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Decay Conditions for Antiplane Shear of a High-Contrast Multi-Layered Semi-Infinite Elastic Strip

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Abstract: The antiplane shear of a semi-infinite multi-layered elastic strip with traction free faces and edges subject to prescribed stress is studied. A high contrast is assumed in the stiffnesses of two types of homogeneous isotropic layers. Explicit conditions on the edge load are derived, ensuring the decay of stress components at the distance of order strip thickness. One of these conditions corresponds to the canonical Saint-Venant's principle, manifesting the self-equilibrium of the load. The rest of the decay conditions consider the presence of high contrast and are of an asymptotic nature, in contrast to the exact former condition. The number of asymptotic conditions is the same as that of soft layers. An example of the implementation of the proposed decay conditions for calculating the solution for the interior (outside of a boundary layer zone) domain of a three-layered semi-strip, considering geometric asymmetry, is presented.

Keywords: Saint-Venant's principle; decay conditions; multi-layered structures; high contrast; asymptotic

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Citation: Prikazchikova, L. Decay Conditions for Antiplane Shear of a High-Contrast Multi-Layered Semi-Infinite Elastic Strip. *Symmetry* **2022**, *14*, 1697. https://doi.org/10.3390/sym14081697

Academic Editors: Raffaele Barretta and Jan Awrejcewicz

Received: 30 June 2022
Accepted: 10 August 2022
Published: 16 August 2022

1. Introduction

The classical Saint-Venant's principle, as stated by Love [1] is formulated as follows: "According to this principle the strains that are produced in a body by the application, to a small part of its surface, of a system of forces statically equivalent to zero force and zero couple, are of negligible magnitude at distances which are large compared with the linear dimensions of the part". For our current purposes, we employ an equivalent formulation, namely that the stresses produced by the self-equilibrated load applied along the edge become negligibly small at distances larger than the edge size. The static equilibrium of an isotropic elastic semi-strip under the action of a self-equilibrated load along the edge, see [2], is of the utmost importance among other examples, demonstrating this principle. The particular value of this approach is related to the fact that it allows for important generalizations using perturbation along a small parameter. For example, plane and anti-plane boundary layers are constructed on the basis of this method, resulting in the derivation of asymptotic boundary conditions in elastic plates and shells, e.g., see [3–5] and also [6,7] and references therein. Moreover, it is possible to derive low-frequency corrections for static decay conditions, corresponding to dynamic generalizations of the Saint-Venant's principle for a semi-infinite strip; see [8,9].

Another important asymptotic generalization of the aforementioned problem for an isotropic semi-strip is the problem of layered composites with high-contrast in the stiffness of the layers. In this case, boundary layers corresponding to self-equilibrated static loads might decay quite slowly and may propagate along the whole strip; see [10]. In dynamics, additional low-frequency modes may appear as a result of high contrast in material or the geometrical properties of the layers; see [11–14].

In the paper [14] the canonical problem for the antiplane shear of a three-layered semi-infinite strip is considered, resulting in the derivation of two decay conditions. The first corresponds to the conventional Saint-Venant's principle, assuming self-equilibrium of a shear edge load. The additional condition is of an asymptotic nature and precludes shear deformation over a soft inner layer.

In this paper, the decay conditions obtained in [14], are extended to a multi-layered, high-contrast laminate with alternating soft and stiff layers, with the faces of the outer stiff layers subject to traction free boundary conditions. As might be expected, one of the resulting decay conditions is associated with the Saint-Venant's principle, whereas the number of additional decay conditions is equal to the number of soft layers. The latter conditions generalize the results in [14].

The paper is organized as follows. The problem is formulated in Section 2. The decay condition corresponding to the Saint-Venant's principle is derived in Section 3. The additional asymptotic decay conditions are obtained in Section 4. Finally, an illustrative example for a three-layered asymmetric laminate is presented in Section 5.

2. Statement of the Problem

Consider a laminate composed of N isotropic layers of thickness $2h_q$, $q = 1, 2, \ldots, N$. The laminate is constructed as a series of alternating soft and stiff layers, with the outer layers being stiff; see Figure 1. In the case of antiplane shear, equations of equilibrium for each layer in the domain $0 \leqslant x_1 < \infty$, $-h_q < x_{2q} < h_q$ can be written as follows:

$$\frac{\partial \sigma_{13}^q}{\partial x_1} + \frac{\partial \sigma_{23}^q}{\partial x_{2q}} = 0, \qquad q = 1, 2, \ldots, N, \tag{1}$$

where σ_{j3}^q, $j = 1, 2$ are stresses in layer q, defined as:

$$\sigma_{13}^q = \mu_q \frac{\partial u^q}{\partial x_1} \quad \text{and} \quad \sigma_{23}^q = \mu_q \frac{\partial u^q}{\partial x_{2q}}. \tag{2}$$

In the above, $u^q = u^q(x_1, x_{2q})$ are displacements orthogonal to $x_1 x_2$ plane; μ_q are Lamé parameters, such that $\mu_q = \mu_1$ for odd values of q and $\mu_q = \mu_2$ for even ones, corresponding to stiff and soft layers, respectively. A small parameter, arising from the high contrast of the layers' stiffnesses, is introduced as

$$\mu = \frac{\mu_2}{\mu_1} \ll 1. \tag{3}$$

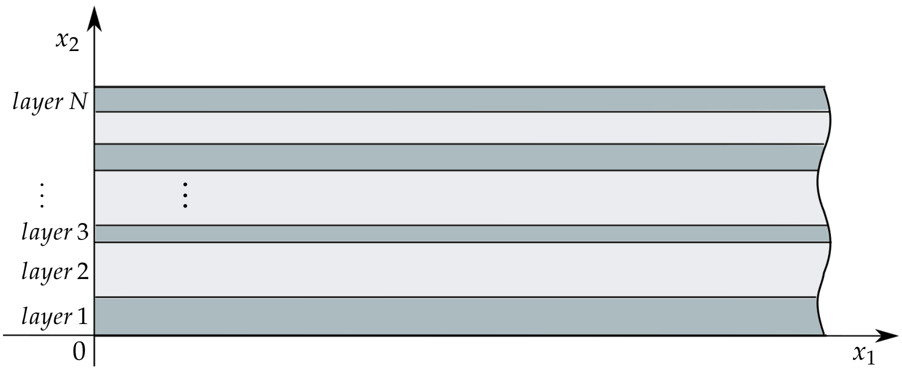

Figure 1. N-layered semi-strip.

In what follows, we consider a laminate with traction-free faces

$$\sigma_{23}^1\big|_{x_{21}=-h_1} = 0, \qquad \sigma_{23}^N\big|_{x_{2N}=h_N} = 0, \tag{4}$$

and prescribed shear load $p(x_2)$ along the edge $x_1 = 0$

$$\sigma_{13}^q\big|_{x_1=0} = p(x_{2q} + H_q), \quad q = 1, 2, \ldots, N, \tag{5}$$

where the local variables x_{2q}, $-h_q \leqslant x_{2q} \leqslant h_q$ and the global one x_2 are related through

$$x_{2q} = x_2 - H_q, \tag{6}$$

with

$$H_1 = h_1, \quad H_q = h_q + 2\sum_{n=1}^{q-1} h_n, \quad q = 2, 3, \ldots, N. \tag{7}$$

The continuity conditions of the interfaces are given by

$$u_q\big|_{x_{2q}=h_q} = u_{q+1}\big|_{x_{2q+1}=-h_{q+1}} \quad \text{and} \quad \sigma_{23}^q\big|_{x_{2q}=h_q} = \sigma_{23}^{q+1}\big|_{x_{2q+1}=-h_{q+1}}, \tag{8}$$

for $q = 1, 2, \ldots, N-1$. The aim of the paper is to formulate the so-called decay conditions on the load $p(x_2)$, such that

$$\sigma_{13}^q\big|_{x_1=\infty} = 0. \tag{9}$$

Moreover, we derive conditions for a 'strong' decay of the boundary layer with the decay region, localized over $O(h)$ edge vicinity, where $h \sim h_1 \sim h_2 \sim \ldots \sim h_N$, with the sign $\sim$ staying for asymptotic equivalence. Thus, in what follows, we assume

$$\frac{\partial}{\partial x_1} \sim \frac{\partial}{\partial x_{2q}} \sim \frac{1}{h}. \tag{10}$$

3. Classical Decay Condition

We are going to integrate an equation of motion for each layer q over its domain $0 \leqslant x_1 < \infty$, $-h_q < x_{2q} < h_q$ using the continuity conditions (8), together with the boundary conditions (4), for outer layers. Starting with the first layer $q = 1$ we obtain

$$\begin{aligned}&\int_0^\infty \int_{-h_1}^{h_1} \left(\frac{\partial \sigma_{13}^1}{\partial x_1} + \frac{\partial \sigma_{23}^1}{\partial x_{21}} \right) dx_1 dx_{21} = \int_{-h_1}^{h_1} \sigma_{13}^1\big|_{x_1=0}^{\infty} dx_{21} + \int_0^\infty \sigma_{23}^1\big|_{x_{21}=-h_1}^{h_1} dx_1 = \\ &- \int_{-h_1}^{h_1} p(x_{21} + H_1) dx_{21} + \int_0^\infty \sigma_{23}^1\big|_{x_{21}=h_1} dx_1 = 0,\end{aligned} \tag{11}$$

resulting in a relation

$$\int_0^\infty \sigma_{23}^1\big|_{x_{21}=h_1} dx_1 = \int_{-h_1}^{h_1} p(x_{21} + H_1) dx_{21}. \tag{12}$$

Similarly, for the upper layer $q = N$, we have

$$\int_0^\infty \sigma_{23}^{N-1}\big|_{x_{2N-1}=-h_{N-1}} dx_1 = -\int_{-h_N}^{h_N} p(x_{2N} + H_N) dx_{2N}. \tag{13}$$

Now, integrating for the interior layers $q = 2, 3, \ldots, N-1$, we arrive at

$$\int_0^\infty \sigma_{23}^q\big|_{x_{2q}=h_q} dx_1 = \int_0^\infty \sigma_{23}^{q-1}\big|_{x_{2q-1}=h_{q-1}} dx_1 + \int_{-h_q}^{h_q} p(x_{2q} + H_q) dx_{2q}. \tag{14}$$

It is possible to re-arrange the latter, expressing integrals for stresses in layers $q-1$ through the integrals for edge loading, starting from (12). This results in the following expression

$$\int_0^\infty \sigma_{23}^q\big|_{x_{2q}=h_q} dx_1 = \int_0^{H_q+h_q} p(x_2) dx_2, \quad q = 2, 3, \ldots, N-1. \tag{15}$$

Next, using this relation for $q = N - 1$ together with (13), the classical Saint-Venant's condition for the decay of stresses away from the loaded edge can be derived in the form

$$\int_0^H p(x_2)dx_2 = 0, \tag{16}$$

where H is a thickness of a strip given by

$$H = 2\sum_{n=1}^{N} h_n. \tag{17}$$

The essence of the problem considered is that similar to the treatment in [14], we may expect more decay conditions that are specific to the considered high-contrast setup.

4. Additional Decay Conditions

To derive additional conditions, let us integrate products of the equation of motion and the vertical coordinate for each soft layer, namely, for $q = 2, 4, \ldots, N-1$. Hence, we have

$$\begin{aligned}
&\int_0^\infty \int_{-h_q}^{h_q} x_{2q}\left(\frac{\partial \sigma_{13}^q}{\partial x_1} + \frac{\partial \sigma_{23}^q}{\partial x_{2q}}\right)dx_1 dx_{2q} = \\
&\int_{-h_q}^{h_q} x_{2q}\sigma_{13}^q\big|_{x_1=0}^{\infty}\, dx_{2q} + \int_0^\infty \int_{-h_q}^{h_q} x_{2q}\frac{\partial \sigma_{23}^q}{\partial x_{2q}} dx_1 dx_{2q} = \\
&-\int_{-h_q}^{h_q} x_{2q} p(x_{2q} + H_q)dx_{2q} + h_q \int_0^\infty \left(\sigma_{23}^q\big|_{x_{2q}=h_q} + \sigma_{23}^q\big|_{x_{2q}=-h_q}\right)dx_1 \\
&\qquad - \int_0^\infty \int_{-h_q}^{h_q} \sigma_{23}^q dx_1 dx_{2q} = 0.
\end{aligned} \tag{18}$$

In the above, the integral

$$I = \int_0^\infty \int_{-h_q}^{h_q} \sigma_{23}^q dx_1 dx_{2q} \tag{19}$$

is asymptotically negligible in comparison to the integral

$$J = \int_{-h_q}^{h_q} x_{2q} p(x_{2q} + H_q)dx_{2q}. \tag{20}$$

Indeed, expressing the stress through displacement in (19) and evaluating the inner integral, we obtain

$$I = \mu_2 \int_0^\infty u_q\big|_{x_{2q}=-h_q}^{h_q} dx_1. \tag{21}$$

Next, from the continuity condition, we can see that $u_q\big|_{x_{2q}=h_q} = u_{q+1}\big|_{x_{2q+1}=-h_{q+1}}$, also $u_{q+1}\big|_{x_{2q+1}=-h_{q+1}} \sim \frac{h\sigma_{23}^{q+1}}{\mu_1}$, since $\sigma_{23}^{q+1} = \mu_1 \frac{\partial u_{q+1}}{\partial x_{2q}}$, where $\dfrac{\partial u_{q+1}}{\partial x_{2q+1}} \sim \frac{u_{q+1}}{h}$ due to the original assumption (10) on the localization of the boundary layer over $O(h)$ edge vicinity. In addition, it is obvious that $\sigma_{23}^{q+1} \sim p(x_{2q+1})$, thus $u_q\big|_{x_{2q}=h_q} \sim \frac{hp}{\mu_1}$ and similarly $u_q\big|_{x_{2q}=-h_q} \sim \frac{hp}{\mu_1}$. Again, considering the aforementioned remark of the localization area, we can finally obtain that $I \sim h^2 p\mu$.

At the same time, it is obvious that $J \sim h^2 p$, resulting in the sought-for estimation $I \sim \mu J$. Therefore, neglecting the last term in (18), and using continuity conditions (8) for stresses, we arrive at the following conditions for $q = 2, 4, \ldots, N-1$

$$\int_{-h_q}^{h_q} x_{2q} p(x_{2q} + H_q)dx_{2q} - h_q \int_0^\infty \left(\sigma_{23}^q\big|_{x_{2q}=h_q} + \sigma_{23}^{q-1}\big|_{x_{2q-1}=h_{q-1}}\right)dx_1 = 0. \tag{22}$$

Now, using previously derived relations (15), together with the classical decay condition (16) and relations (6), the latter may be re-written in the form

$$\int_{H_q-h_q}^{H_q+h_q}(x_2-H_q)p(x_2)dx_2 - h_q\int_0^{H_q-h_q}p(x_2)dx_2 + h_q\int_{H_{q+1}-h_{q+1}}^{H}p(x_2)dx_2 = 0, \tag{23}$$

where $q = 2,4,\ldots,N-1$.

5. Example

Consider a three-layered asymmetric semi-infinite strip. In this case, decayy conditions (16) and (23) reduce, respectively, to

$$\int_0^H p(x_2)dx_2 = 0, \qquad H = 2\sum_{n=1}^{3}h_n, \tag{24}$$

and

$$\int_{H_2-h_2}^{H_2+h_2}(x_2-H_2)p(x_2)dx_2 - h_2\int_0^{H_2-h_2}p(x_2)dx_2 + h_2\int_{H_3-h_3}^{H}p(x_2)dx_2 = 0. \tag{25}$$

Let us now consider the problem (1), (4), (5) and (8) with $N = 3$. First, by taking displacements as $u_q = U_q(x_{2q})e^{-kx_1}$, $q = 1,2,3$, where $k > 0$, we can obtain a set of linear equations with the coefficients, given by the matrix

$$M = \begin{pmatrix} C_1 & 0 & 0 & S_1 & 0 & 0 \\ 0 & 0 & C_3 & 0 & 0 & -S_3 \\ S_1 & S_2 & 0 & C_1 & -C_2 & 0 \\ 0 & S_2 & S_3 & 0 & C_2 & -C_3 \\ C_1 & -\mu C_2 & 0 & -S_1 & -\mu S_2 & 0 \\ 0 & \mu C_2 & -C_3 & 0 & -\mu S_2 & -S_3 \end{pmatrix}, \tag{26}$$

where

$$C_q = \cos(kh_q), \quad S_q = \sin(kh_q), \quad q = 1,2,3, \tag{27}$$

with $\mu = \mu_2/\mu_1 \ll 1$ as above. The characteristic equation follows from the condition $\operatorname{Det} M = 0$ and finally takes the form

$$\mu\{\tan(2h_1k) + \tan(2h_3k)\} + \mu^2\tan(2h_2k) - \tan(2h_1k)\tan(2h_2k)\tan(2h_3k) = 0, \tag{28}$$

where $\mu = \mu_2/\mu_1$. There is a small root given at the leading order by

$$k \approx \frac{1}{2}\sqrt{\frac{(h_1+h_3)\mu}{h_1h_2h_3}}, \tag{29}$$

corresponding to a slowly decaying solution

$$u_A = A\,e^{-kx_1}\begin{cases} -\dfrac{h_3}{h_1}, & 0 \leqslant x_2 \leqslant 2h_1, \\ \dfrac{x_2(h_1+h_3) - 2h_1^2 - 2h_3(h_1+h_2)}{2h_1h_2}, & 2h_1 \leqslant x_2 \leqslant 2h_1+2h_2, \\ 1, & 2h_1+2h_2 \leqslant x_2 \leqslant H, \end{cases} \tag{30}$$

due to the effect of high contrast in stiffnesses of the layers, see also the discussion in [10]. It is well-known that such small roots are not a feature of non-contrast laminates. The associated stress state corresponds to the interior solution but not to a boundary layer localized in the narrow vicinity (of order of the laminate's thickness) at the edge of a semi-strip.

Another solution to the original problem, which is not localized near the narrow vicinity of the edge $x_1 = 0$, takes the traditional polynomial form

$$u_B = Bx_1 + C. \tag{31}$$

Figures 2 and 3 demonstrate dimensionless displacements u_A/A and u_B/C at $x_1 = 0$ for $h_1 = 0.03$ m, $h_2 = 0.02$ m, $h_3 = 0.01$ m and $\mu = 0.1$. The first one shows a homogeneous transverse shear deformation of the soft layer and rigid-body motions of the outer stiff layers, while the second one corresponds to the rigid-body motion of a semi-strip. For the former, the slope of the straight line in the Figure 2 is related to the value of the stiffness contrast; see also [14] for further details.

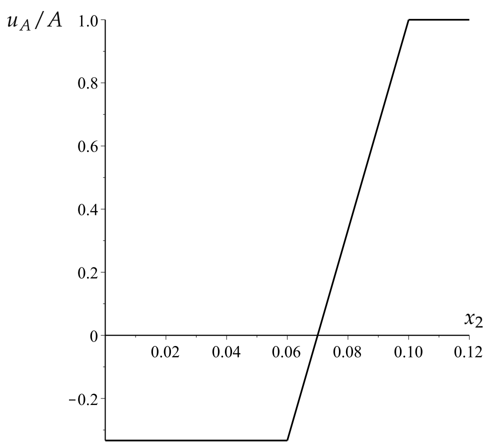

Figure 2. Dimensionless displacement variation (30) across the width of a semi-strip.

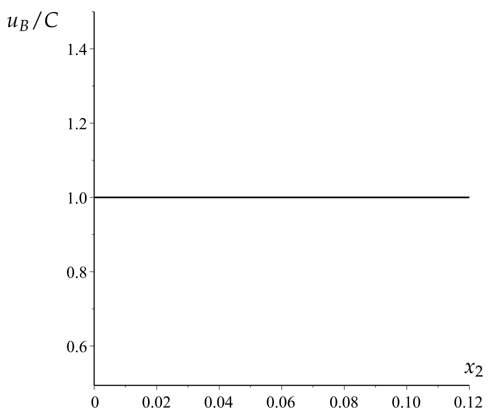

Figure 3. Dimensionless displacement variation (31) across the width of a semi-strip.

The stress components at the edge $x_1 = 0$ corresponding to the displacements (30) and (31) are given, respectively, by

$$\sigma_{13}^A = A\frac{\mu_1}{2}\sqrt{\frac{(h_1+h_3)\mu}{h_1h_2h_3}} \times \begin{cases} \dfrac{h_3}{h_1}, & 0 \leqslant x_2 \leqslant 2h_1, \\ -\mu\dfrac{(x_2(h_1+h_3) - 2h_1^2 - 2h_3(h_1+h_2))}{2h_1h_2}, & 2h_1 \leqslant x_2 \leqslant 2h_1+2h_2, \\ -1, & 2h_1+2h_2 \leqslant x_2 \leqslant H, \end{cases} \tag{32}$$

and

$$\sigma_{13}^{B} = B\mu_1 \begin{cases} 1, & 0 \leqslant x_2 \leqslant 2h_1, \\ \mu, & 2h_1 \leqslant x_2 \leqslant 2h_1 + 2h_2, \\ 1, & 2h_1 + 2h_2 \leqslant x_2 \leqslant H, \end{cases} \tag{33}$$

see Figures 4 and 5 for the scaled stresses $\sigma_{13}^{A}/A\mu_1$ and $\sigma_{13}^{B}/B\mu_1$.

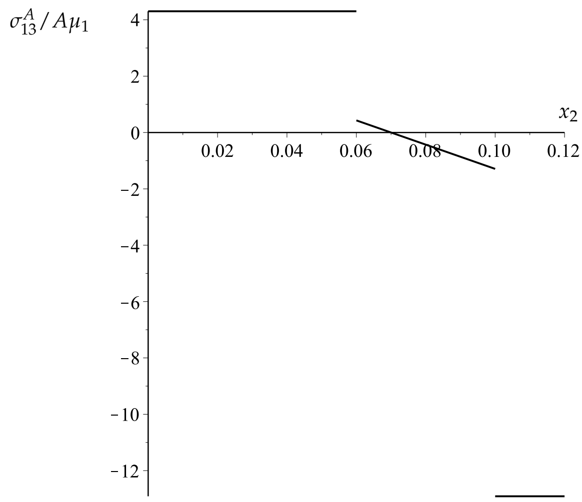

Figure 4. Scaled stress variation (32) across the width of a semi-strip.

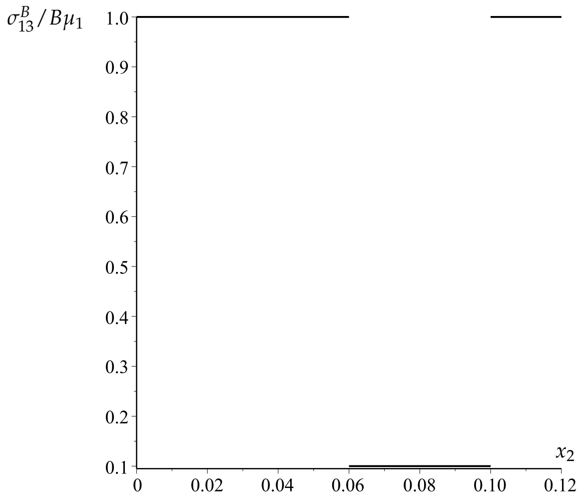

Figure 5. Scaled stress variation (33) across the width of a semi-strip.

Next, assume that the stress $\sigma_{13} = Q(x_2)$ is prescribed along the edge. To determine the constants A_0 and B_0 in the formulae (30)–(31), characterising the sought-for interior stress field, we substitute the deviation $p = Q - Q_{interior}$, where $Q_{interior} = \sigma_{13}^{A} + \sigma_{13}^{B}$ into the decay conditions (24)–(25). As a result, we have, at leading order,

$$\begin{aligned} A = & \frac{1}{\mu_1\sqrt{\mu}}\sqrt{\frac{h_1}{h_2 h_3 (h_1+h_3)^3}}\bigg(2h_2h_3\int_0^{H_2-h_2} Q dx_2 - 2h_1h_2\int_{H_3-h_3}^{H} Q dx_2 \\ & - \int_{H_2-h_2}^{H_2+h_2}\Big(x_2(h_1+h_3) - 2h_1^2 - 2h_3(h_1+h_2)\Big) Q dx_2\bigg) \end{aligned} \tag{34}$$

and

$$B = \frac{1}{2(h_1+h_3)\mu_1}\int_0^H Q dx_2. \tag{35}$$

At the same time, a constant C, corresponding to rigid body motion will remain undefined.

6. Conclusions

The sought-for $\frac{N+1}{2}$ end decay conditions for an N-layered, high-contrast, semi-infinite strip are found in a simple closed form. The decay condition (16) corresponds to the conventional Saint-Venant's principle, stating the overall self-equilibrium of the prescribed end load. The remaining decay conditions, given by (23), in contrast to the aforementioned condition (16), are not exact. Each of them has an asymptotic error $O(\mu)$. They aim to preclude slowly decaying solutions due to specific behaviours within each of the $\frac{N-1}{2}$ soft layers. The central point in the asymptotic derivation consists of the evaluation of the integral (19), which was proved to be negligible at leading order.

The derived decay conditions were developed to formulate the boundary conditions for the interior solution, starting from the Saint-Venant's principle. The example presented in the paper briefly illustrates this procedure; for more details, see [4,6,15]. It is worth noting that such boundary conditions are also valid at leading order for dynamics problems, since the low-frequency corrector to decay conditions only comes at a higher order, e.g., [9].

The proposed approach allows for various useful generalizations. It may be extended to a similar plane-strain problem. A multi-parametric analysis inspired by the high contrast in the layers thickness is also of obvious interest. Finally, the laminates of different layouts, e.g., with soft outer layers, might be also studied.

Funding: This research received no external funding.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: Not applicable.

Acknowledgments: Fruitful discussion with D. Prikazchikov is greatly acknowledged.

Conflicts of Interest: The author declares no conflict of interest.

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