E_p^{\rm sc} = 0.03$, $E_p^{\rm PBE0} = -0.07$, and $E_p^{\text{HSE}} = 0.11 \text{ eV}$, with sc-hybrid, PBE0 and HSE, respectively, and for hole small polarons we obtained $E_p^{\rm sc} = 0.41, E_p^{\rm PBE0} = 0.23$, and $E_p^{\rm HSE} = 0.46$ eV. Although the results obtained with hybrid functionals are only qualitative estimates, since we did not optimize the geometry at the respective levels of theory, several conclusions can be drawn. First we note that hole polarons are not expected to be stable, irrespective of the functional used. The stability of the electron polaronic configurations depends on the mixing parameter α , as observed by Kweon et al.²⁹ for electron polarons in $BiVO_4$. Since in the case of sc-hybrids, the energy of the nonpolaronic configurations was only 0.03 eV higher than the polaronic one, we investigated whether stable polarons could be found in other geometries. We interpolated the polaronic and nonpolaronic geometries obtained at the DFT+U level, and we found some electron polaronic configurations about 70 meV more stable than the nonpolaronic one. Given the accuracy of the sc-hybrid functional in reproducing the electronic structure of Ta_3N_5 ,⁸ these results suggest that electron small polarons may occur in bulk Ta_3N_5 , with low formation energy, and small polaron hopping transport may coexist with band transport for electrons. On the other hand, hole small polarons are not favorable and the transport mechanism for holes is band-like.

Within an adiabatic hopping approximation, the small polaron mobility μ_{pol} can be estimated using the Einstein relation

$$\mu_{pol} = \frac{eD}{k_B T} = \frac{eR^2 n}{k_B T} \nu \exp\left[-\frac{E_a}{k_B T}\right]$$
(1)

where D is the diffusion coefficient, k_B is the Boltzmann constant, T is the temperature, R is the distance between transfer sites, n is the number of accepting sites, ν is the longitudinal optical phonon frequency and E_a is the activation energy. Fig. 3(a) shows the barrier (or activation energy), estimated at the DFT+U level with U(Ta) = 7 eV, to move an electron small polaron between two equivalent sites. Taking the nearest equivalent atoms to the polaron site, we considered electron polaron hopping along paths A and B, as shown in Fig. 3(b), and the intermediate configurations were calculated by interpolating the atomic positions of the initial and final geometries. We found $E_a = 0.21$ eV along path A and $E_a = 0.25$ eV along path B, which are of similar magnitude to those reported, e.g. for BiVO₄ (between 0.2 and 0.5 eV, depending on the mixing parameter α in the HSE functional)²⁹ and for TiO₂ (~ 0.3 eV).²⁷ Using the largest optical phonon frequency from our phonon calculations,³² $\nu = 2.67 \times 10^{13}$ Hz, and T = 300 K we obtained $\mu_{pol} \approx 9.96 \times 10^{-4}$ cm²/V·s along path A and $\mu_{pol} \approx 8.37 \times 10^{-4} \text{ cm}^2/\text{V} \cdot \text{s}$ along path B. These mobilities are at least three orders of magnitude lower than the mobility measured in Ta_3N_5 films,⁷ namely 1.3 to 4.4 cm^2/V ·s. Hence we conclude that electron small polaron may occur in bulk Ta₃N₅, but the main transport mechanism for electrons in this material is expected to be band-like.

B. Effects of impurities and strain on the band transport mobilities of Ta_3N_5

We now turn to the discussion of band transport. The band transport mobility can be affected by several different scattering mechanisms, including acoustic phonons, optical phonons and ionized impurity scattering. The carrier effective mass (m^*) is an important factor influencing all of these mechanisms; for example, the mobility due to acoustic phonon



FIG. 3. (a) Energy variation for electron polaron hopping along paths A and B (shown in panel (b)) computed using U(Ta) = 7 eV.

scattering^{40,41} is proportional to $\frac{1}{m_I^*(m_b^*)^{3/2}}$, where $m_I^* = 3(1/m_x^* + 1/m_y^* + 1/m_z^*)^{-1}$ and $m_b^* = (m_x^* m_y^* m_z^*)^{1/3}$ are the inertial and band effective masses, respectively. Therefore, large effective masses may be responsible for low band transport mobilities. In our previous work⁸ we estimated the effective masses of electrons (m_e^*) and holes (m_h^*) from quadratic fits of the band structure and we found large effective masses along some directions, e.g. $m_e^* = 2.70m_0$ and $m_h^* = 3.56m_0$ along the Γ Y direction.

Here we investigated the effects of substitutional impurities (Nb, V, and P) and strain on the electronic structure and carrier effective masses. Before doing so, we computed the elastic constants of Ta_3N_5 using the energy-strain method;^{42,43} as can be seen in Table SII, our results are in good agreement with those calculated in Ref. 36.

We considered a $1 \times 1 \times 1$ supercell with 32 atoms. For substitutional impurities we investigated Nb and V replacing Ta atoms (Nb_{Ta} and V_{Ta}) and P replacing N atoms (P_N). We considered one and two impurities per supercell and examined different substitutional sites; the most stable configurations are shown in Fig. S2.

Table I lists the band gap and the carrier effective masses in Nb_{Ta}-Ta₃N₅, V_{Ta}-Ta₃N₅ and P_N-Ta₃N₅ systems computed at the PBE level of theory. We did not find any significant improvement in the effective masses, with respect to the pristine system, that could lead to a substantial enhancement in the carrier mobilities. For low concentration of Nb_{Ta} we found that the hole effective mass along the Γ Y direction decreased by ~ 3%, which may lead to an increase of ~ 6% in the hole mobility due to acoustic phonon scattering (here we are estimating changes in the mobility considering only changes in the effective masses).

Regarding the band gaps, we found significant reduction in the band gaps in the presence of substitutional V and P atoms. For high concentration of P_N the system becomes metallic at the PBE level of theory. For V_{Ta} we found that the band gap reduces by ~ 28% and 68% with low and high impurity concentration, respectively; this is mainly due to the presence of V 3d states (see Fig. S3), which have lower energy than Ta 5d orbitals, at the bottom of the conduction band. A reduction of 28% due to the presence of V_{Ta} may bring the optical band gap of Ta₃N₅ (which is ~ 2.1 eV⁸) close to ~ 1.5 eV, which is lower than the minimum required to split water (~ 1.9 eV is an optimal value⁴⁴ considering thermodynamic losses and reaction overpotentials). Therefore, our results suggest that the presence of V_{Ta} and P_N does not improve the band transport properties of Ta₃N₅ and V_{Ta} -Ta₃N₅ and P_N -Ta₃N₅ are

System	$E_g (eV)$	m^*/m_0 (electron/hole)		
		ΓХ	ΓҮ	ΓZ
Ta_3N_5	1.43	0.21/0.66	2.70/3.56	0.85/0.26
low impurity concentration				
Nb _{Ta} -Ta ₃ N ₅	1.32	0.24/0.66	2.70/3.46	0.92/0.26
V_{Ta} - Ta_3N_5	1.03	0.48/0.59	3.37/4.85	1.42/0.28
P_N - Ta_3N_5	0.83	0.23/0.98	3.16/4.76	0.85/0.32
high impurity concentration				
Nb_{Ta} - Ta_3N_5	1.18	0.24/0.73	2.83/3.31	0.95/0.27
V_{Ta} - Ta_3N_5	0.45	0.46/0.58	3.53/3.83	1.25/0.32
P _N -Ta ₃ N ₅	metallic	_	_	_

TABLE I. Calculated band gaps, E_g , and carrier effective masses of Nb_{Ta}-Ta₃N₅, V_{Ta}-Ta₃N₅ and P_N-Ta₃N₅ with low (1 impurity/supercell, corresponding to a concentration of ~ 8.3%) and high (2 impurities/supercell, corresponding to a concentration of ~ 16.7%) impurity concentration. The calculations reported below are at the PBE level of theory.

not suitable for photoelectrochemical water splitting. On the other hand, the presence of substitutional Nb atoms in Ta_3N_5 may potentially enhance the performance of this material since it leads to a small increase in the hole mobility, keeping the band gap in the appropriate range for visible light absorption.

We next investigated the effects of strain on the band structure and carriers effective masses of Ta₃N₅ bulk. We considered (i) uniaxial strain along *a*, *b* and *c* directions (see Fig. S4(d)): the cell was strained in one direction and allowed to fully relax along the perpendicular directions; (ii) biaxial strain in the *ab*, *ac* and *bc* planes: the in-plane lattice parameters were equally strained and lattice relaxation was allowed in the direction perpendicular to the plane; and (iii) hydrostatic strain: the cell was equally strained in the *a*, *b*, and *c* directions. We considered strains in the range of -1.5% to 1.5%, likely to be realized experimentally since similar strains have been reported, e.g. in TiO₂ (tensile strain up to 4% on rutile TiO₂⁴⁵ and compressive strain about 1.83% in anatase TiO₂ nanostructures⁴⁶) a material with elastic constants and bulk modulus comparable to those of Ta₃N₅.^{47,48} A compressive strain of 2.7% was also achieved in In_{0.12}Al_{0.88}N grown on a GaN buffer;⁴⁹ firstprinciples calculations at the LDA level of theory predicted the bulk modulus of In_{0.12}Al_{0.88}N to be between 196 and 214 GPa,⁵⁰ similar to that computed for Ta₃N₅ (244 GPa).³⁶

At the DFT-PBE level of theory, strain-free Ta_3N_5 bulk has an indirect band gap of 1.43 eV, with the valence band maximum (VBM) located at the Γ point and conduction band minimum (CBM) located at Y point.⁸ We found that hydrostatic strains between

-1.5% and 1.5% induce small changes in the band gap, less than 1.5% (see Fig. S5 (c)), while uniaxial strain along the *b* direction induces changes up 5.6\% (Fig. S5 (a)). For biaxial strain along *a* and *c* directions (Fig. S5 (b)), we found that the band gap decreases up to 6.5\% under tensile strain and increases up to 6.1% with compression. These small changes in the band gap do not compromise the appropriateness of this material for water splitting, since we expect the optical band gap of strained Ta₃N₅ to be between 1.96 and 2.23 eV.

Finally, we estimated the carrier effective masses in the strained systems. We found (see Figs. S6, S7, and S8) that the effective masses of both electrons and holes significantly change. Under compressive uniaxial strain of 1.5% along the *b* direction, for example, we found a reduction up to ~ 26% in the hole effective mass and up to ~ 13% in the electron effective mass. We estimate that the reduction in the effective masses may lead to an increase of ~ 33% and ~ 8% in the electron and hole mobility, respectively, due to acoustic phonon scattering. In the case of biaxial strain, we estimated that the reduction of ~ 16% in the electron effective mass under 1.5% tensile strain along *a* and *c* directions may result in an improvement of 31% in the electron mobility, while an increase up to 15% in the hole mobility can be achieved under compressive strain along *b* and *c* directions. Our results therefore suggest that the band transport properties of Ta₃N₅ can be improved by straining the material.

IV. CONCLUSIONS

In summary, we performed first-principles calculations to investigate the charge transport properties of bulk Ta₃N₅. We investigated the formation of small polarons using both DFT+U and hybrid functionals and we found that electron small polaron may occur, while hole small polarons are not stable. The computed electronic small polaron mobilities are at least three orders of magnitude smaller than measured mobilities,⁷ suggesting that polaron transport may coexist with band transport but it is not the main transport mechanism for electrons. Hence, our results indicate that band transport is the main transport mechanism for both electrons and holes in Ta₃N₅. We also showed that the overall large effective masses of electrons and holes may be reduced with applied strain. In particular, electron mobility may be increased by ~ 30% and hole mobility by 15% under moderate biaxial strains in the range of -1.5 to 1.5%.

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