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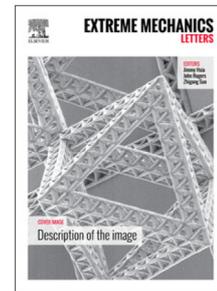
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Localized Necking of a Dielectric Membrane

Yibin Fu¹, Luis Dorfmann², Yuxin Xie³

¹Department of Mathematics
Keele University, Staffordshire ST5 5BG, UK

²Department of Civil and Environmental Engineering
Tufts University, MA, USA

³Department of Mechanics
Tianjin University, Tianjin 300072, China

Abstract

We first revisit the problem of localized bulging of an inflated rubber tube, and show that the associated bifurcation condition is equivalent to the vanishing of the determinant of the Hessian of the strain-energy density function when the latter is viewed as a function of the internal volume v and the axial stretch λ_z . This led us to conjecture and verify that when a dielectric plane membrane is subjected to the combined action of an electric field and in-plane mechanical stretching, localized necking will occur when the determinant of the Hessian of the free-energy function becomes zero. One situation in which this bifurcation condition is satisfied is when the nominal electric field as a function of the nominal electric displacement reaches a maximum under a dead load. This corrects the widespread mis-conception that when the nominal electric field reaches a maximum, a so-called pull-in instability would occur whereby the membrane thins down uniformly, leading to wrinkle formation or dielectric failure. We highlight the fact that the above bifurcation condition can be satisfied, and hence localized necking may occur, even if the nominal electric field does not have a maximum at all. This happens, for instance, when a rectangular membrane is uni-axially stretched and then has its four edges fixed before it is subjected to an electric field.

Keywords: Nonlinear electroelasticity, dielectric membranes, stability, bifurcation.

1. Introduction

Dielectric elastomers have received much attention in recent decades due to their use in high-performance actuators for a variety of applications [1, 2]. An important technical problem involving such materials is the loss of stability when a dielectric membrane coated with soft electrodes is subjected to in-plane stretching as well as an electric field. The three main strands of research addressing this problem are (i) studies based on the stability criterion that requires the Hessian of the total energy function to be positive definite [3, 4, 5, 6, 7], (ii) analysis of possible bifurcation of the uniformly deformed configuration into periodic patterns [8, 9, 10], and (iii) stability analysis based on approximate models [11, 12, 13]. The main aim of this paper is to correct two widespread misconceptions that have often appeared in the literature. The first one is that when the voltage reaches a maximum in uniform loading, corresponding to marginal violation of the Hessian stability criterion under dead mechanical loading, the membrane will thin down uniformly, leading sometimes to dielectric failure. This is commonly referred to as pull-in instability [14] in the literature. The second one is that since the Hessian stability criterion refers to stability with respect to spatially uniform perturbations, it only has limited applicability and so other stability criteria might be more relevant. We show that when the Hessian stability criterion is marginally violated, i.e. when the determinant of the Hessian matrix vanishes, localized necking will take place and will, in the absence of any constraint, rapidly evolve into a two-phase deformation, as sketched in Fig. 1. When deformation is restricted at the boundaries, the well-defined two-phase configuration in Fig. 1 will be replaced by wrinkles. The two-phase states have been observed experimentally [15], and explained theoretically and numerically [16, 17, 18, 19, 20], but the role played by necking has not been clarified although necking has previously been conjectured [21]. We emphasize that although necking is only a transient behaviour, understanding the role played by necking is more than of academic interest since the onset of necking is extremely sensitive to imperfections [22], which must be taken into account when comparing theoretical results with experimental data, or designing actuators that work on the verge of the voltage maximum [23, 24].

We note that the localized necking under investigation is different in character from the ‘charge localization instability’ examined in [25]. For the material model considered in [25] that also assumed zero in-plane stress, the voltage against the total charge has a maximum at Φ_c , say, attained when the charge is equal to Q_c . For each $\Phi < \Phi_c$, there correspond two values of Q , Q_L and Q_S say, and there is a certain value of Φ (Φ_0 say) such that $Q_L + Q_S = Q_c$. Based on this fact the authors argued that at $Q = Q_c$, the uniform state under voltage Φ_c can snap into a piecewise uniform state with lower voltage Φ_0 that is formed by joining two uniform states together, the two uniform states having total charges Q_L and Q_S , respectively, and different thicknesses. Thus, the localization considered in [25] corresponds to a finite jump in voltage, but stability of the resulting piecewise homogeneous deformation with respect to variations of the size of the localized region is not considered. In fact, the size of the localized region is undetermined in the analysis and assumed to be comparable with the membrane thickness.

We present our results in the following three sections. In Section 2 we reformulate the bifurcation condition for localized bulging and observe the analogy between the bulging problem and the current necking problem. The condition for localized necking is then stated. This bifurcation condition is verified in Section 3 for the case when an ideal dielectric plane membrane is subjected to uni-axial tension as well as an electric field. We also construct the necking solution explicitly in order to show that the bifurcation condition is not only necessary but also sufficient. The paper is concluded in Section 4 with additional remarks.

2. Reformulation of the bifurcation condition for localized bulging

We first reformulate the bifurcation condition for the localized bulging of a hyperelastic rubber tube that is subjected to the combined action of an internal pressure P and a net axial force T ; see Fig. 2. This localization phenomenon is now well-understood and we refer to [26] for a comprehensive review of the relevant literature. For instance, it is known that loss of stability is manifested by the sudden (dynamic) formation of a localized bulge, which quickly evolves into a stable two-phase state, the so-called propagation or Maxwell stage. The bifurcation condition for localized bulging is given by

$$\begin{vmatrix} \partial P / \partial \lambda_a & \partial P / \partial \lambda_z \\ \partial T / \partial \lambda_a & \partial T / \partial \lambda_z \end{vmatrix} = 0, \quad (1)$$

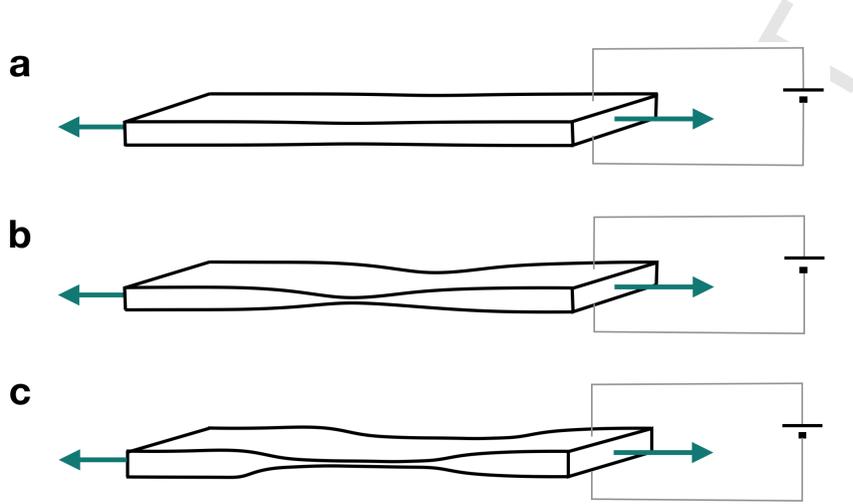


Figure 1: A schematic of necking of a dielectric membrane under plane-strain uni-axial tension and electricity actuation: a. Uniform deformation; b. Initiation of necking; c. Fully-developed necking in the absence of constraints.

where λ_a and λ_z are the azimuthal stretch on the inner surface and the axial stretch, respectively. This bifurcation condition is valid for tubes of arbitrary thickness. The vector (P, T) is not exactly conjugate to the deformation vector (λ_a, λ_z) , rather it is conjugate to (v, λ_z) , where $v = \lambda_a^2 \lambda_z$ is the ratio of deformed internal volume over the initial undeformed volume. However, in view of the relations

$$\frac{\partial}{\partial \lambda_a} = 2\lambda_a \lambda_z \frac{\partial}{\partial v}, \quad \frac{\partial}{\partial \lambda_z} = \frac{\partial}{\partial \lambda_z} + \lambda_a^2 \frac{\partial}{\partial v},$$

it can easily be seen that Eq. (1) is equivalent to

$$J(P, T) \equiv \begin{vmatrix} \partial P / \partial v & \partial P / \partial \lambda_z \\ \partial T / \partial v & \partial T / \partial \lambda_z \end{vmatrix} = 0, \quad (2)$$

where $J(P, T)$ denotes the Jacobian of J . This bifurcation condition thus has a clear physical interpretation: the condition is satisfied when locally the relation between force and deformation cannot be inverted uniquely. We observe that the bifurcation condition is independent of the conditions imposed at the ends of the tube, but whether localized bulging can take place or not does depend on the loading path, i.e. how inflation is carried out. If, for instance, one end is closed and is free of any constraints or dead weight, then $T = 0$ defines the loading path in the (λ, λ_z) -plane. It can be used to

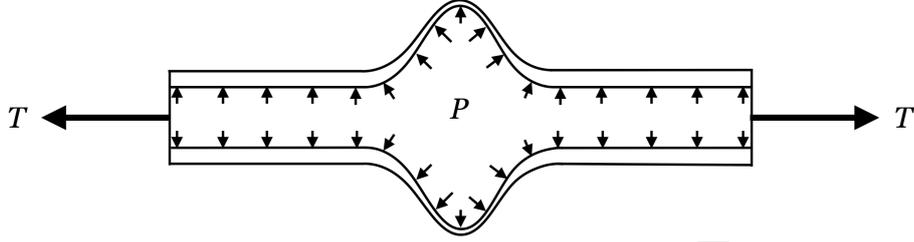


Figure 2: A schematic of localized bulging of an inflated membrane tube.

express λ_z in terms of λ , and then (2) reduces to $dP/dv = 0$, that is, the initiation pressure for localized bulging coincides with the maximum pressure in uniform inflation. The existence of such a maximum pressure is known as a *limiting point* instability [27, 28], and is the counterpart of the pull-in instability for the current electroelastic problem. Another less obvious result that follows from (2) is that if λ_z is fixed during inflation, which is the case for commonly used arterial models, localized bulging can occur even if P is a monotonic function of v (so that a limiting point instability does not exist).

To keep the mathematical complexity to a minimum, we now specialize to the case of a membrane tube. The total energy per unit length in the axial direction, is then given by

$$\Pi(v, \lambda_z) \equiv 2\pi R \tilde{w}(v, \lambda_z) - T(\lambda_z - 1) - P\pi R^2(v - 1), \quad (3)$$

where R is the averaged radius in the undeformed configuration, and $\tilde{w}(v, \lambda_z) = w(\sqrt{v/\lambda_z}, \lambda_z)$ with $w(\lambda, \lambda_z)$ denoting the membrane strain-energy per unit surface area in the undeformed configuration and λ the azimuthal stretch in the mid-surface.

Equilibrium requires $\partial\Pi/\partial v = 0$ and $\partial\Pi/\partial\lambda_z = 0$, from which we obtain

$$P = \frac{2}{R} \frac{\partial\tilde{w}}{\partial v}, \quad T = 2\pi R \frac{\partial\tilde{w}}{\partial\lambda_z}. \quad (4)$$

With the use of these expressions, the bifurcation condition (2) becomes $\det H = 0$, where H is the Hessian matrix of the strain-energy function defined by

$$H(v, \lambda_z) = \begin{bmatrix} \partial^2\tilde{w}/\partial v^2 & \partial^2\tilde{w}/\partial v\partial\lambda_z \\ \partial^2\tilde{w}/\partial v\partial\lambda_z & \partial^2\tilde{w}/\partial\lambda_z^2 \end{bmatrix}.$$

Thus, the bifurcation condition for localized bulging can be stated alternatively as the vanishing of the determinant of the Hessian of the strain-energy

function $\tilde{w}(v, \lambda_z)$. This led us to immediately conjecture that the bifurcation condition for necking in a dielectric membrane is that the determinant of the Hessian of the total energy function should vanish. The necking and bulging problems are mathematically similar although it has been much more difficult to observe necking than bulging. The positive definiteness of the Hessian matrix for a dielectric membrane was stated as a criterion for stability [3], but it has previously not been established that loss of stability will necessarily mean the onset of localized necking. As a matter of fact, the prevalent view is that since the Hessian stability criterion refers to stability with respect to homogeneous perturbations, its marginal violation would signal the onset of the ‘pull-in’ phenomenon, that is, the membrane would thin down uniformly, leading either to wrinkle formation or dielectric failure [29].

3. Bifurcation condition for localized necking

As outlined in [30], and elaborated in [31] and [32], the bifurcation condition for localization can be derived by examining the eigenvalue problem governing the incremental deformations based on the small-on-large approach. For a dielectric plate, the necessary incremental equations are presented in [8] where the bifurcation condition for a periodic buckling solution proportional to e^{ikx_1} is determined. Here $i = \sqrt{-1}$ and k denotes the mode number in the x_1 -direction. In the current context, a solution in the form of $e^{\alpha x_1}$ is preferred with α being the spectral parameter. For most of the problems in elasticity, $\alpha = 0$ is an eigenvalue due to the translational invariance in x_1 , and there also exist some real eigenvalues when there is no deformation or the deformation is sufficiently small. We denote the two real eigenvalues closest to zero by $\alpha = \pm\alpha_0$, and follow their movements as the load is increased. The general theory of Kirchgässner [30] states that under certain assumptions, a localized solution will bifurcate from the trivial solution when $\alpha_0 = 0$, that is when 0 becomes a triple eigenvalue.

Obviously, the above limit can also be obtained from the bifurcation condition derived in [8] by taking the limit $k \rightarrow 0$, which is the approach used in the subsequent analysis.

As an illustrative example, consider an ideal neo-Hookean dielectric membrane whose total energy, after scaling by the shear modulus, is given by

$$\Omega^*(\mathbf{F}, \mathbf{D}_l) = \frac{1}{2}(I_1 - 3) + \frac{1}{2}\varepsilon^{-1}I_5, \quad (5)$$

where ε is the permittivity scaled by the inverse of the ground state shear modulus, \mathbf{F} and \mathbf{D}_l are the deformation gradient and the Lagrangian electric displacement, respectively, and I_1 and I_5 are the two invariants defined by $I_1 = \text{tr } \mathbf{C}$, $I_5 = \mathbf{D}_l \cdot \mathbf{C} \mathbf{D}_l$, and $\mathbf{C} = \mathbf{F}^T \mathbf{F}$. Full details of the nonlinear theory of electroelasticity are given in [33, 34]. We choose a coordinate system with basis vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$, and the coordinate x_3 measuring distance from the midplane in the thickness direction. Consider the dielectric membrane in a state of plane strain subjected to an electric field in the thickness direction. The primary deformation, signified henceforth by an overbar, and the Lagrangian electric field have the forms

$$\bar{\mathbf{C}} = \lambda^2 \mathbf{e}_1 \otimes \mathbf{e}_1 + \mathbf{e}_2 \otimes \mathbf{e}_2 + \lambda^{-2} \mathbf{e}_3 \otimes \mathbf{e}_3, \quad \bar{\mathbf{E}}_l = E_{l3} \mathbf{e}_3,$$

where λ is the principal stretch in the x_1 -direction and E_{l3} the electric field component scaled by the shear modulus. We also define the Lagrangian and Eulerian electric displacements D_{l3} and D_3 , and Eulerian electric field E_3 (again scaled by the shear modulus). They are related by $E_3 = \lambda E_{l3}$ and $D_3 = \lambda^{-1} D_{l3}$. For an ideal dielectric the permittivity ε is constant and the electric displacement has the only nonzero component $D_3 = \varepsilon E_3$.

The total energy now reduces to

$$\omega(\lambda, D_{l3}) \equiv \frac{1}{2}(\lambda^2 + \lambda^{-2} - 2) + \frac{1}{2}\varepsilon^{-1}\lambda^{-2}D_{l3}^2,$$

from which we obtain

$$\begin{cases} s = \frac{\partial \omega}{\partial \lambda} = \lambda - \lambda^{-3} - \varepsilon^{-1}\lambda^{-3}D_{l3}^2, \\ E_{l3} = \frac{\partial \omega}{\partial D_{l3}} = \varepsilon^{-1}\lambda^{-2}D_{l3}, \end{cases} \quad (6)$$

where s denotes the nominal stress in the x_1 -direction.

Setting to zero the determinant of the Hessian matrix of ω yields the conjectured bifurcation condition

$$3 + \lambda^4 - \varepsilon^{-1}D_{l3}^2 = 0. \quad (7)$$

To verify its validity, we expand the exact bifurcation condition in [8] for small k , as indicated earlier. The incremental deformation is decomposed into extensional and flexural modes. Symmetry assumptions suggest that

modes relevant to localized necking are extensional. Then, the bifurcation condition for periodic extensional modes can be expanded as

$$3 + \lambda^4 - \varepsilon^{-1}D_{l3}^2 + \frac{2k^2}{3} \{10 + 9\lambda^2 + 7\lambda^4 + 3\lambda^6 + \lambda^8 - \varepsilon^{-1}D_{l3}^2(4 + 3\lambda^2 + \lambda^4)\} + O(k^4) = 0, \quad (8)$$

where we have taken the half membrane thickness to be unity, i.e. the half-thickness is used as length unit. Taking the limit $k \rightarrow 0$ in (8) then recovers the bifurcation condition (7).

As in the localized bulging problem, the bifurcation condition (7) is again independent of how the membrane is loaded. When the bifurcation and loading conditions are depicted by two graphs in the (λ, D_{l3}) -plane, localized necking will occur only if the two have an intersection.

Consider three different ways of loading. First, keep E_{l3} constant and vary the nominal stress s . It follows that $\partial s / \partial \lambda = \lambda^{-4}(3 + \lambda^4 - \varepsilon^{-1}D_{l3}^2)$, which vanishes when (7) is satisfied. Hence, the bifurcation condition for necking is satisfied precisely when s reaches its maximum. Alternatively, keep the nominal stress s constant and vary E_{l3} . Then, (6) defines λ as a function of D_{l3} , and implicit differentiation yields

$$\left(1 + 3\lambda^{-4} + \frac{3}{\varepsilon}\lambda^{-4}D_{l3}^2\right) \frac{d\lambda}{dD_{l3}} - \frac{2}{\varepsilon}\lambda^{-3}D_{l3} = 0,$$

which results in an expression for $d\lambda/dD_{l3}$. Using this and (6)₂ we then find that dE_{l3}/dD_{l3} is equal to

$$\left(1 + 3\lambda^{-4} + \frac{3}{\varepsilon}\lambda^{-4}D_{l3}^2\right)^{-1} \varepsilon^{-1}\lambda^{-6}(3 + \lambda^4 - \varepsilon^{-1}D_{l3}^2).$$

Therefore, the bifurcation condition (7) is satisfied when E_{l3} reaches its maximum.

Finally, consider the case where the membrane is first stretched and then, with λ held constant, an electric field is applied. In this case, $dE_{l3}/dD_{l3} = \varepsilon^{-1}\lambda^{-2} > 0$ and so E_{l3} is monotonic. Localized necking may still take place since the bifurcation condition (7) can be satisfied by

$$E_{l3} = \varepsilon^{-1/2}\lambda^{-2}\sqrt{3 + \lambda^4}.$$

We highlight the fact that in the purely mechanical case the bifurcation condition (7) cannot be satisfied by most material models, a notable exception being the Varga material [32]. The negative nature of the third term in

(7) implies that in the presence of an electric field, localized necking should be possible for all material models.

To demonstrate that the above bifurcation condition is not only necessary but also sufficient for localized necking to take place, we may apply the perturbation procedure explained in [32] to obtain the localized necking solution explicitly. Consider the case when E_{l3} is held fixed and λ is varied (or equivalently the nominal stress is varied). We choose this case because bifurcation takes place when the nominal stress reaches a maximum, and there is a clear geometrical and physical interpretation. We denote the solution of the bifurcation condition (7) by λ_{cr} , and consider variations of λ in a small neighborhood of λ_{cr} in the form

$$\lambda = \lambda_{\text{cr}} + \zeta \lambda_0, \quad (9)$$

where ζ is a positive small parameter and λ_0 is a constant. We expect that the solution structure is the same as in the purely mechanical case, i.e. the amplitude equation has the same universal form. Thus, to leading order, the components of the incremental displacement on the top surface $x_3 = 1$ associated with the necking solution are given by $u_1 = \zeta^{1/2} A(X)$, $u_3 = -\zeta A'(X)$, where $X = \zeta^{1/2} x_1$ is a stretched variable describing the lengthscale over which the near-critical necking solution varies, and the prime denotes differentiation. The amplitude function A satisfies an equation of the form

$$A'''' + c_1 \lambda_0 A' + c_2 A'^2 = 0, \quad (10)$$

where the constants c_1 and c_2 are determined as follows. Eq. (10) admits the homogeneous solution $A' = -c_1 \lambda_0 / c_2$, which corresponds to a total stretch $(\lambda_{\text{cr}} + \zeta \lambda_0)(1 + u_{1,1}) \approx (\lambda_{\text{cr}} + \zeta \lambda_0)(1 + \zeta A') \approx \lambda_{\text{cr}} + \zeta \lambda_0 + \lambda_{\text{cr}} \zeta A'$. Since this stretch and the one given by (9) correspond to the same nominal stress and the stress-stretch graph is locally parabolic near $\lambda = \lambda_{\text{cr}}$, the two stretches must lie on opposite sides of, and be equi-distance from, $\lambda = \lambda_{\text{cr}}$. Hence, $c_2 = \lambda_{\text{cr}} c_1 / 2$.

To determine the coefficient c_1 we substitute a sinusoidal wrinkling solution of the form $A = e^{ikx_1} = e^{ikX/\sqrt{\zeta}}$ into the linearized form of (10) to obtain $c_1 \lambda_0 \zeta = k^2$. On the other hand, by substituting (9) into (8) and keeping only leading-order terms, we obtain $3\lambda_{\text{cr}}^2 \lambda_0 \zeta = k^2$. Combining these two expressions then yields $c_1 = 3\lambda_{\text{cr}}^2$.

Hence, equation (10) has an explicit localized solution

$$A_X = -\frac{3\lambda_0}{\lambda_{\text{cr}}} \text{sech}^2\left(\frac{1}{2} \sqrt{-c_1 \lambda_0} X\right).$$

Since $c_1 > 0$, this solution exists only for $\lambda_0 < 0$, and thus the necking bifurcation is subcritical.

4. Conclusion

The inflation of a hyperelastic rubber tube and the electric actuation of an electrodes-coated dielectric membrane share many common features, and the associated mathematical problems are analogous to each other. This analogy has previously been exploited to explain the formation of multi-phase states in a plane dielectric membrane [16, 17, 18, 19, 20]. In this paper we have drawn upon more recent results concerning the initiation condition for localized bulging, and shown that when the Hessian of the free energy function ceases to be positive definite localized necking will take place first. The Hessian based stability criterion can be violated in a number of ways although almost all previous studies have focussed on the dead-load case for which marginal violation of the stability criterion corresponds to the electric field reaching a maximum. We have given one example in which the electric force does not have a maximum, but localized necking may still be possible. It is possible to manufacture dielectric materials (e.g. through fibre reinforcement) so that the electric field does not have a maximum even under a dead load. In this case localized necking will not occur under the dead load, but it may still take place if it is the stretch that is fixed. A more detailed explanation of this striking fact for the bulging problem can be found in [35].

The necking phenomenon is usually unstable and is quickly followed by a two-phase deformation when the deformation is not restricted, or wrinkle formation if the deformation is restricted, or even electric breakdown. In the presence of material and geometric imperfections, which are inevitable in practice, the critical stretch and/or electric field may be significantly less than the theoretical value associated with the perfect case because of the sub-critical nature of the initiation process. This fact should be taken into account when designing actuators that operate on the verge of the pull-in instability without electric breakdown [36, 37]. We have focussed on the simplest case of a plane dielectric membrane, and drawn upon the analogy between the current problem and the localized bulging problem. Interestingly, the two problems can also appear together in a single problem when a dielectric membrane tube is coated with electrodes on its inner and outer surfaces, and is subjected to the combined action of an internal pressure, axial force, and an electric field. Inevitably, such a tube would exhibit a large

variety of bifurcation behaviour [38, 39, 40]. It is hoped that our current result will help understand and classify the bifurcation behaviour of such more complex dielectric structures.

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