Social Capital and the Status Externality[*]

Jun-ichi Itaya[†]   Chris Tsoukis[‡]

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Abstract

This paper investigates how the presence of social capital affects the externality arising from status-seeking preference as a parable for inefficient antagonistic behavior. It is assumed that the stock of social capital is accumulating through joint social interaction between rational individuals who are forward looking. Using a differential game, we show that although the presence of social capital mitigates the tendency of overconsumption over time, social capital ends up declining to zero. It is also shown that the benefits from social capital enhance the motivation of individuals to accumulate social capital thereby leading to deter overaccumulation and thus possibly improving social welfare.

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[†]Corresponding author. Graduate School of Economics and Business Administration, Hokkaido University, Sapporo 060-0809, Japan. Tel: +81-11-706-2858; Fax: +81-11-706-4947; E-mail: itaya@econ.hokudai.ac.jp

[‡]Economics and Finance, Keele Management School, Darwin Building, Keele University, Staffordshire, ST5 5BG, UK; Tel: +44 (0)1782 733101; E-mail: c.tsoukis@keele.ac.uk

1 Introduction

1.1 Motivation

Social capital is a concept with a long history in the social sciences, and the subject of a rapidly expanding literature in economics. In varying contexts, it has been shown to affect positively growth and development (Knack and Keefer 1997; Knack and Zak, 1999; Dasgupta and Serageldin, 2000; Grootaert and van Bastelear, 2002; Putnam, Leonardi and Nanetti, 1993), institutional design and performance (Djankov, Glaeser, La Porta, Lopez-de-Silanes, and Shleifer, 2003) and financial development (Guiso, Sapienza, and Zingales, 2004). In his seminal contribution, Putnam (1993) found a correlation between measures of civic engagement and governmental quality across Italian regions. Similarly, Guiso, Sapienza and Zingales (2004) find a correlation between civic engagement and financial development, again for Italy. LaPorta, Lopez-de-Silanes, Shleifer and Vishny (1999) find cross-country correlation between a measure of trust and judicial efficiency (positive) and government corruption (negative). But, despite the lavish attention to the concept, a precise definition has remained elusive. One implication of the lack of conceptual clarity is that formal treatment of social capital is sparse in the literature, with important exceptions of course; as a result of both conceptual ambiguity and lack of formal analysis, empirical measurement has proved difficult (Durlauf, 2002). Our overall objective in this paper is to provide an analytical framework of social capital, starting from a clear definition. This Introduction discusses the concept in detail (sub-Section 1.2), related literature (1.3), our contribution (1.4) and the structure of the paper (1.5).

1.2 What is 'social capital'?

Landmark contributions in sociology such as Coleman (1990), Putnam (1993, 2000) and Fukuyama (1997) place shared norms, beliefs and trust that facilitate economic (and wider) activity and promote efficiency, at the heart of social capital. Putnam, Leonardi, and Nanetti (1993), Bowles and Gintis (2002) and Routledge and von Amsberg (2002) define social capital as the social or governance structures that facilitate cooperation. However, defining social capital by its supposed effect (cooperation) seems like putting the cart in front of the horses;

a clear definition should go first, then the effects investigated. Additionally, in some of these analyses, social capital is a by-product of exchange, social interaction or other decision such as location; there is no associated investment decision. Such a definition does not satisfy Solow's (1995) four essential criteria for a concept of capital: distinctiveness (particularly from human capital), measurability (in principle), and clear process of accumulation and depreciation.[1]

In a recent attempt to define the concept more precisely. Guiso, Sapienza and Zingales (2012) define social capital as '. . . shared beliefs and values that help a group overcome the free rider problem in the pursuit of socially valuable activities.' [2] While plausible, this appears unduly restrictive: One may think of instances when a more general group of activities may alleviate more inefficiently negative externalities than the free rider such as status-related 'rat races'. In the same spirit, Durlauf and Fafchamps (2005) suggest that: (a) social capital generates positive externalities for members of a group; (b) these externalities are achieved through shared trust, norms, and values and their consequent effects on expectations and behaviour; (c) shared trust, norms, and values arise from informal forms of organizations based on social networks and associations. The first two aspects of this definition are virtually identical to the Guiso et al. (2012) definition; requirement (c), however, appears a little restrictive in that beneficial collective action, even informal and unintended, may happen in the wider community, outside of such associations.[3]

A further point that requires clarification is the distinction between a pure social capital and socially-related but privately-owned capital. Social capital is often defined in such a way that it entails a private as well as a social benefit via various externalities. E.g., Glaeser, Laibson and Sacerdote (2002, p. F439) define social capital as 'social characteristics that yield a market as well as non-market yield'; as they themselves acknowledge however, such social skills (to use the everyday term) include those of '. . . the stereotypical used car salesman who has lots of individual social capital (i.e., he's good at selling lemons to naïve consumers), but who generates little net social capital because of his negative social capital externalities'.

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[1] Very useful surveys on social capital include Dasgupta (2003), Durlauf and Fafchamps (2005) and Guiso, Sapienza and Zingales (2012).

[2] 'Civic capital', in their terminology.

[3] Element (c) may in fact impart an element of circularity in so far as the defining characteristic of networks is shared norms.

Thus, this type of skill should more appropriately be considered as a socially-related, private skill or form of human capital; social capital properly defined should be closer to what Glaeser et al. (2002) call 'aggregate' social capital, which nets out all the externalities from individual capital. Likewise, membership in an organization or network, a common definition of social capital, that confers a private benefit and perhaps positive externalities among the group but little on no wider social benefit, should also be excluded from a proper definition of social capital for the same reasons.

In the light of the above, we propose that any definition should start from the words themselves: social and capital: capital formed through costly (in an opportunity-cost sense) investment, e.g., education; but it may well be intangible, such as shared norms, beliefs, networks and institutions; and social, contributed to by many but not owned by anyone in particular. A further point concerns the exact nature of the benefit that accrues at societal level. Activities such as communal policing and cleaning of streets, while socially beneficial, are arguably more in the nature of public services arising from voluntary contributions (of time and/or effort) rather than taxation. So, we define:

Definition:

'Social capital is the set of all community-related formal and informal engagements, beliefs, values, norms and practices that:

(a)   help society achieve Pareto-superior outcomes, and

(b)   require resources (time and/or effort) to build.'

Examples may include contributions to charity, ethical behavior and norms such as politeness and mutual help, environmentally friendly norms and practices, quality of governance and institutions (Tabellini, 2008), etc. All these require investment such as formal and informal education and/or personal involvement and time. The beneficial effects arise firstly from the alleviation of inefficiencies related to status-seeking, conspicuous, corrupt or aggressive behaviour, other rat races, free-riding, etc. Additionally, social capital may confer utility directly: Helliwell, Aknin, Shiplett, Huang and Wang (2017) find that a higher social capital is associated with a greater subjective well-being.[4] Indeed, one may think of plausible reasons

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[4]This is an effect akin to the 'warm glow' type of benefit from giving (Andreoni, 1990) except it it stems from the level of social capital rather than the individual contribution to it.

why there may be utility derived directly from social capital in the form of a lower crime rate or greater social ties, mutual trust and cooperation, reducing monitoring costs, reducing contractual costs and so on, in addition to the mitigation of the negative externalities.

1.3: Related literature

Due perhaps to conceptual ambiguity, existing formal treatment of the concept of social capital is rather sparse. In an elegant early contribution, Glaeser, Laibson and Sacerdote (2002) model the investment-into-social capital decision in an explicitly optimizing way; a clear progress over earlier analyses. However, as alluded to above, their interface between individual and social capital is problematic as they fail to incorporate the externalities. Additionally, by relying on atomistic agents who take the aggregate social capital as exogenous, they ignore the feedback effect of optimal investment decisions regarding social capital made by other individuals. Thus, they cannot incorporate the interpersonal complementaries across individuals in accumulating social capital accumulation: The likelihood of my free riding interacts with the extent of aggregate free riding.

Routledge and von Amsberg (2002) offers a careful game-theoretic analysis of social capital-as-cooperation; the problem with this approach has been mentioned above. In a model where growth arises from productivity changes that require labor re-allocation, social capital has welfare effects. Without social capital, maximum growth arises from frictionless re-allocation, yet this re-allocation will involve less likelihood of stable (repeated) trade relationships and therefore cooperation is harder to sustain. Since social capital is defined as the sustenance of cooperation, welfare may be higher with less labor mobility.

Without calling it social capital, Tabellini (2008) models the willingness to cooperate which develops from the costly education by parents to their children in order for the latter to acquire more cooperative values. This is a complementary approach to ours in that there is 'something' that is sustained by costly investment and that helps society achieve more efficient outcomes; Tabellini (2008) calls it cooperation, we call it social capital. Varvarigos and Xin (2015) also presents a formal framework which is however more narrowly focused on the theme of trust.[5] Closer to the spirit of our analysis, Chou (2006) presents a model in which social capital facilitates standard human capital formation. Interestingly, this paper

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[5]While trust may arguably be a component of social capital, it is characterised by reciprocity; thus, trust is either shared widely or not. One may think of examples of social capital in such a way that one person may

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shows that there is a possibility of a poverty trap with little social capital and development. Poverty traps is a wide-ranging theme in macroeconomics and growth where lots of avenues have been explored but rather little convergence seemed to have been reached (see Azariadis and Stachurski, 2005).

1.4: Our contribution

Guided by the above definition, we propose a novel formal framework to model social capital as a community-related state variable, formed through costly investment, that alleviates the social inefficiencies associated with negative externalities. As explained below, we advance the existing literature in several ways. In order to focus attention on social capital without unnecessary complications, the model is a simple macro setup with flexible labor; it is static except the evolution of social capital that induces dynamics. Analytically, investment in social capital relies on individual contributions of time; this time is costly in an opportunity-cost sense and, implicitly, enhances social activities such as formal and informal education. Social capital is also subject to gradual depreciation. Thus, it satisfies all of Solow's (1995) criteria. In a nutshell, social capital is an aggregate state variable that alleviates unsociable/sub-optimal behavior. This is a novel treatment based on conceptual clarity; as we show, it yields novel insights, thus satisfying the additional criterion of usefulness.

To capture inefficiencies, the utility function exhibits a status motive (alternatively known as 'keeping up with the Joneses') that induces zero-sum behavior (see, e.g., among many others: Alonso-Carrera, Caballé and Raurich, 2004; Alvarez-Cuadrado, Monteiro and Turnovsky, 2004; Futagami and Shibata,1998; Tsoukis, 2007; Tournemaine and Tsoukis, 2009). But the emphasis on status is only for analytical convenience; very similar behavior and effects includes corruption and aggression, ostentatious behavior, rat races, as well as free-riding. Our framework is perfectly compatible with a wider interpretation, in which social capital restrains individuals from any unsociable (i.e., inefficient) behavior. In one version of our setup, we allow for direct personal satisfaction from social capital as well as its effects in alleviating negative externalities, in line with the evidence in Helliwell et al. (2017).

Methodologically, our framework uses dynamic non-cooperative game theory and mod-

contribute to it while another may not, thus, reciprocity is not a necessary feature. In much of our analysis below, we shall assumed symmetry, thus reciprocity is imposed.

els the formation of social capital as a Markov Nash equilibrium. We believe we are the first to model social capital as a state variable. This treatment of social capital clarifies the relationship between aggregate private and social capital, a difficulty that plagued some earlier work as discussed. We capture the dynamic strategic considerations among forward-looking individuals. This allows us to incorporate the status-related externalities and also to take into account the temptation to free-ride; crucial complementarities between individuals are thereby captured. This is an improvement over previous optimizing analyses in that the interface between the individual decision to contribute to social capital and aggregate dynamics is made explicit. As such, our framework resolves the tension between the traditional individual-optimizing approach of economics and more sociological analyses that view the community level as the appropriate decision-making unit for social capital (see Glaeser, Laibson and Sacerdote, 2002).

We derive several results related to equilibrium labor, social capital and welfare in prisoner's dilemma-type of situations with repeated interaction. We also contribute to poverty traps by showing conditions under which economies get trapped in low social capital. Our first specific result is that, in the case where social capital alleviates externalities but does not confer direct utility of the 'warm glow' type, there exist (possibly, a continuum of) MPE strategies which almost always causes overconsumption compared to the Pareto-efficient consumption level. Secondly, the most interesting result of this analysis, is that such overconsumption inevitably leads social capital to zero. In this case, the society almost always fails to accumulate social capital, and it stays in a poverty trap in the sense that the unchecked negative externalities cause net welfare to be low. In other words, a situation akin to a prisoners' dilemma arises. This pessimistic result is lifted when (a necessary condition) social capital enters directly individuals' utility function. According to our third result, if the degree of the disutility of labor takes intermediate values, getting out of a poverty trap is possible.

1.5: Structure of the paper

The organization of the paper is as follows. The next section describes the basic model, and then characterizes the dynamics of the model. Section 3 conducts the same analysis by extending the basic model to include peculiar benefits from social capital. Section 4 performs an analysis regarding welfare comparison between the command economy and the decentralized economy. Section 5 concludes the paper with a discussion and suggestions for

future research. Additional mathematical proofs will be given in Appendices A and B.

2 The Model

There are n (≥ 2) individuals. The instantaneous utility of individual i depends positively on consumption, c_i, and social status $c_i/\bar{c}$, where $\bar{c} = (\Sigma_{j=1}^{n} c_j)/n$ represents average consumption. The objective function of individual i is a discounted-sum of utilities over an infinite-time horizon:

$$u_i = \int_0^\infty \left[\log c_i - \beta l_i + (1 - \theta_i(S)) \log \left(\frac{c_i}{\bar{c}} \right) \right] e^{-\rho t} dt, \quad \rho > 0, \quad i = 1, 2, ..., n, \quad (1)$$

where the function $\log(c_i/\bar{c})$ captures the status externality, and β represents the weight on the disutility arising from labor supply. Specification of the second term in (1) would capture the fact the utility of each individual increases in the utility that the other individuals assign to her or his achievement in excess of average.[6] In addition, the variable $\theta_i(S)$ represents the perception of social capital by individual i: $\theta_i(S) : R_+ \mapsto [0, 1]$; that is, the extent to which the status externality is internalized due to the spirit of social capital, and thus $1 - \theta_i(S)$ represents the weight put on status-seeking motive.

Individuals decide at each point in time how much they engage in social interaction as well as how much to consume. Their choices are constrained by a time constraint. Consumption is also constrained by income, which is earned from working at a wage rate w according to the following flow budget constraint for individual i at each point in time:

$$c_i = w l_i,$$

while the time constraint is given by

$$1 \geq l_i + a_i,$$

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[6]Although it appears that larger S reduces the utility in (1), it is not a case because we will show that the presence of status-seeking preferences leads individuals to overconsumption which in turn deteriorates their well-being; however, the tendency of overconsumption which would be larger than the Pareto-efficient level is gradually weakened as a result of the accumulation of social capital, thereby generating positive externalities on individual utility.

where a_i is the time allocated to social activities which contribute to accumulating the stock of social capital over time and l_i is the amount of labor supply. For notational simplicity, we assume not only that the relative price between private consumption and leisure (i.e., the wage rate w), which is set equal to 1, but also that the time endowment is also normalized to be one through time. As a result, combining both constraints yields

$$c_i = 1 - a_i. \tag{2}$$

The aggregate stock of social capital will continuously change over time according to

$$\dot{S} = S(\Sigma_{i=1}^n a_i) - \delta S, \tag{3}$$

where δ is a constant depreciation rate of social capital ($0 < \delta < 1$). The emergence of S in the production of social capital reflects the fact that the existing social capital stock has spillover effects on the building of social capital.

Each individual maximizes the intertemporal utility u_i in (1) by selecting the time paths of c_i and a_i subject to (2) and the given initial level of social capital, S_0 (≥ 0), taking into account the others' strategies a_j ($j = 1, 2, ..., n : j \neq i$). We define a subgame perfect Nash equilibrium of this n-player differential game denoted by $\Gamma(S_0, 0)$ in the next subsection, where S_0 represents a historically given initial stock of social capital.

2.1 Solution Concept

We solve the differential game $\Gamma(S_0, 0)$ using the notion of a stationary Markov Perfect Equilibrium (MPE), which is appealing because, in ruling out all direct strategic interactions, it allows use of optimal control tools. To state this concept of equilibrium in a mathematically rigorous way, we reproduce a series of definitions given by Dockner et al. (2000) with notational modifications:

Definition 1 *(Dockner et al., 2000, Definition 3.1). A control path $c_i : [0, \infty) \mapsto \mathbb{R}_+$ is **feasible** for the game $\Gamma(S_0, 0)$ if the initial value problem defined by $(1) - (3)$ has a unique, absolutely continuous solution $S(.)$ such that the constraints $S(t) \in \mathbb{R}_+$ and $c_i(t) \in [0, 1]$ hold for all t and the integral in (1) is well defined.*

As the game $\Gamma(S_0, 0)$ is stationary, we can focus on equilibria supported by stationary strategies. For analytical simplicity, we further restrict ourselves to stationary Markov strategies throughout the paper.

Definition 2 *(Dockner et al.,2000, p.97). A stationary Markov strategy is a mapping ϕ^i : $\mathbb{R}_+ \mapsto [0, 1]$, so that the time path of the control is $c_i(t) = \phi^i(S(t))$.*

Hence, stationary Markov strategies are functions only of the current state. Then we can define:

Definition 3 *The n-tuple of functions $(\phi^1, \phi^2, ..., \phi^n)$ is a stationary Markov Nash equilibrium if for each $i \in \{1, 2, .., n\}$ an optimal control path $c_i(t)$ of the problem $\Gamma(S_0, 0)$ exists and is given by the stationary Markov strategy $c_i(t) = \phi^i(S(t))$.*

We use the further strengthening of Markov Nash equilibrium, that is, subgame perfectness, to characterize an equilibrium path:

Definition 4 *(Dockner et al., 2000, Definition 4.4). The n-tuple of functions $(\phi^1, \phi^2, ..., \phi^n)$ is a Markov Nash equilibrium of the game $\Gamma(S_0, t)$. The Markov Nash equilibrium is Markov perfect equilibrium (MPE) if for each $(S, t) \in \mathbb{R}_+ \times [0, \infty)$, the subgame $\Gamma(S, t)$ admits a Markov Nash equilibrium $(\psi^1, \psi^2, ..., \psi^n)$ such that $\psi^i(\hat{S}, s) = \phi^i(\hat{S}, s)$ for all $i \in \{1, 2, ..., n\}$ and all $(\hat{S}, s) \in \mathbb{R}_+ \times [t, \infty)$.*

Since the subgame $\Gamma(S, t)$ is stationary, $\Gamma(S, 0) = \Gamma(S, t)$ and thus all stationary Markov Nash equilibria are MPE.

The model described above is a differential game in which each player's strategies are its consumption and social interaction (or labor supply), which is measured by the time devoted to social activities, while the state variable of the game is the aggregate stock of social capital S. Since we focus on the (stationary) Markov-perfect equilibrium (MPE) defined above, we assume that each agent's strategies, c_i and a_i, are functions of the current stock level of aggregate social capital, S, alone. This means that the value function of the i-th agent's optimization problem at time t can be written as

$$V_i(S_t) \equiv \max \int_t^\infty e^{-\rho(\tau - t)} u_i(c_i(\tau), l_i(\tau), S_t) \, d\tau. \tag{4}$$

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We make the following assumption:

Assumption 1: The value function of individual i, $V_i(S) = \max_{\{c_i(.),\, l_i(.)\}} J^i_{\phi^{-i}}(c_i(.), l_i(.))$: $R_+ \to R$, is locally Lipschitz continuous.

2.2 Candidate Markov Perfect Equilibrium Strategies

The value function (4), after substitution of $l_i = c_i$ in (1), satisfies the Hamilton-Jacobi-Bellman (HJB) equation such as

$$\rho V_i(S) = \max_{\{c_i\}} \left\{ \log c_i - \beta c_i + (1 - \theta_i(S)) \log\left(\frac{c_i}{C/n}\right) + V_i'(S)\left[S\Sigma_{j=1}^n (1 - c_j) - \delta S\right] \right\} \quad (5)$$

for all $t \geq 0$, where $C \equiv \Sigma_{j=1}^n c_j$. In solving the maximization problem defined in the right-hand-side of (5) at each moment in time, the i-th individual takes the other players' strategies, $\{c_j\}_{j \neq i}$ $(j = 1, 2, .., n)$, as given. The first-order conditions for i's choice of consumption are given by

$$\frac{1}{c_i} - \beta + (1 - \theta_i(S))\frac{1}{c_i/C}\frac{\Sigma_{j=1,\, j \neq i}^n c_j}{C^2} - V_i'(S) S \begin{cases} = 0 \implies c_i \in (0, 1], \\ > 0 \implies c_i = 1, \end{cases} \quad (6)$$

or equivalently,

$$c_i(S) = \frac{1 + (1 - \theta_i(S))\left(\Sigma_{j=1,\, j \neq i}^n c_j/C\right)}{\beta + V'(S) S}. \quad (7)$$

According to (6), each individual, when choosing c_i, trades the marginal utility gains from an increase in current consumption and the associated status-seeking consumption against the sum of the utility loss from increased labor supply and the marginal loss in the discounted value of the future stream of *positive* externalities arising from the social capital stock in such a way that the accumulation of social capital stock prevents from excessive consumption. If the marginal utility from an increase in current consumption is larger than the payoff loss implied by the decrease in a_i for all levels of $c_i \in (0, 1]$, then i will rationally devote all resources to consumption. However, i never chooses $c_i = 0$ due to the property of logarithm utility functions. Moreover, close inspection of (6) reveals the following features; The higher the disutility of labor (i.e., β), the lower the labor supply; consequently, instantaneous consumption immediately falls through the budget constraint (2), while instantaneous consumption is also stimulated by the presence of status-seeking competition which is captured by the term

$\Sigma_{j=1,\ j\neq i}^n c_j/C$; The accumulation of social capital diminishes the *positive* externality of relative consumption on its own consumption, which is captured by $1 - \theta S$, thereby depressing consumption, while it generates the intertemporal externality captured by the term $V'(S)S$, which may increase or decrease consumption depending on the sign of $V'(S)$.

To get a closed-form solution, we have to impose a special form on the function $\theta_i(S)$ such as

$$\theta_i(S) = \begin{cases} \theta_i S & \text{if } S < \bar{S}_i, \\ 1 & \text{if } \bar{S}_i \leq S, \end{cases} \tag{8}$$

where $\bar{S}_i = 1/\theta_i$. We call $\bar{S}_i$ "a satiation level of the social capital stock" perceived by individual i.

Equation (6) gives the MPE solutions which are expressed as $\{c_i(S)\}$ for $i = 1.2.,...,n$. Substituting these optimal solutions back into the HJB equation (5) associated with agent i, together with (8), we obtain

$$\rho V_i(S) = \log c_i(S) - \beta c_i(S) + (1 - \theta_i S)\log\left(\frac{c_i(S)}{C(S)/n}\right) + V_i'(S)\left[S\Sigma_{j=1}^n(1 - c_j(S)) - \delta S\right]. \tag{9}$$

By the use of the envelop theorem, we find that differentiating both sides of (9) with respect to S gives

$$\rho V_i'(S) = \frac{c_i'(S)}{c_i(S)} - \beta c_i'(S) - \theta_i \log\left(\frac{nc_i(S)}{C(S)}\right) + (1 - \theta_i S) \cdot$$
$$\frac{C(S)}{c_i(S)}\frac{c_i'(S)C(S) - c_i(S)\Sigma_{j=1}^n c_j'(S)}{[C(S)]^2} + V_i''(S)\left[S\Sigma_{j=1}^n(1 - c_j(S)) - \delta S\right]$$
$$+ V_i'(S)\left[\Sigma_{j=1}^n(1 - c_j(S)) + S\Sigma_{j=1}^n\left(-c_j'(S)\right) - \delta\right]. \tag{10}$$

Since we have started our analysis assuming identical individuals and since the state equation (3) is symmetric with respect to their controls, a natural focus is placed on symmetric equilibria. The symmetry assumption allows us to drop the subscript i in the subsequent discussion, and we will suppress this index unless strictly necessary for expositional clarity. Since under the assumption of symmetry $\log(C(S)/nc(S)) = 0$ (recalling $C(S) = nc(S)$) and $c'(S)nc(S) - c(S)nc'(S) = 0$, we obtain

$$\rho V'(S) = \left[\frac{1}{c(S)} - \beta\right] c'(S) + V''(S)\left[Sn(1 - c(S)) - \delta S\right]$$
$$+ V'(S)\left[n(1 - c(S)) - Snc'(S) - \delta\right]. \tag{11}$$

After the tedious manipulation, we can rewrite (11) as follows (see Appendix A):

$$c'(S) = -\frac{1}{S} \cdot$$
$$\frac{\rho\left[1 - \beta c(S) + (1 - \theta S)\frac{n-1}{n}\right] + \theta\frac{n-1}{n}S\left[n(1 - c(S)) - \delta\right]}{\frac{1}{c(S)}\left[(n - c(S)) - \delta\right]\left[1 - \beta c(S) + (1 - \theta S)\frac{n-1}{n}\right] + (1 - \theta S)\frac{n-1}{n} + \beta\left[n(1 - c(S)) - \delta\right]}. \tag{12}$$

2.3 Dynamics and Steady State

We will draw the representatives of Markov strategies in a control and state space in order to characterize qualitative solutions to the nonlinear differential equation (12). The solutions satisfying (12) are only candidates and, in some cases, many of them do not qualify for a MPE. Those solutions must pass further tests to qualify for a MPE satisfying the properties stated later. To stress the candidate status, we introduce an additional notation, $\phi(S)$, instead of $c(S)$. Let us first denote by C_2 the loci where $\phi'(S)$ goes to plus/minus infinity, and by C_3 the loci where $\phi'(S)$ equals zero in the (S, c) space:

$$C_1 := \{(S, c) : \dot{S} = Sn(1 - \phi(S)) - \delta S = 0\},$$
$$C_2 := \{(S, c) : \phi'(S) \to \pm\infty\}, \tag{13}$$
$$C_3 := \{(S, c) : \phi'(S) = 0\}.$$

First, we can obtain the steady state locus C_1 by setting $\dot{S} = 0$; that is,

$$c = 1 - \frac{\delta}{n}, \tag{14}$$

which implies that the steady-state locus C_1, denoted by $\dot{S} = 0$ in Fig. 1, is a horizontal line at $c = 1 - (\delta/n)$ in the (S, c) space. The arrows illustrated in Fig. 1 indicate that for initial values of $\phi(S)$ above the steady state locus, $c = 1 - (\delta/n)$, the social capital stock is declining in time, while for those of $\phi(S)$ below the steady state locus the social capital stock
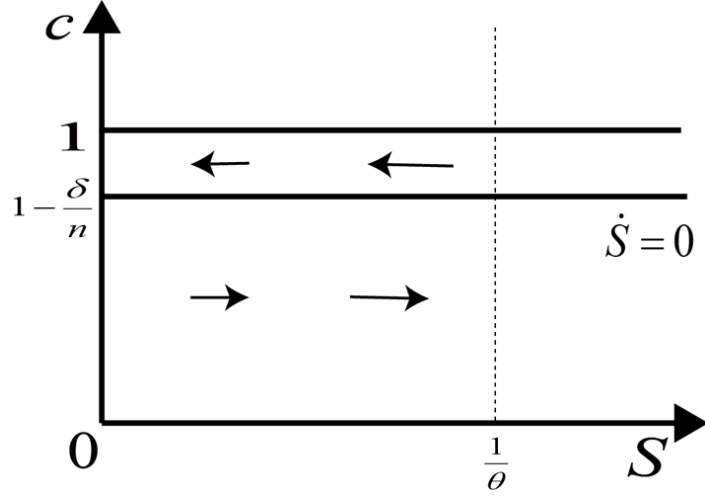
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Figure 1: Steady state locus

is increasing in time. It is also important to note that there is an upper bound on S (i.e., $S \leq (1/\theta)$), as illustrated by the dotted vertical straight line at $1/\theta$ in Fig. 1.[7] Beyond this upper bound, there is no incentive for every individual to accumulate more social capital, because further social capital accumulation reduces utility.

Next, by setting the denominator in (12) equal to zero, we can obtain the locus of the curve C_2:

$$\frac{1}{c} \left[(n-c) - \delta \right] \left[1 - \beta c + (1 - \theta S) \frac{n-1}{n} \right] + (1 - \theta S) \frac{n-1}{n} + \beta \left[n(1-c) - \delta \right] = 0, \quad (15)$$

which we call "the non-invertibility locus" following Rowat (2007). After some manipulation, we can rewrite (15) as follows:

$$\beta c^2 (1-n) - c + (n-\delta) \left[1 + (1 - \theta S) \frac{n-1}{n} \right] = 0, \quad (16)$$

whose solutions for c are given by $c \geq 0$:

$$c = \frac{\sqrt{1 + 4\beta (n-1)(n-\delta) \left[1 + (1 - \theta S) \frac{n-1}{n} \right]} - 1}{2\beta (n-1)}, \quad (17)$$

[7]Since we have made the assumption of symmetry, the satiation levels of the social capital capital perceived by all individuals are common among them, so that $\bar{S}_i = \bar{S}$ for $\forall i$.

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which entails that

$$\lim_{S \to 0} c = \frac{\sqrt{1 + 4\beta(n-1)(n-\delta)\dfrac{2n-1}{n}} - 1}{2\beta(n-1)} \gtreqless 1, \text{ and} \qquad (18)$$

$$\lim_{S \to 1/\theta} c = \frac{\sqrt{1 + 4\beta(n-1)(n-\delta)} - 1}{2\beta(n-1)} > 0.$$

An important point is to be noticed that if the solution curve of $\phi(S)$ crosses the curve C_2, it becomes discontinuous at this intersection point; consequently, it is ruled out as a solution (see the next subsection in further details).

Inspection of (17) further reveals not only that the curve C_2 may cross the vertical axis above or below the budget line $c = 1$ because

$$\frac{\sqrt{1 + 4\beta(n-1)(n-\delta)\dfrac{2n-1}{n}} - 1}{2\beta(n-1)} - 1 \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{(n-\delta)(2n-1)-n}{n(n-1)}, \qquad (19)$$

but also that the curve C_2 may cross the vertical axis above or below the steady state line $1 - (\delta/n)$ because

$$\frac{\sqrt{1 + 4\beta(n-1)(n-\delta)\dfrac{2n-1}{n}} - 1}{2\beta(n-1)} - \left(1 - \frac{\delta}{n}\right) \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{2n}{n-\delta}. \qquad (20)$$

It can also be verified that the curve C_2 always crosses the upper boundary $S = 1/\theta$ *below* the budget line $c = 1$ because

$$\frac{\sqrt{1 + 4\beta(n-1)(n-\delta)} - 1}{2\beta(n-1)} - 1 < 0, \qquad (21)$$

while the curve C_2 may cross the upper boundary $\bar{S} = 1/\theta$ below or above the steady state line $1 - (\delta/n)$ because

$$\frac{\sqrt{1 + 4\beta(n-1)(n-\delta)} - 1}{2\beta(n-1)} - \left(1 - \frac{\delta}{n}\right) \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{n}{n-\delta}. \qquad (22)$$

Taking into account of (19), (20), (21) and (22) in conjunction with the following facts:

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$$\frac{(n-\delta)(2n-1)-n}{n(n-1)} - \frac{n}{n-\delta} = \frac{n^3 - 4n^2\delta - n^2 + 2n\delta^2 + 3n\delta - \delta^2}{n(n-1)(n-\delta)} \gtreqless 0,$$

$$\frac{(n-\delta)(2n-1)-n}{n(n-1)} - \frac{2n}{n-\delta} = \frac{-\delta\left(\delta - 3n + 4n^2 - 2n\delta\right)}{n(n-1)(n-\delta)} < 0,$$

$$\frac{n}{n-\delta} - \frac{2n}{n-\delta} < 0, \text{ and}$$

$$\frac{2n-1}{n} - \frac{n}{n-\delta} = \frac{n^2 - (2\delta+1)n + \delta}{(n-\delta)\,n} \gtreqless 0,$$

we can draw several geometric representations for the curve C_2 such as Figs. $2-8$, corresponding to various parameter values of n, β and δ.

On the other hand, the locus C_3 is obtained by setting the numerator in (12) equal to zero. Solving it for c gives the following locus:

$$c = \frac{(n-\delta-\rho)\,\theta\dfrac{n-1}{n}S + \rho\dfrac{2n-1}{n}}{\rho\beta + \theta\,(n-1)\,S}, \tag{23}$$

with

$$\lim_{S\to 0} c = \frac{2n-1}{\beta n}, \quad \lim_{S\longrightarrow\infty} c = (n-\delta-\rho)\frac{1}{n} < 1, \text{ and } \lim_{S\longrightarrow\frac{-\rho\beta}{\theta(n-1)}} c = \infty.$$

Taken together, it turns out that the nonlinear curve C_3 is an *orthogonal hyperbolic curve* in the (S,c) space which intersects with the vertical axis at $\dfrac{2n-1}{\beta n}$, as illustrated in Figs. $2-8$. Note, moreover, that the intercept of the curve C_3 lies above (below) the intercept of the budget line $c = 1$ on the vertical axis if $\beta < \dfrac{2n-1}{n}$ ($\beta > \dfrac{2n-1}{n}$) because

$$\frac{2n-1}{\beta n} - 1 = \frac{(2-\beta)n - 1}{\beta n} \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{2n-1}{n},$$

while the curve C_3 crosses the vertical axis below (above) the steady state line (i.e., , $c = 1-\dfrac{\delta}{n}$) if $\beta > \dfrac{2n-1}{n-\delta}(\beta < \dfrac{2n-1}{n-\delta})$ because

$$\frac{2n-1}{\beta n} - \left(1 - \frac{\delta}{n}\right) = \frac{2n - n\beta + \beta\delta - 1}{\beta n} \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{2n-1}{n-\delta}.$$

Lastly, the curve C_3 crosses the line $1/\theta$ above (below) the steady sate line if $\beta < \dfrac{n}{n-\delta}$ $\left(\beta > \dfrac{n}{n-\delta}\right)$, because

$$\frac{(n-\delta-\rho)\dfrac{n-1}{n} + \rho\dfrac{2n-1}{n}}{\rho\beta + (n-1)} - \left(1 - \frac{\delta}{n}\right) = \rho\frac{-(n-\delta)\beta + n}{[\rho\beta + (n-1)]\,n} \gtreqless 0 \text{ if and only if } \beta \lesseqgtr \frac{n}{n-\delta}.$$

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Collecting the arguments discussed so far, we draw several geometric representations for the curves C_2 and C_3, depending on the combinations of various parameter values of β, n and δ. Two remarks are made in order. First, based on the information regarding signs of (12) which are determined according to the respective regions divided by the curves C_2 and C_3, together with the signs indicated by Fig.1, we can construct several phase diagrams indicating how the variables c and S move as illustrated in Figs $2-8$.[8] Second, it is important to note that *not* all strategies satisfying (12), which are drawn in these figures, are solution curves. In the next subsection, we will provide further conditions to pin down the MPE strategies characterizing optimal consumption paths.

2.4   Refining Candidate Strategies

Unfortunately, not all interior strategies satisfying the HJB equation (5) are qualified as MPE strategies defined in Definition 4. Dockner et al. (2000) have provided sufficiency conditions for the existence of MPE. So we have to test whether those candidate strategies satisfy the sufficiency conditions in Theorem 4.1 of Dockner et al. (2000).

The solution paths must satisfy the following **three** requirements: The first requirement is that the strategies must be defined over the *entire* feasible state space (i.e., $S \in [0, 1/\theta]$ in the present model) and not only for its subset. Rowat (2006) claims that we have to take the definition of subgame perfection *seriously*, as long as we cannot explain how an initial

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[8]At this point, it may be worthy pointing out whether the intercept of curve C_2 may be greater or less than that of curve C_3. To identify this, we take the difference of their intercepts to yields

$$\frac{\sqrt{1 + 4\beta(n-1)(n-\delta)\frac{2n-1}{n}} - 1}{2\beta(n-1)} - \frac{2n-1}{\beta n}$$

$$= \frac{\sqrt{1 + 4\beta(n-1)(n-\delta)\frac{2n-1}{n}} - 1 - 2(2n-1)\left(1 - \frac{1}{n}\right)}{2\beta(n-1)},$$

whose numerator can be manipulated by squaring both sides of the above expression to yield:

$$1 + 4\beta(n-1)(n-\delta)\frac{2n-1}{n} - \left[1 + 2(2n-1)\left(1 - \frac{1}{n}\right)\right]^2 =$$

$$4(2n-1)\frac{1}{n}(n-1)\left[\beta(n-\delta) - \left(2n-2+\frac{1}{n}\right)\right] \gtreqless 0 \text{ if } \beta \gtreqless \frac{2(n-1)+(1/n)}{n-\delta}n.$$

The above result implies that as long as $\beta > \frac{2(n-1)+(1/n)}{n-\delta}$, the intercept of the curve C_2 is greater than that of the curve C_3; otherwise, the opposite holds.

condition of the state variable was reached in the first place. To show how this requirement is used to refine candidate strategies, let us take up Fig. 2, which displays representatives of those integral curves that are categorized into three typical types of the families of strategies designated by the integral curves ϕ_j, $j = 1, 2$, and ϕ^*. The arrows attached those integral curves indicate the evolution of S over time. Applying the first requirement, the family of strategies ϕ_2 can be ruled out, because those *interior* solutions do not cover the whole range of S as seen in Fig. 2. Nevertheless, it may be possible to extend the families of those strategies by patching with the corner strategy $\phi = 1$ to the entire domain of S *when those strategies hit the boundary $c = 1$*, as Rowat (2004) did. To check this possibility, consider a case where the candidate interior strategy hits the boundary line $c = 1$. We can show that the resulting patched strategy could cover the entire domain of the state variable S, **only if** the interior strategy ϕ^* is connected with the corner strategy $\phi = 1$ at the points lying within the appropriate range indicated by the following lemma:

Lemma 1 *The corner strategy $\phi = 1$ is not a MPE strategy if the value function $(B.2)$ (see Appendix B) possesses a non-zero constant of integration and satisfies $\phi(0) = 1$. The corner strategy $\phi = 1$ is a MPE strategy if the value function $(B.2)$ possesses a zero constant of integration and the domain of the state variable S is given by $\left[0, \frac{1}{\theta}\left(1 - \frac{n(\beta - 1)}{(n-1)}\right)\right]$.*

The proof is relegated to Appendix B. With the aide of this lemma, we are able to identify which interior strategy ϕ^* can or cannot be patched with the corner strategy $\phi = 1$, *when that interior strategy hits the boundary line $c = 1$*. However, note that the range indicated by Lemma 1 is valid only when $\beta < (2n-1)/n$; otherwise, the extreme right end of that interval, $\frac{1}{\theta}\left(1 - \frac{n(\beta - 1)}{(n-1)}\right)$, is *negative*. Making use of this result, we can easily see that condition $\beta < (2n-1)/n$ is not satisfied in all figures except for Fig.6. As a result, the corner solution $\phi = 1$ is ruled out and hence the patched solution is not possible in those figures.[9] On the other hand, it follows from (6) that as long as $\lim_{S \to 0} V'(S) \geq 0$,

$$c(S)|_{S=0} = \frac{\dfrac{2n-1}{n}}{\beta + \lim_{S \to 0}\left[V'(S)S\right]} \leq 1 \text{ if } \beta \geq \frac{2n-1}{n}. \tag{24}$$

Since it is seen that condition $(2n-1)/n \leq \beta$ holds in all figures except for Fig.6, $c(S) < 1$ for $S \in (0, 1/\theta]$ as long as $\lim_{S \to 0} V'(S) \geq 0$.

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[9]Note also that there is no MPE strategy in Fig.6 as shown later.

The second requirement is that the strategies should be **continuous** over the entire range of a state variable, i.e., $S \in [0, 1/\theta]$. More specifically, when the strategies cross the non-invertible curve C_2, they ceases to be continuous at that crossing point, and thus such strategies should be eliminated as an equilibrium solution. For this reason, the strategies ϕ_2 in Fig. 2 are once again disqualified as MPE ones.

The third requirement is that the value function associated with the solution should be bounded. This condition is clearly met for the candidate strategies because they either reach zero or a bounded value of social capital stock together with the specification of the objective function (1). In this view, although it appears that the families of the strategy ϕ_1 are MPE ones in Fig. 2, they are **not**. The reason is as follows. When the optimal strategy ϕ_1 reaches the satiated locus $1/\theta$, it has to stay at that hitting point on that locus by freely disposing of the extra resource which has not been consumed, because further social capital accumulation more than $S = 1/\theta$ is to reduce individual's utility. However, this is a **suboptimal** choice for every individual because the individuals can augment their utility by increasing their consumption levels rather than disposing of that extra resource. Such a choice causes their consumption levels to immediately jump to the steady state consumption level, i.e., $c = 1 - (\delta/n)$. Unfortunately, such a jumping solution entails discontinuity at that point thus violating the second requirement. Alternatively, we may say that the strategy ϕ_1 is no longer qualified as the MPE strategy, because each individual who adopts the strategy ϕ_1 has an incentive of deviating from this strategy. Taken together, we can conclude that the families of the strategies ϕ_1 as well as ϕ_2 are **not** equilibrium strategies in Fig. 2. The same argument applies to the strategies ϕ_1 as well as ϕ_2 appearing in other figures.

In spite of these requirements, there may still be multiple solution curves because there may be a continuum of the interior strategies located *slightly above* the strategy ϕ^* can reach the vertical axis *without touching or crossing the curve C_2 as well as the budget constraint $c = 1$ during the transition path to zero social capital* in Figs. 2, 3 and 8. In contrast, there are cases which do not allow for the existence of any MPE solution at all as illustrated in Figs. 4, 6 and 7, because the curve C_2 prevents any interior strategy from reaching the vertical axis.

Taken together, the results are summarized as follows:

Proposition 5 *Consider the differential game* $\Gamma(Z_0, 0)$ *and assume* $\lim_{S \to 0} V'(S) \geq 0$:

$$Either \; if \; \beta \quad \geq \quad max \left\{ \frac{(n-\delta)(2n-1)-n}{n(n-1)}, \frac{n}{n-\delta}, \frac{2n-1}{n} \right\} \; or$$

$$if \frac{n}{n-\delta} \quad \geq \quad \beta \geq max \left\{ \frac{(n-\delta)(2n-1)-n}{n(n-1)}, \frac{2n-1}{n} \right\},$$

then there may exist a multiple (a continuum) of MPE strategies. The solution paths corresponding to those MPE strategies lead a social capital stock to decline to nil, while the associated consumption levels monotonically rise through time.

As to Proposition 1, there are several remarks in order. First, Proposition 1 implies that there may be multiple MPE strategies unless "the non-invertibility locus" C_2 prevents the solution path from reaching the vertical axis. More precisely, if "the non-invertibility locus" C_2 crosses not only the vertical axis above the boundary line $c = 1$ but also the satiated locus $S = 1/\theta$ below the steady state line as illustrated in Figs. 4, 6 and 7, there is no MPE strategy. The potential existence of multiple MPE solutions would be consistent with the results of Itaya and Shimomura (1993), and Rubio and Casino (2002) which also provides multiplicity of equilibrium strategies as well as the uncountable many long run equilibria associated with those multiple nonlinear MPE strategies. This similarity between them stems from the *boundedness* of the entire space of a state variable imposed by the satiated level of social capital together with the lack of the transversality condition.

Second, the optimal consumption paths associated with those MPE strategies all display a *downward sloping* in the level of social capital stock. This property obviously reflects the fact that larger stocks of social capital tend to deter excessive consumption which is induced by status-seeking preferences enhanced by status-seeking competition in our setting.

Third, most impotently, all figures indicate that any consumption level supported by the MPE strategy (except for the consumption level at the steady state) is always greater than the steady state one, so that the accumulation of social capital *never* takes place, irrespective of the size of an initial social capital stock. In other words, all of the MPE strategies *always* lead the social capital stock to decline to nil; consequently, either if there were initially a small amount of social capital stock in social capital-poor societies or if there were initially a sizable amount of social capital in social capital-rich societies but it is less than the satiated level

$(= 1/\theta)$, overconsumption driven by status-seeking preferences enhanced by its competition inevitably prevails, so that it eventually vanishes in the long run. As a consequence, the society by itself is unable to escape from a *social capital-poverty trap.* We may also say that such social capital-poor societies are captured in a poverty trap when status-seeking preferences are present. The reason for this pessimistic result is simply that those negative externalities are strong enough to overweigh the positive externality arising from social capital accumulation.[10]

In such a situation, government intervention should be called upon to help the societies escape from such a poverty trap. Since the society by itself can never spontaneously and autonomously accumulate social capital, governments and/or outside authorities have to enforce or regulate the members of the society to accumulate more social capital up to the satiated level by curtailing individual consumption. Put differently, in the societies where status-seeking preferences are commonly observed, the government must play a very active role in a way that corrects or regulates the behavior of rational and forward-looking individuals to large extent so as to curtail their consumption levels through time which leads to the satiated steady state level. Alternatively, the government may implement "a capital big push" in a way of immediately building up the satiated level instantaneously or initially before social capital starts accumulating.

Fourth, the main cause for the above-mentioned "Tragedy of the Commons" is that the incentive to accumulate social capital is too weak. To induce individuals to promote social capital accumulation, we need to introduce individual's pleasure or peculiar benefits from social capital in addition to voluntarily restraining overconsumption caused by the presence of social-status preferences enhanced by social-status competition. This may correspond to the warm-glow motivation in the model of voluntary provision of public goods (see, e.g.,

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[10]Social capital poverty trap also has been captured by Chou (2006) in the endogenous growth model with the Romer (1990)'s R&D function augmented with social capital spillover. He shows that if the spillover effect on one person's decision to accumulate social capital varies with the level of social capital in the community, such an economy has two possible steady states: one of which is a stable social capital-poor equilibrium and the other of which is an unstable social caipital-rich equilibrium and calls the first equilibrium a social capital poverty trap. However, the emergence of a poverty trap stems from a non-concave reduced-form function of social capital which implicitly embodies the social capital spillover. In contrast, our model has microeconomic foundations which explain how the spillover effect of social capital operate across individuals.
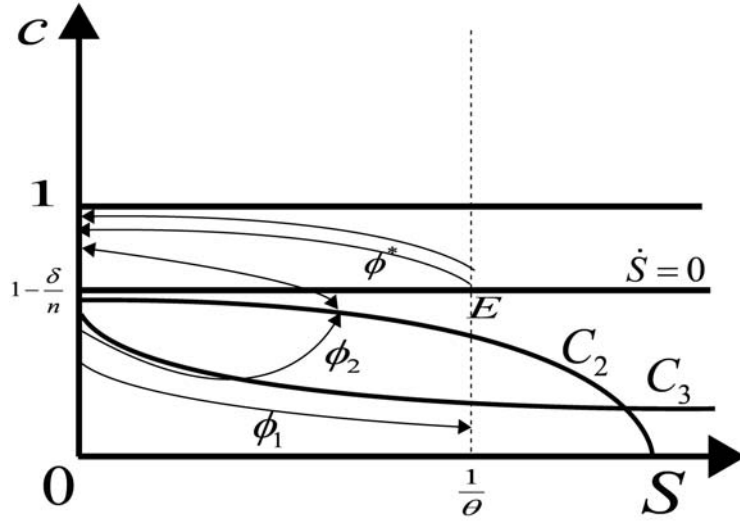
Figure 2: $\beta \geq \dfrac{2n}{n-\delta}$

Andreoni, 1990) which also enhances an incentive of donors to supply public goods. We extend the present model to include some sort of peculiar benefits from social capital stock in Section 3.

3 Peculiar Benefits from Social Capital

In this section, we incorporate the benefit from social capital, which enhances the incentive of individuals to accumulate social capital. To do this, we modify the intertemporal utility function such as:

$$u_i = \int_0^\infty \left[\log c_i - \beta l_i + \gamma S + (1 - \theta_i(S)) \log\left(\frac{c_i}{\bar{c}}\right)\right] e^{-\rho t} dt, \quad \rho > 0, \quad i = 1, 2, ..., n, \quad (25)$$

where the parameter γ represents the strength of preferences towards social capital, while the rest of the model is the same as before. The presence of social capital in the utility function may be viewed as the quality of social environment which depends on accumulated social capital (see, e.g., Antoci, et al., 2014).[11] The function, after substitution of $l_i = c_i$ in (25),

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[11]Alternatively, we may assume that individuals have preferences for socializing. In this formulation, the choice variable of a_i directly enters the utility function instead of the state variable S. Although this modification does not alter our results essentially, the analysis becomes extremely complicated.
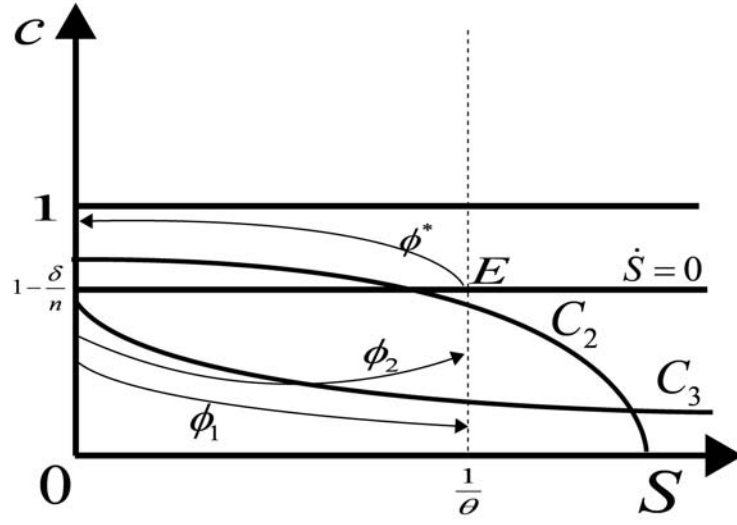
Figure 3: $\dfrac{2n}{n-\delta} \geq \beta \geq \max\left\{\dfrac{2n-1}{n-\delta}, \dfrac{(n-\delta)(2n-1)-n}{n(n-1)}\right\}$
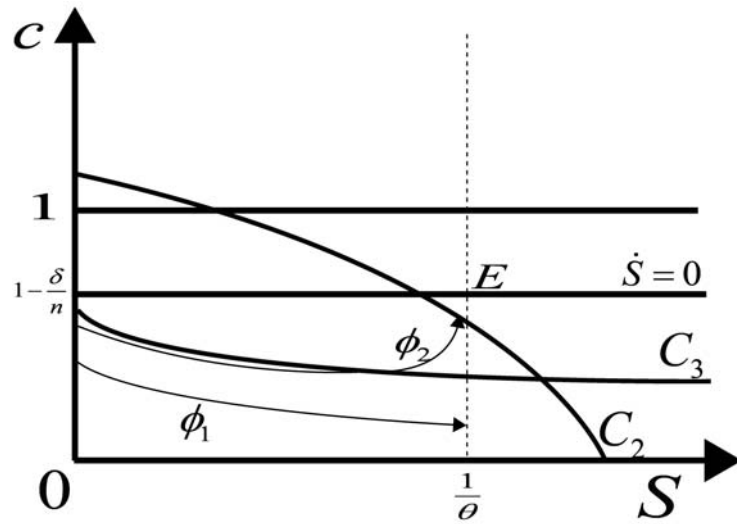

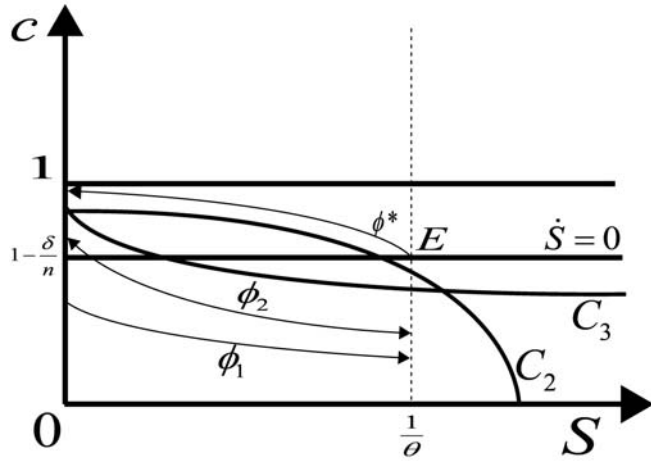Figure 4: $\dfrac{(n-\delta)(2n-1)-n}{n(n-1)} \geq \beta \geq \dfrac{2n-1}{n-\delta}$

Figure 5: $\dfrac{2n-1}{n-\delta} \geq \beta \geq \max\left\{\dfrac{(n-\delta)(2n-1)-n}{n(n-1)}, \dfrac{2n-1}{n}, \dfrac{n}{n-\delta}\right\}$
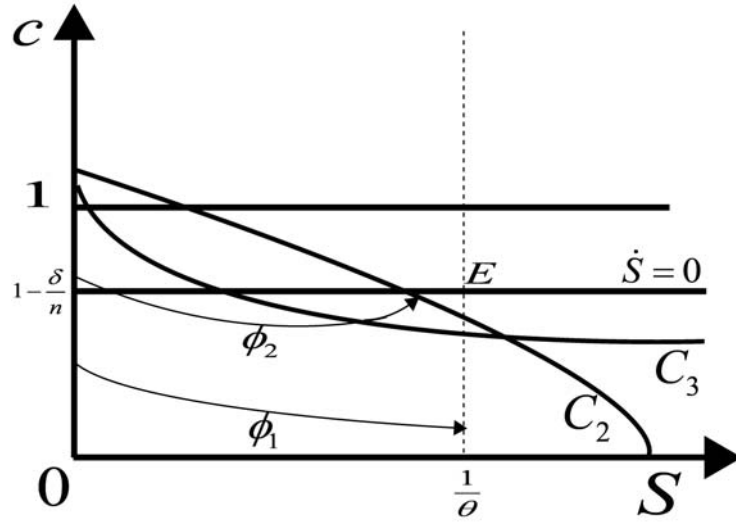

Figure 6: $\min\left\{\dfrac{2n-1}{n}, \dfrac{(n-\delta)(2n-1)-n}{n(n-1)}\right\} \geq \beta \geq \dfrac{n}{n-\delta}$
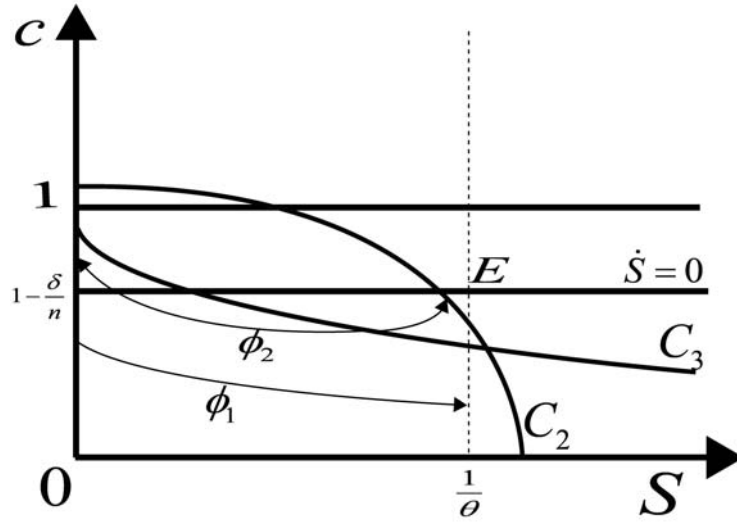
Figure 7: $\dfrac{(n-\delta)(2n-1)-n}{n(n-1)} \geq \beta \geq \max\left\{\dfrac{2n-1}{n}, \dfrac{n}{n-\delta}\right\}$
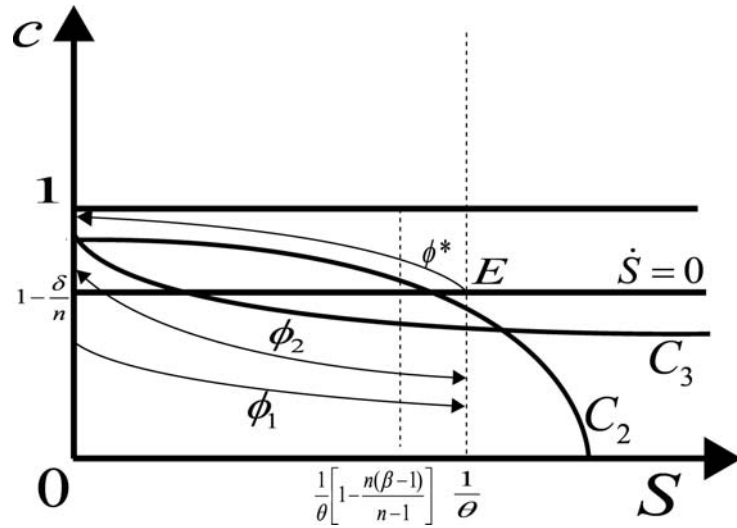

Figure 8: $\dfrac{n}{n-\delta} \geq \beta \geq \max\left\{\dfrac{2n-1}{n}, \dfrac{(n-\delta)(2n-1)-n}{n(n-1)}\right\}$

the Hamilton-Jacobi-Bellman (HJB) equation is rewritten as:

$$\rho V_i(S) = \max_{\{c_i\}} \left\{ \log c_i - \beta c_i + \gamma S + (1 - \theta_i(S)) \log \left(\frac{c_i}{C/n} \right) + V_i'(S) \left[S(\Sigma_{j=1}^n (1 - c_j)) - \delta S \right] \right\}$$

(26)

for all $t \geq 0$. The first-order conditions for i's choice of consumption are the same as before.

Substituting these optimal solutions (i.e., as $\{c_i(S)\}$ for $i = 1, 2., ..., n$) back into the HJB equation (26) associated with agent i, together with (8), we obtain

$$\rho V_i(S) = \log c_i(S) - \beta c_i(S) + \gamma S + (1 - \theta_i S) \log \left(\frac{c_i(S)}{C(S)/n} \right) + V_i'(S) \left[S\Sigma_{j=1}^n (1 - c_j(S)) - \delta S \right].$$

(27)

By the use of the envelop theorem and focusing on symmetric equilibria, we can finally obtain

$$c'(S) = -\frac{1}{S} \cdot$$

$$\frac{\rho \left[1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right] + \theta \frac{n-1}{n} S \left[n(1 - c(S)) - \delta \right] - \gamma}{\frac{1}{c(S)} \left[(n - c(S)) - \delta \right] \left[1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right] + (1 - \theta S) \frac{n-1}{n} + \beta \left[n(1 - c(S)) - \delta \right]}.$$

(28)

In view of (28), it turns out that the curves C_1 and C_2 are the same as before, the only curve C_3 is slightly different from the one in (23). Setting the numerator in (28) equal to zero, we can get a new expression for the curve C_3:

$$c = \frac{(n - \delta - \rho) \theta \frac{n-1}{n} S + \rho \frac{2n-1}{n} - \gamma}{\rho \beta + \theta (n-1) S},$$

(29)

with

$$\lim_{S \to 0} c = \frac{\rho \frac{2n-1}{n} - \gamma}{\rho \beta}, \quad \lim_{S \to \infty} c = (n - \delta - \rho) \frac{1}{n} < 1, \quad \text{and} \quad \lim_{S \to \frac{-\rho\beta}{\theta(n-1)}} c = \infty.$$

Taken together, it turns out that the curve C_3 is an *orthogonal hyperbolic curve* in the (S, c) space which approaches the vertical line $S = \rho\beta/\theta(n-1)$. The most important difference is that the curve C_3 crosses the line $1/\theta$ below the steady sate line if $\beta > \frac{n - (\gamma/\rho)}{n - \delta}$, because

$$\frac{(n - \delta - \rho) \frac{n-1}{n} + \rho \frac{2n-1}{n} - \gamma}{\rho\beta + (n-1)} - \left(1 - \frac{\delta}{n} \right) = \frac{1}{n} \frac{\rho[n - (n - \delta)\beta] - \gamma}{\rho\beta + (n-1)} \lesseqgtr 0$$

$$\text{if and only if } \beta \gtreqless \frac{n - (\gamma/\rho)}{n - \delta}.$$

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Collecting the arguments discussed so far, we can draw Fig. 9. It follows from the above condition that if $\dfrac{n}{n-\delta} \geq \beta \geq \dfrac{n-(\gamma/\rho)}{n-\delta}$, the curve C_2 will be located *above* the steady state line at $S = 1/\theta$, while the curve C_3 is located *below* the steady state line at $S = 1/\theta$, as illustrated in Fig. 9. As a result, therefore, the presence of the peculiar preferences towards social capital shifts the curve C_3 downwards, so that the satiated steady state point E will end up being in between these two curves at $S = 1/\theta$. This feature makes it possible that there is the *unique* optimal consumption path supported by the MPE strategy ϕ^* drawn in Fig. 9. Along this MPE solution curve, social capital can spontaneously and automatically reach the satiated steady state point E. In particular, it should be stressed that **only when** the degree of disutility (i.e.. β) should take intermediate values such as $\beta \in \left[\dfrac{n}{n-\delta}, \dfrac{n-(\gamma/\rho)}{n-\delta} \right]$, there exists the unique optimal consumption path supported by the MPE strategy which converges to the satiated steady state level of consumption; see Fig.9. This feature can be explained as follows: excessively higher values of β induce the individuals to deter consumption which in turn gives rise to overaccumulation of social capital thus preventing from reaching the steady state level of consumption corresponding to point E; conversely, excessively lower values of β stimulate individual's consumption too much which in turn gives rise to underaccumulation of social capital thereby leading to zero social capital. Note also that consumption does not change monotonically through time. More precisely, as shown in Fig.9, there is an unstable steady state equilibrium in which consumption monotonically rises as social capital declines to zero when the initial stock of social capital is less than S^E, while consumption falls and then rises to its steady state level as social capital increases to its satiated level when the initial stock of social capital is greater than S^E. It is important to note not only that there will be multiple MPE strategies that lead social capital to zero, but also that larger values of γ (i.e., the social capital stock becomes more attractive) or lower values of ρ allow for a wider range of values of β so as to satisfy the range of $\dfrac{n}{n-\delta} \geq \beta \geq \dfrac{n-(\gamma/\rho)}{n-\delta}$, whereby improving the likelihood of the existence of MPE strategies. Lower values of ρ imply that the individual becomes more patient and thus more cooperative to sustain the unique MPE strategy thereby making it possible to escape from a social capital poverty trap.
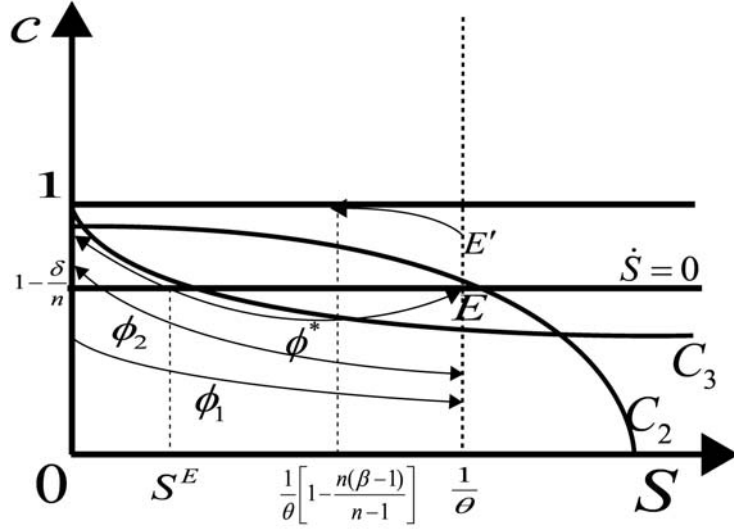
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Figure 9: $\dfrac{n}{n-\delta} \geq \beta \geq \dfrac{n-(\gamma/\rho)}{n-\delta}$

4   Welfare Comparison

In this section, we first compare between social welfare in a hypothetical planner's economy and in a decentralized economy in which social capital is accumulated through voluntary effort (social activity) made by individuals through time.

First, in the planner's economy, the government is able to attain the first-best resource allocation by directly controlling resource allocations in order to fully internalize a negative externality arising from the aspire for relative consumption. The planner's economy is able to make the efficient use of all the available instruments so as to completely eliminating the externality associated with status-seeking competition and thus restore the first-best allocation. This amounts to maximizing the following utility function over an infinite horizon:

$$u_i = \int_0^\infty \left[\log c_i - \beta l_i\right] e^{-\rho t} dt, \tag{30}$$

subject to $c_i = l_i$, whose optimal solution is given by $c^P = 1/\beta$. Note that the the Pareto efficient consumption level is *constant* in the model without peculiar preferences towards social capital.

We compare between the Pareto-efficient level of consumption $(= 1/\beta)$ and the consump-

tion level realized in the decentralized economy. To this end, we first consider a case where

$$\frac{1}{\beta} < 1 - \frac{\delta}{n},$$

which can be rewritten as

$$\beta > \frac{n}{n - \delta}.$$

This condition holds true in Figs. $2 - 7$. Moreover, Figs. $2 - 7$ show that the consumption levels associated with the optimal strategy ϕ^*, including (possibly a continuum of) other optimal strategies, in the decentralized economy are always larger than the Pareto-efficient one $(= 1/\beta)$; that is, overconsumption *always* occurs during the transition path as well as in the long-run equilibrium (i.e., $S = 0$).

Conversely, if the opposite inequality holds; that is, $\beta < n/(n - \delta)$, it holds true in Fig. 8 alone. As illustrated in Fig.8, the long-run consumption supported by *the MPE strategy ϕ^** (i.e., $S = 0$) in the decentralized economy should be realized within the range of

$$\left[\frac{\sqrt{1 + 4\beta(n - 1)(n - \delta)\frac{2n - 1}{n}} - 1}{2\beta(n - 1)}, 1 \right],$$

whose left-end point is given by the intersection of the non-invertible curve C_2 with the vertical axis; otherwise, *the MPE strategy ϕ^* will cross the curve C_2*. We compare the minimum long-run consumption level with the Pareto-efficient one $(= 1/\beta)$ thus yielding

$$\frac{\sqrt{1 + 4\beta(n - 1)(n - \delta)\frac{2n - 1}{n}} - 1}{2\beta(n - 1)} - \frac{1}{\beta} > 0 \text{ for } \beta \in \left[\frac{n^2}{(n - \delta)(2n - 1)}, \frac{n}{n - \delta} \right],$$

which reveals that the long-run consumption (corresponding to $S = 0$) supported by the strategy ϕ^* (including other optimal strategies) in the decentralized economy is also greater than the Pareto efficient level in Fig.8. Note, however, that when the strategy ϕ^* starts from initial values of S close to $1 - (\delta/n)$, earlier consumption levels are clearly less than the Pareto efficient one $(= 1/\beta)$.

In the presence of peculiar benefits from social capital the equilibrium path of consumption attained in the decentralized economy is given by (6). In this model the Pareto-efficient consumption $\hat{c}^P$ is characterized by

$$\hat{c}^P = \frac{1}{\beta + \lambda S} < \frac{1}{\beta}, \tag{31}$$

$$\dot{\lambda} - \rho\lambda = -[\gamma + \lambda(n(1 - c) - \delta)]. \tag{32}$$

28

It is clear that the presence of peculiar benefits from social capital, i.e., a positive value of γ, makes the Pareto-efficient consumption level smaller compared to the previous one c^P. Since social capital is accumulated up to the satiated steady state level in the decentralized economy when $\frac{n}{n-\delta} \geq \beta \geq \frac{n-(\gamma/\rho)}{n-\delta}$, the long-run consumption level ends up being equal to $c^{DE} = 1 - (\delta/n)$; consequently, it may or may not be greater than the Pareto-efficient consumption level $\hat{c}^P = 1/(\beta + \lambda S)$.

To sum up, we have the following lemma:

Proposition 6 *The long-run consumption level in the decentralized economy is greater than the Pareto-efficient one, while in the presence of peculiar preferences towards social capital the long-run consumption level associated with the unique MPE strategy in the decentralized economy may or may not be larger than the Pareto-efficient consumption level.*

5 Concluding Remarks

This paper proposes a novel formal framework to model social capital as an aggregate state variable that alleviates unsociable/sub-optimal behavior. By clarifying a concept that has been the subject of ambiguity, our analysis yields novel insights and advances on existing literature. To capture inefficiencies, the utility function exhibits a status motive (alternatively known as 'keeping up with the Joneses') that induces zero-sum behavior. But status is only an example of negative externalities; our framework is compatible with a broader set of unsociable (i.e., inefficient) behavior instead of status. In one version of our setup, we allow for direct personal satisfaction from social capital. We use dynamic non-cooperative game theory and model the formation of social capital as a Markov Nash equilibrium, allowing for dynamic strategic considerations among forward-looking individuals.

The first message of this paper is that completely rational behavior inevitably leads to the undesirable social capital poverty trap, because the negative externality arising from status-seeking preferences is strong enough to offset the positive externality arising from social capital accumulation. Social capital has public good characteristics, so it is likely to be underprovided because of incomplete collective internalization of the positive externality inherent in its formation, which causes an intertemporal free-riding behavior. As a result, the accumulation of social capital fails to deter inefficiently high consumption due to the

status-seeking behavior. Thus, repeated social interaction may not be enough to resolve free-rider problems and to deter opportunistic behavior; this is in sharp contrast to findings in the literature on repeated games. Furthermore, the magnitude of the positive externality generated by social capital accumulation plays a key role in determining whether the game may be either a Prisoner's Dilemma game or the Coordination game depending on how much benefit from coordination accrues to each individual.

The second message is that, for higher stocks of social capital to become possible, we have to enhance the attractiveness of social capital so as to dominate the negative externality arising from status-seeking preferences. On the other hand, if the domain is strictly restricted due to the existence of the satiated level of social capital, there may be a continuum of MPE strategies. However, regardless of whether there are multiple or a continuum of MPE strategies, in this case, our results indicate that zero social capital remains the only MPE strategy. From policy a perspective, governments should provide incentives by enhancing the direct utility flowing from social capital.

The model presented in this paper can be developed further in several directions. First, the weight function put on status-seeking preferences could be relaxed to allow for a whole domain of R_+; for example, negative exponential functions. Under this extended setting, it is questionable whether globally defined Markovian perfect equilibrium strategies may or may not exist, because it is highly likely that they will ultimately pass the non-invertible curve. Nevertheless, with help of Rowat's patching method, it might be possible to define globally defined strategies. Second, introducing asymmetric agents into the present model would enable us to compare the results of the present model with those of the static conflict model played by asymmetric agents. Such a model could allow for the emergence of the term representing status-seeking preferences in equilibrium because the ratio between individual consumption and average consumption is not equal to one and thus never vanishes even in equilibrium. This may potentially allow for a variety of MPE strategies and richer outcomes. Another interesting research agenda is to investigate non-Markovian equilibria supported by history-dependent strategies such as trigger ones in the present model, which may also support cooperative subgame perfect equilibria (see Benhabib and Radner, 1992). Fourth, Putnam (2000) has made a distinction between bridging social capital in which bonds of connectedness are formed across diverse social groups, and bonding social capital that cements

only homogenous groups. The endogenous choices of individuals would be informative on how much time they spend on enhancing closed networks, which contributes to the accumulation of *bonding* social capital, and to the formation of open networks which contributes to accumulation of *bridging* social capital.

Appendix A

Under the assumption of symmetry the first-order condition (6) becomes

$$\frac{1}{c(S)} - \beta + \frac{1}{c(S)}(1-\theta S)\frac{n-1}{n} - V'(S)S = 0. \tag{A.1}$$

Substituting (A.1) into (11) and further rearrangement yields

$$
\begin{aligned}
\rho V'(S) &= -\frac{1}{c(S)}(1-\theta S)\frac{n-1}{n}c'(S) + V''(S)\left[Sn(1-c(S)) - \delta S\right] \\
&\quad + V'(S)\left[n(1-c(S)) - S(n-1)c'(S) - \delta\right],
\end{aligned}
$$

$$
\left[\rho + \delta - n(1-c(S)) + S(n-1)c'(S)\right]V'(S) + \frac{c'(S)}{c(S)}(1-\theta S)\frac{n-1}{n} =
$$

$$
V''(S)\left[Sn(1-c(S)) - \delta S\right]. \tag{A.2}
$$

We once again differentiae (6) (i.e., $V_i'(S) = \frac{1}{c_i(S)S}\left[1 - \beta c_i(S) + (1-\theta_i S)\frac{\Sigma_{j=1,\ j\neq i}^{n} c_j(S)}{C(S)}\right]$) with respect to S and using $\Sigma_{j=1,\ j\neq i}^{n} c_j'(S)C(S) - \Sigma_{j=1,\ j\neq i}^{n} c_j(S)C'(S) = 0$ by symmetry, we obtain

$$
V''(S) = \frac{1}{c(S)S}\left[-\beta c'(S) - \theta\frac{n-1}{n} - \left(1 - \beta c(S) + (1-\theta S)\frac{n-1}{n}\right)\left(\frac{c'(S)}{c(S)} + S^{-1}\right)\right]. \tag{A.3}
$$

Substituting (A.1) and (A.3) into (A.2) yields

$$
\left[\rho + \delta - n(1-c(S)) + S(n-1)c'(S)\right]\frac{1}{c(S)S}\left[1 - \beta c(S) + (1-\theta S)\frac{n-1}{n}\right] =
$$

$$
+\frac{c'(S)}{c(S)}(1-\theta S)\frac{n-1}{n} = \tag{A.4}
$$

$$
\frac{1}{c(S)S}\left[-\beta c'(S) - \theta\frac{n-1}{n} - \left(1 - \beta c(S) + (1-\theta S)\frac{n-1}{n}\right)\left(\frac{c'(S)}{c(S)} + S^{-1}\right)\right].
$$

$$
\left[Sn(1-c(S)) - \delta S\right].
$$

31

Multiplying both sides by $c(S)S$ and collecting the terms related to $c'(S)$ on the left-hand side of (A.4) gives

$$c'(S) \left[\left(S(n-1) + \frac{Sn}{c(S)} - Sn - \frac{\delta S}{c(S)} \right) \left(1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right) \right.$$
$$\left. + S(1 - \theta S) \frac{n-1}{n} + \beta (Sn(1 - c(S)) - \delta S) \right] =$$
$$- \left[(\rho + \delta - n(1 - c(S))) + S^{-1} (Sn(1 - c(S)) - \delta S) \right] \left[1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right]$$
$$- \theta \frac{n-1}{n} [Sn(1 - c(S)) - \delta S].$$

After rearrangement,, we obtain

$$c'(S) \left[\frac{1}{c(S)} ((n - c(S)) S - \delta S) \left(1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right) \right.$$
$$\left. + S(1 - \theta S) \frac{n-1}{n} + \beta (Sn(1 - c(S)) - \delta S) \right] =$$
$$- \rho \left[1 - \beta c(S) + (1 - \theta S) \frac{n-1}{n} \right] - \theta \frac{n-1}{n} [Sn(1 - c(S)) - \delta S],$$

which gives (12).

Appendix B The proof of Lemma 1

Proof. When all individuals play $\phi = 1$, the HJB equation (5) becomes

$$\rho V(S) = -\beta + V'(S)(-\delta S). \tag{B.1}$$

Integrating (B.1) yields

$$V(S) = -\frac{\beta}{\rho} + A S^{-\frac{\rho}{\delta}}, \tag{B.2}$$

where A is an arbitrary constant of integration. In follows from (B.2) that when $A > 0$, $\lim_{S \to 0} V(S) = \infty$. This unbounded limit, together with the fact that $S(t)$ always converges from any initial value of $S \in [0, \infty)$ to zero, contradicts the bounded above instantaneous objective function in (1). On the other hand, the first-order condition (6) for the corner solution $\phi(S) = 1$ together with (B.2) yields

$$1 - \beta + (1 - \theta S) \frac{n-1}{n} > A \left(-\frac{\rho}{\delta} \right) S^{-\frac{\rho}{\delta}}, \tag{B.3}$$

which implies that as $S \to 0$, the right-hand side of (B.3) diverges toward plus infinity when $A < 0$. As a result, $A < 0$ and $\phi(0) = 1$ are contradictory. Taken together, the only candidate with $A = 0$ remains as a possible candidate. In this case, the corner strategy $\phi(S) = 1$ must be consistent with (B.3); consequently,

$$1 - \beta + (1 - \theta S)\frac{n-1}{n} > 0,$$

which can be rewritten as follows:

$$S < \frac{1}{\theta} + \frac{n(1-\beta)}{\theta(n-1)} < \frac{1}{\theta},$$

which completes the proof. ∎

References

Alonso-Carrera, J., Caballé, J., Raurich, X. (2004). Consumption externalities, habit formation and equilibrium efficiency. Scandinavian Journal of Economics, 106 (2), 231-251.

Andreoni, J. (1990) Impure altruism and donations to public goods: A theory of warm-glow giving. Economic Journal, 100, 464-477.

Alvarez-Cuadrado, Monteiro, F., G., Turnovsky, S. (2004). Habit formation, catching up with the Joneses, and economic growth. Journal of Economic Growth, 9 (1), 47-80.

Benhabib, J., Radner, R. (1992). The joint exploitation of a productive asset: A game-theoretic approach. Economic Theory, 2, 155–190.

Bowles, S., Gintis, H. (2002). Social capital and community governance. Economic Journal, 112 (483), 419–436.

Chou, Y. K. (2006). Three simple models of social capital and economic growth. Journal of Socio-Economics, 35, 889–912.

Coleman, J. S. (1990). The Foundations of Social Theory. Cambridge, Mass: Harvard University Press.

Dasgupta, P. (2003). Social capital and economic performance: Analytics. In: Ostrom, E., Ahn, T. (Eds.), Foundations of Social Capital. Edward Elgar, Cheltenham, 309–339.

Djankov, S., Glaeser, E., La Porta, R., Lopez-de-Silanes, F., Shleifer, A. (2003). The new comparative economics. Journal of Comparative Economics, 31, 595–619.

Dockner, E., Jorgensen, S., Long, N.V., Sorger, G. (2000). Differential Games in Economics and Management Science. Cambridge University Press.

Durlauf, S. N. (2002). On the Empirics of Social Capital. Economic Journal, 112, 459-479.

Durlauf, S.N., Fafchamps. M. (2005). Social capital. In: Handbook of Economic Growth, vol 1, North Holland, Amsterdam.

Fukuyama, F. (1997). Social capital. Tanner Lecture on Human Values.

Futagami, K., Shibata, A. (1998). Keeping one step ahead of the Joneses: Status, the distribution of wealth and long run growth. Journal of Economic Behavior and Organization, 36, 109-126.

Glaeser, E. L., D. Laibson, Sacerdote, B. (2002). An economic approach to social capital. Economic Journal, 112, 437-458.

Grootaert, C., van Bastelear, T. (Eds.) (2002). The role of social capital in development: An Empirical Assessment. Cambridge University Press, Cambridge.

Guiso, L., Sapienza, P., Zingales, L. (2004). The role of social capital in financial development. American Economic Review, 94 (3), 526–556.

Guiso, L., Sapienza, P., Zingales, L. (2012). Is social capital the missing link? *Handbook of Social Economics*, eds (Jess Benhabib, Alberto Bisisn, Matthew Jackson), (pp. 417-480), Elsevier. http://faculty.chicagobooth.edu/luigi.zingales/papers/research/Civic_Capital_as_the

Helliwell, J. F., Aknin, L.B., Shiplett, H., Huang, H., Wang, S. (2017). Social capital and prosocial behaviour as sources of well-being. NBER Working Paper No. 23761.

Itaya, J.-I. and C. Tsoukis (2018). Social capital and the status externality. CESifo Working Paper.

Knack, S., Keefer, P. (1997). Does social capital have an economic payoff: A cross country investigation, Quarterly Journal of Economics, 112, 1251-1288.

Knack, S., Zak, P.(1999). Trust and growth. Working Paper, Claremont University .

LaPorta, Lopez-de-Silanes, Shleifer, A., Vishny, R. (1999). The quality of government. Journal of Law, Economics and Organization, 15(1), 222-279.

Putnam, R., D. (1993). Making Democracy Work, Civic Traditions in Modern Italy. Princeton: Princeton University Press.

Putnam, R., D. (2000). Bowling Alone: The Collapse and Revival of American Community. New York: Simon and Schuster.

Romer, P. M. (1990). Endogenous technological change. Journal of Political Economy, 98 (5), 71-102.

Routledge, B.R., von Amsberg, J. (2002). Social capital and growth. Journal of Monetary Economics, 50 (1), 167-193.

Rowat, C. (2007). Nonlinear strategies in a linear quadratic differential game with bounded controls. Journal of Economic Dynamics and Control, 31, 3179-3202.

Rubio, S.J., Casino, B. (2002). A note on cooperative versus non-cooperative strategies in international pollution control. Resource and Energy Economics, 24, 251–261.

Tabellini, G. (2008). The scope of cooperation: Values and incentives. Quarterly Journal of Economics, 123, (3), 905–950.

Tsoukis, C. (2007). Keeping up with the Joneses, growth, and distribution. Scottish Journal of Political Economy, 54 (4), 575-600.

Tsoukis, C., Tournemaine, F. (2013). Status in a canonical macro model: Status, growth, and inequality. The Manchester School, 81 (2), 65–92.

Varvarigos, D., Xin, G. (2015). Social interactions, the evolution of trust, and economic growth. Working Paper No. 15/05, University of Leicester.