

1 Deep geothermal energy in northern 2 England: insights from 3D finite difference 3 temperature modelling

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5 Authorship statement

- 6 • Louis Howell: compiled metadata, constructed model, wrote manuscript
- 7 • Christopher Brown: helped construct model and write manuscript
- 8 • Stuart Egan: helped construct model and write manuscript

9 Code availability

10 Source code is available via [https://github.com/lphowell/Geothermal-](https://github.com/lphowell/Geothermal-Modelling/tree/master/Geothermal_NEngland)
11 [Modelling/tree/master/Geothermal_NEngland](https://github.com/lphowell/Geothermal-Modelling/tree/master/Geothermal_NEngland) or by contacting the lead author.

12 Highlights

- 13 • Subsurface temperature and heat flow maps for northern England are produced by
14 temperature modelling
- 15 • These maps are more resolute and geologically more realistic relative to equivalent
16 contoured maps for the UK
- 17 • Temperature models highlight ‘hot spots’ associated with granite intrusions and geological
18 structure
- 19 • This technique comprises a potentially useful tool for deep geothermal energy exploration

20 Key words

21 Geothermal Energy; Temperature; Finite Difference; Numerical Modelling

22 Abstract

23 Many of the most widely used deep geothermal resource maps for the UK are produced by
24 contouring around sparsely distributed and often unreliable data points. We thus present a
25 MATLAB-based 3D finite difference temperature modelling methodology, which provides a means
26 for producing more resolute and geologically realistic versions of these maps. Our case study area in
27 northern England represents an area where both sedimentary basins and radiothermal granite
28 bodies comprise potential geothermal resources. We divide our 3D model into geological units,
29 which are then assigned separate thermal properties. Assuming conductive heat transfer and
30 steady-state and fixed boundary conditions, we calculate 3D regional subsurface temperature. Due
31 to our averaging technique for thermal properties, the resolution of our geological model is scarcely
32 compromised with respect to similar finite element methods. One predicted ‘hot spot’ at 1 km depth
33 in the central part of our case study area corresponds with the granitic North Pennine Batholith.
34 Other shallow hot spots correspond with thermally insulating sedimentary rock units and geological
35 structures that incorporate these units. Predictive heat flow density maps highlight areas with
36 accelerated surface heat flow associated with shallow conductive basement rock and heat producing
37 granite bodies. Our predicted subsurface temperatures show broad similarities with measured

38 equilibrium borehole temperatures. Inaccuracies may relate to convective heat transfer involving
39 fault systems, or input variables relating to the geological model. Our predictive subsurface
40 temperature and heat flow density maps are more resolute and geologically realistic relative to pre-
41 existing contoured maps. The method presented here represents a useful tool for understanding
42 controls on subsurface temperature distribution and geothermal potential.

43 1. Introduction

44 Geothermal may provide one alternative energy resource as part of a worldwide effort to
45 reduce our reliance on fossil fuels and combat climate change (Zhang *et al.*, 2019). Nonetheless, the
46 UK lags its neighboring north-western European counterparts with regards to harnessing its deep
47 geothermal potential. This is reflected by the fewer number of geothermal boreholes drilled (Gluyas
48 *et al.*, 2018), the smaller contribution of geothermal towards the combined energy mix (BP Energy
49 Outlook, 2019), smaller research output, and the now somewhat outdated subsurface temperature
50 and heat flow maps for the UK (e.g. Downing and Gray, 1986a, 1986b; Lee *et al.*, 1987; Busby, 2010,
51 2014; Busby *et al.* 2011). These maps are commonly constructed by contouring around sparsely
52 distributed and sometimes unreliable data points (Rollin, 1995), rendering them often irresolute and
53 inaccurate (Fig. 1). Despite increasing interest in UK geothermal, as several recent and ongoing
54 projects testify to (Younger *et al.*, 2016; Adams *et al.*, 2019; Monaghan *et al.*, 2019; Paulillo *et al.*,
55 2020), the reliance on these quasi-resource maps remains a cause for concern.

56 Where data is either sparse or unreliable, predictive modelling may comprise a useful tool
57 (Pérez-Zárte *et al.*, 2019). Numerically based 3D regional subsurface temperature models help
58 communicate regional geothermal potential (e.g. Cacace *et al.*, 2010; Calcagno *et al.*, 2014; Fuchs
59 and Balling, 2016). Such models typically implement elaborate, but often complex and,
60 consequently, less reproducible finite element techniques (e.g. Cacace and Jacquy, 2017). Finite
61 difference analyses offer less computationally intensive alternatives to these methods. Although the
62 resolution and accuracy of finite difference models are limited by the typically rectangular nodal
63 arrangements of finite difference grids, for smaller problems, such as for the (<1 km) area around a
64 geothermal well head, a finite difference grid can be sufficiently scaled to compromise between
65 both model accuracy and rapid model convergence (e.g. Croucher *et al.*, 2020; Keller *et al.*, 2020).
66 Finite difference techniques are also adopted for subsurface temperature problems where the
67 geological uncertainty is greater than the model resolution, such as for the deep lithosphere and
68 mantle (e.g. Fullea *et al.*, 2009). However, for intermediate scale problems, such as for subsurface
69 temperature and heat flow density mapping (e.g. Fig. 1), a combination of the often inflexible finite
70 difference temperature grids, and the coarse model resolutions required to reduce run times, can
71 render such methods too inaccurate (*cf.* Gibson *et al.*, 2008).

72 We present an innovative 3D finite difference thermal modelling method that is used to
73 predict deep subsurface temperature and heat flow density in northern England. Due to our
74 averaging techniques for thermal conductivity and radiogenic heat production values, the resolution
75 of our geological model is effectively far greater than the temperature model's coarse nodal spacing.
76 Consequently, the accuracy of our model is not compromised to reduce computational intensity. We
77 document formulae and include MATLAB script with supplementary information for 3D steady-state
78 conductive heat transfer. Comparisons are made between results from our simulations and
79 measured borehole temperatures and heat flow densities. This technique represents some key
80 influences of complex geological structure on subsurface temperature distribution. Its main
81 strengths are its robustness, simplicity and reproducibility relative to more elaborate finite element
82 techniques. Compared to other finite difference techniques, our methodology offers more resolute

83 and geologically more realistic solutions. We present and discuss the UK's first deep 3D temperature
84 model and associated geothermal resource maps.

85 2. Study area: northern England

86 Our case study area comprises an area of the UK where both sedimentary basins and ancient
87 granite bodies comprise potential geothermal resources (Gluyas *et al.*, 2018). Together, it comprises
88 the northern part of the Lake District, the north-east of England and the Scottish borders (Fig. 2). The
89 primary energy demand for this region is roughly along the north-east coast and includes Newcastle-
90 upon-Tyne and Sunderland. Besides Carlisle, the remainder of our study area is amongst the most
91 sparsely populated areas of England. Ideally for the purposes of our study, this is an area that has
92 had widely documented but ultimately unsuccessful geothermal exploration (Gluyas *et al.*, 2018).

93 Despite the magnitude of recent investments in geothermal exploration in northern England
94 (Manning *et al.*, 2007; Hirst, 2012; Younger *et al.*, 2016), what we know about deep subsurface
95 temperatures and heat flow in the region is based upon somewhat outdated quasi-resource maps
96 (e.g. Downing and Gray, 1986a; Busby *et al.*, 2011) (Fig. 1). In our study area, for example, maps
97 depicting temperature at 1 km depth are based on contours around just six temperature data points
98 (Fig. 3). These data are situated predominantly within the Carboniferous basins of the region and
99 only two of these are equilibrium measurements (Burley *et al.*, 1984). On further inspection of these
100 maps and the UK Geothermal Catalogue (Burley *et al.*, 1984), heat flow density maps for this region
101 are based on contours around just 9 data points (Fig. 1b). Based on the type of conductivity and
102 temperature measurement, amongst other factors, Rollin (1995) graded the reliability of these data
103 with quality functions from 0 to 1, with 1 being good and 0 being poor. The highest grade awarded
104 for a data point in our study area was 0.65. Just five data points surpassed 0.25.

105 3. Data

106 A 3D subsurface geological model of northern England comprises the primary dataset of our
107 study (Fig. 4). A structural model of the Carboniferous-Permian basins of our study area is based on
108 the seismic interpretations of Chadwick *et al.* (1995) (*cf.* Terrington and Thorpe, 2013). The structure
109 of pre-Carboniferous basement bound Caledonian granites are based upon the gravity
110 interpretations of Kimbell *et al.* (2010). The bases of these granite intrusions are assumed flat at 9
111 km depth (*cf.* Kimbell *et al.*, 2010) (Fig. 4). Our geological model does not include the Cheviot
112 granites or other granites along the Southern Uplands, which are located beyond the northern
113 margin of our study area. Our model's geological boundaries are derived from several 2D nodal grids
114 of elevation values based on these metadata. The boundaries that are derived from these data are
115 extrapolated to fill a 110 km by 150 km grid. The coordinates at which elevation values are given
116 each correspond to separate nodes within our temperature grid and are uniformly spaced.

117 The geological boundaries within our model separate geological units, which are assigned
118 distinct thermal properties (Table 1). Thermal conductivity of the crust is a function of temperature
119 and pressure, as well as composition (Norden *et al.*, 2020); therefore, conductivity of middle-lower
120 crustal rock decreases with depth (and temperature). Thermal properties for basement rock and
121 basin fill are based on numerous literary sources (e.g. Čermác and Rybach, 1982; Downing and Gray,
122 1986; Norden and Förster, 2006; Manning *et al.*, 2007; Norden *et al.*, 2008; Vilá *et al.*, 2010; Younger
123 *et al.*, 2016; Busby, 2019). Borehole temperatures for comparison with our modelled subsurface
124 temperature grid are derived from the UK Geothermal Catalogue (Burley *et al.*, 1984) and published
125 literature (e.g. Younger *et al.*, 2016). Typically, finite difference techniques dictate that the thermal
126 property matrices within temperature models are divided into a series of variably sized cuboids, the

127 volume of which are defined by the temperature grids nodal spacing (e.g. Fulla *et al.*, 2009).
128 However, in Section 4.3 we detail how more geologically realistic thermal property matrices may be
129 derived from a geological model, whilst still implementing a less computationally intensive finite
130 difference methodology and coarse nodal spacing.

131 4. Methods

132 A summary of our modelling approach is illustrated in Figure 5. These methods may be
133 amended depending on the characteristics of geological models or the specifications of subsurface
134 temperature models, although the crux of this technique may remain unchanged. We recommend
135 that the meshing process is treated separately from temperature simulation, to reduce memory
136 drainage and ultimately reduce temperature convergence times.

137 4.1 Governing equations

138 To calculate subsurface temperature, we solve a steady-state conductive heat equation, or
139 diffusion equation according to Fourier's law. The diffusion equation operates on the basis of energy
140 conservation and relates heat flow (q) to temperature gradients (∇T). In its differential form, it can
141 be given as:

$$142 \quad q = -k \nabla T$$

143 (Eq. 1)

144 where k is the bulk rock thermal conductivity tensor. Temperature change experienced by each
145 node within the temperature grid is equal to the heat conducted into or out of a node, plus
146 radiogenic heat production (Q). Thus the following relationship between change in heat flow (∇q)
147 and time (t) can be determined:

$$148 \quad (\rho c) \frac{\partial T}{\partial t} = -\nabla q + Q$$

149 (Eq. 2)

150 where ρ is the bulk rock density and c is the bulk specific heat capacity. When Equation 1 is
151 substituted into Equation 2, the equation for transient diffusion is given:

$$152 \quad (\rho c) \frac{\partial T}{\partial t} = \nabla(k \nabla T) + Q$$

153 (Eq. 3)

154 Under steady-state conditions, any transient effect is neglected. Therefore, the equation can be
155 rearranged further as thus:

$$156 \quad \nabla(k \nabla T) = -Q$$

157 (Eq. 4)

158 This equation is solved for the temperature using a 3D implementation of the finite difference
159 methodology with algorithms developed using the MATLAB (Mathworks) numerical computing
160 environment.

161 4.2 Boundary conditions and model validation

162 The solution to Equation 4 using the finite difference method requires definition of
163 boundary conditions. For subsurface thermal modelling, we adopt an upper boundary (surface)

164 temperature of 10 °C, in concurrence with UK annual mean average air temperature (Busby *et al.*,
165 2009). The lower boundary temperature at the base of our model represents a more irreconcilable
166 problem. The base of the lithosphere is at a depth of approximately 125 km beneath much of north-
167 western Europe and is represented by the 1333 °C isotherm (Sclater and Christie, 1980).

168 To validate the differential solution against an analytical solution in one-dimension and
169 determine the likely lithosphere-scale geothermal structure of our case study area, we reiterate the
170 linear equation until an asymptotic solution, our modelled geothermal gradient, is reached (Fig. 6).
171 When adopting a uniform grid spacing of 1 km, the modelled geothermal gradient approaches its
172 steady state solution after approximately 10,000 iterations. To reduce convergence time, the
173 temperature matrix can be populated with a pre-defined temperature distribution (e.g. Bayer *et al.*,
174 1997) or be thermally conditioned using temperatures from previous model simulations. Besides
175 boundary temperatures, thermal conductivity has a primary control on the geothermal gradient. The
176 decreased geothermal gradient with depth, after 30 km, reflects the increased thermal conductivity
177 of mantle rock relative to crustal rock below the Moho boundary (e.g. Čermác and Rybach, 1982)
178 (Table 1). With the addition of radiogenic heat production, the modelled geothermal gradient forms
179 a convex upwards curve.

180 The lateral boundaries of our 3D model, in the x and y directions, are closed. Thus $\delta T/\delta x =$
181 0 , and $\delta T/\delta y = 0$. This implies no heat is transferred beyond the lateral boundaries of the model and
182 that these boundaries represent surfaces of symmetry. Neither of these assumptions fit reality but
183 they provide approximations for complex geological structures. To reduce the potentially
184 detrimental effects of these boundaries, a wide aspect model ratio is necessary. Increasing the
185 dimensions of the temperature model to three decreases convergence time by the nodal widths of
186 the model in both the x and y directions, by 150 km and 110 km respectively for our model of
187 northern England. To reduce computational intensity, therefore, we adopt a shallow lower boundary
188 condition of 665.6 °C at 30 km depth, in concurrence with results from our one-dimensional
189 lithosphere-scale model (Fig. 6), and assume the resolution of our model in terms of node spacing
190 within the temperature grid is 500 m.

191 4.3 Approximation of geological model

192 The shortcomings of a finite difference model relate to its inflexibility. In implementing a
193 finite difference methodology, the value for radiogenic heat production of a single node comprises
194 heat production for the entire cubic rock volume for which that node represents. Likewise, for
195 thermal conductivity, one value calculated between two adjacent nodes represents the combined
196 conductivity for that transect of rock, which is 500 m long in this instance. Where the modelled rock
197 volume is structurally complex or characteristically heterogeneous, therefore, thermal properties for
198 individual temperature nodes may be misrepresentative, rendering the temperature model
199 inaccurate. These issues are exacerbated when coarse model resolutions are necessary, as they are
200 here. We thus demonstrate how more representative 3D thermal property matrices may be derived
201 from structurally complex geological models.

202 Thermal properties for distinct points within the bounds of our 3D temperature model
203 reflect the corresponding depths of those points at specific x and y coordinates relative to the
204 depths of geological boundaries in a geological model. Depending on the preassigned distance
205 between temperature nodes (∇i), the corresponding depth of a temperature node in a geological
206 model is determined by:

$$207 \text{depth} = (z - 1) \nabla i$$

208 (Eq. 5)

209 Where z is a reference to the depth corresponding to the position of a given node within the
210 temperature matrix.

211 Geological boundaries separate the numerous units of our geological model, which are assigned a
212 series of distinct thermal properties (Table 1). So that we may avoid removing any of our geological
213 model that is situated above sea level, the depths of geological horizons are given relative to surface
214 elevation.

215 4.3.1 Thermal conductivity matrices

216 We overcome resolution issues for thermal conductivity tensors between adjacent
217 temperature nodes, i.e. $k_{i+1/2}$ and $k_{i-1/2}$, by finding the harmonic mean (Hantschel and Kauerauf,
218 2009) of multiple thermal conductivity values at uniformly spaced points between the respective
219 nodes. Depending on the interval spacing resolution (res) of sampled k points relative to
220 temperature node spacing (∇i), the distance between these sampling points (ss) is determined as:

$$221 \quad ss = \nabla i / res$$

222 (Eq. 6)

223 We adopt a resolution 50 times that of our temperature node spacing so that $ss = 10$ m.

224 For each node within our temperature matrix there are references to depths of geological
225 boundaries at corresponding x and y coordinates of our geological model. The precision of these
226 depth values is not fixed to the resolution of our temperature model. Therefore, determining
227 thermal conductivity values for distinct points at x and y coordinates between vertically adjacent
228 temperature nodes based on their corresponding depths within a geological model is
229 uncomplicated. However, as inputted spatial data for geological boundaries are limited to the x and
230 y coordinates of our temperature matrix, we may not apply this exact method to determine more
231 representative thermal conductivity tensors laterally in between temperature nodes. To avoid
232 inputting finer and more computationally intensive spatial data for geological boundaries, we
233 interpolate depths of geological boundaries between laterally adjacent temperature nodes. These
234 interpolated depths are used as a basis for determining k values in between laterally adjacent
235 temperature nodes. The harmonic mean of these values may then be determined.

236 4.3.2 Radiogenic heat production matrices

237 Poor resolutions for Q value matrices are not as detrimental to the accuracy of predictive
238 subsurface temperature models as k value matrices. Nonetheless, more representative matrices of
239 Q values may be attained by adopting similar approaches to those just described for thermal
240 conductivity. We determine Q values for multiple points up to half the temperature node spacing
241 away from a given temperature node in the x , y and z directions, which is 250 m in this instance. We
242 manage this by adopting the same technique for determining k values at points in between
243 temperature nodes in the z direction, and the x and y directions respectively. The arithmetic mean
244 of these values is then determined (Hantschel and Kauerauf, 2009).

245 Figure 7 illustrates the benefit of deriving more accurate thermal property matrices from
246 geological models in this way. Compared with finding the harmonic mean between just two
247 conductivity values at points corresponding to adjacent temperature nodes, our more accurate
248 thermal conductivity matrix is smoother. Sharp lateral conductivity changes correspond only to
249 steeply dipping beds or fault offsets in this more accurate scenario (Fig. 7a), rather than also

250 shallowly dipping beds or the variable dips of beds with vertical thicknesses less than our
251 temperature node spacing (Fig. 7b).

252 5. 3D temperature simulation

253 Our 3D subsurface temperature model reflects the controls of geological structure on
254 vertical and lateral heat transfer and heat production. Temperatures calculated at depths of less
255 than approximately 5 km are influenced by a combination of sedimentary basin fill and heat
256 producing granite intrusions within the basement. At depths greater than 5 km, the basement has a
257 predominant control on temperature distribution. We ignore parts of our model that are less than
258 10 km away from the lateral boundaries that are more strongly influenced by boundary conditions.

259 5.1 Predicted shallow subsurface temperatures

260 The dominant 'hot spots' at 1 km depth are situated upon the central part of the Alston
261 Block (Fig. 2a), the northern part of the Solway Syncline, the southern part of the Bewcastle
262 Anticline, along the Vale of Eden and along the eastern margins of the Alston Block, and the
263 Stainmore Trough (Fig. 8a). The modelled hot spot at 1 km depth on the central part of the Alston
264 Block, where temperatures reach 46 °C, correlates strongly with the North Pennine Batholith (Fig.
265 2b). However, the absence of any such hot spot in the Lake District, which is underpinned by the
266 Lake District Batholith, at 1 km depth suggests that other factors influence this particular hot spot.
267 We suggest that elevated temperatures on the Alston Block are influenced also by the local, variably
268 thick, and comparatively insulating Carboniferous cover (*cf.* Bott *et al.*, 1972) (Fig. 4). This cover
269 thickens towards the east and incorporates progressively younger and more insulating coal-bearing
270 strata. These trends may account for the preservation of greater heat at 1 km depth towards the
271 vertically adjacent eastern margin of the heat producing North Pennine Batholith, despite the
272 eastwards thinning of this structure here (Kimbell *et al.*, 2010).

273 Owing to the comparatively thick and thermally insulating sedimentary fill preserved in the
274 Vale of Eden Basin and lateral heat transfer from the radiothermal Lake District and North Pennine
275 batholiths, our 3D subsurface temperature model predicts elevated temperatures at 1 km in this
276 region, up to 43 °C (Fig. 8a). The parallel, NNE-SSW orientated Solway Syncline and Bewcastle
277 Anticline provide more interesting thermal anomalies at 1 km depth. The northern part of the
278 Solway Syncline, is comparatively hot at 1 km depth, up to 43 °C. Towards the south where this
279 structure plunges, modelled temperatures at 1 km decrease to less than 39 °C. Conversely, the
280 northern part of the Bewcastle Anticline is coolest, less than 37 °C, where thermally conductive pre-
281 Carboniferous basement rock is shallowest. Where this structure also plunges to the south and
282 preserves progressively thicker and younger insulating Carboniferous strata, temperatures increase
283 up to 43 °C. Some of these thermal trends may be explained by the non-uniform presence and
284 comparative thicknesses of coal-bearing and thermally insulating strata in this part of the
285 Northumberland-Solway Basin. Some other thermal trends, however, may instead be explained by
286 the vertical distributions of variably conductive rock units within the subsurface and the effects of
287 these distributions on geothermal gradients at different depths. Transitioning from relatively
288 insulating to conducting rock units with depth results in a decreased geothermal gradient with
289 depth. The opposite arrangement results in an increased geothermal gradient with depth. Because
290 the thermally insulating Pennine Coal Measures Group is at depths greater than 2 km to the south of
291 the Solway Syncline, towards where the fold plunges, the geothermal gradient at these depths here
292 is greater. Resulting temperatures at shallower depths, 1 km depth, are less. In contrast, in the
293 northern part of the Solway Syncline, the thermally insulating Coal Measures are at depths between

294 0.5 and 2 km. As a result, the geothermal gradient is steepest at these depths and temperatures at 1
295 km are comparatively elevated.

296 5.2 Predicted deep subsurface temperatures

297 Maximum vertical sedimentary basin thickness in our study area is approximately 8 km.
298 Around these depths, little is known about the characteristics of basin fill (*cf.* Chadwick *et al.*, 1995)
299 so differentiating thermal properties is difficult. The two main hot spots for these depths are
300 associated with the radiothermal Lake District and North Pennine batholiths, where temperatures
301 reach up to 154 °C (Fig. 8c). Faintly elevated temperatures at 5 km depth (Fig. 8b) are associated
302 with the Solway Syncline and the eastwards thickening of Carboniferous strata within the northern
303 Pennine Basin. At 7 km depth, elevated temperatures associated with the Solway Syncline are
304 diminished further, as the modelled geotherm equilibrates laterally as it approaches the lower
305 boundary condition (Fig. 8c). Slight local temperature elevations may be associated with the greater
306 thicknesses of Carboniferous strata towards the east of our study area, up to 190 °C. At these
307 depths, however, any other sources of localized temperature anomalies are dwarfed by comparison
308 with anomalies due to the Lake District and North Pennine batholiths.

309 5.3 Predicted isotherm depth

310 By cubically interpolating vertically between temperature nodes, we determine depth to the
311 100 °C isotherm across our study area. Depth to this temperature boundary varies between
312 approximately 2.87 km and 3.51 km below surface in our study area (Fig. 9). The modelled isotherm
313 is shallowest in the Lake District, although boundary conditions may exaggerate these shallow
314 depths. The isotherm is also shallower than 3 km in the Alston Block, in the centre of our study area
315 and towards Newcastle-upon-Tyne, suggesting that the two radiothermal granite intrusions of our
316 study area strongly influence these depths. Markedly shallower depths, between approximately 3
317 km and 3.2 km below surface, for the isotherm are also predicted for the Solway Basin, the Vale of
318 Eden Basin and the eastern part of our study area. In these areas, comparatively thick Pennine Coal
319 Measures Group successions are preserved. The greatest depths to the 100 °C isotherm are
320 predicted in the western and central parts of the Northumberland Basin and in the Southern
321 Uplands.

322 5.4 Predicted heat flow

323 We solve the heat flow equation (Eq. 1), using the modelled temperature difference (∇T)
324 and vertical thermal conductivity (k) (e.g. Fig. 7) between temperature nodes at surface and 500 m
325 below surface, to determine surface heat flow density (Fig. 10). Because the heat flow equation
326 integrates thermal conductivity and temperature gradient, areas where predicted heat flow is
327 comparatively elevated with respect to the remainder of our study area do not perfectly conform to
328 subsurface temperature 'hot spots' (Fig. 8). Instead, areas with elevated surface heat flow density
329 correspond to regions where shallow subsurface temperatures and bedrock conductivity are high,
330 such as on the central and eastern parts of the Alston Block and the Lake District. In these areas,
331 predicted surface heat flow exceeds 90 mW m⁻². Predicted heat flow in our case study area is more
332 strictly aligned to depositional settings during early Carboniferous rifting (e.g. Howell *et al.*, 2019)
333 than subsurface temperature. Comparatively uplifted pre-Carboniferous basement blocks have
334 overall greater heat flow whereas deeper basins, which were typically infilled by thermally insulating
335 sedimentary rock, have overall lower heat flow.

336 6. Model verification

337 To demonstrate the accuracy of our subsurface temperature model, we compare our
338 predictions against results from previous studies, including resource maps based on contouring
339 methods (e.g. Fig. 1), and measured equilibrium borehole temperatures from our case study area.
340 We also consider variations between results from our thermal model and temperature
341 measurements that may not be resolved by adopting our predictive modelling technique.

342 6.1 Comparisons of modelled and measured subsurface temperature data

343 Overall, there is a wide dispersion of temperatures of temperatures at 1 km depth in our
344 study area (Fig. 11a). Our mean modelled temperature at 1 km depth of 41.36 °C indicates an
345 average shallow geothermal gradient of 31.36 °C km⁻¹, which is slightly greater than the UK average
346 of 28 °C km⁻¹, although our study area is widely considered to be geothermally hotter than much of
347 the rest of the UK (Busby *et al.*, 2011). There are broad similarities between the distributions of
348 modelled hot and cold temperature anomalies (Fig. 8) and predicted anomalies based on contouring
349 (Fig. 3).

350 Equilibrium borehole temperature measurements effectively remove drilling induced
351 transient temperature effects (Oxburgh *et al.*, 1972). Analyzing these data, when possible, should be
352 considered an integral part of verifying predictive temperature models. Our predicted subsurface
353 temperatures show strong similarities with measured temperatures from the Rookhope Borehole
354 (Fig. 11d), which are described in detail by Bott *et al.* (1972). In particular, the decreased geothermal
355 gradient after approximately 450 m depth below surface is well reproduced by our modelling
356 methodology. This depth corresponds to the top (Caledonian) basement unconformity, which locally
357 separates overlying and comparatively thermally insulating Carboniferous sediments from the more
358 conductive and radiogenic North Pennine Batholith.

359 There are stronger dissimilarities between our predicted subsurface temperatures and
360 measured equilibrium temperatures from the Newcastle Science Central Deep Geothermal Borehole
361 (Younger *et al.*, 2016) (Fig. 11e). The implementation of our modelling methodology under-predicts
362 the temperature gradient with respect to measured temperatures in this region. This under-
363 prediction could perceptibly be attributed to the spatial variability of thermal properties (*cf.* Fuchs *et al.*,
364 2020), or to the Ninety Fathom and Stublick fault system, which cuts across this region as well as
365 geothermally hotter regions to the west (Fig. 2a). If these faults behave as non-sealing conduits, they
366 may facilitate accelerated heat fluxes via fluid convection (*cf.* Calcagno *et al.*, 2014).

367 The greatest disconnect between predicted and measured equilibrium temperature is
368 associated with the youngest and most scarcely preserved Carboniferous sediments of our study
369 area that are encountered in the Becklees borehole (*cf.* Jones *et al.*, 2011) (Fig. 11f). Like
370 temperatures in the Becklees borehole, our predicted geothermal gradient steepens between 500
371 and 1000 m depth below surface. For predicted subsurface temperatures, this is due to the presence
372 of thermally insulating Pennine Coal Measures Group stratigraphy within our geological model
373 between these depths (Chadwick *et al.*, 1995) (Fig. 4). Instead of encountering a thick succession
374 solely of this insulating rock unit, however, the Becklees borehole encounters approximately 600 m
375 of sandstone-rich and variably porous sedimentary rock belonging to the Warwickshire Group,
376 overlaying an approximately 500 m thick succession of the Pennine Coal Measures Group (Jones *et al.*,
377 2011) (Fig. 12). These overlaying units are likely to be more conductive due to their compositions
378 (e.g. Rybach, 1981) and may provide high permeability pathways for heat convection. Modelled
379 subsurface temperatures may be over-predicted with respect to measured temperatures in the
380 Becklees borehole as a result (Fig. 11f). However, as most of the remainder of Carboniferous

381 sediments in northern England are typically tight (e.g. Younger *et al.*, 2016), we choose to
382 acknowledge these sources of inaccuracy and maintain our simplistic, yet more robust, modelling
383 approach.

384 6.2 Comparisons of modelled and measured heat flow density data

385 Contoured heat flow density maps provide more precise constraints for our temperature
386 model, given the greater density of heat flow data in our case study area (Fig. 1b). The two bullseyes
387 over the Lake District and Alston Block, where heat flow is locally greater than 90 mW m^{-2} , are
388 broadly replicated, as are the lower heat flows in the Northumberland-Solway Basin and Stainmore
389 Trough (Fig. 10). Our temperature simulations offer greater resolution compared with these
390 contoured resource maps. Figure 11d shows a cross-plot for measured heat flow data and modelled
391 data taken from equivalent locations. Overall, there is a positive correlation, suggesting that our
392 modelling technique successfully replicates areas of greater heat flow density. However, the
393 dispersion of modelled heat flow density data falls short of equivalent measured data (also see Fig.
394 11b). This is indicated by the shallow cross-plot gradient of 0.2 (Fig. 11b).

395 At these shallow (<500 m) depths, modelled heat flow inaccuracies could perceptibly be
396 attributed to the neglected influences of superficial deposits, given that in northern England, many
397 heat flow measurements were recorded in the shallowest tens of metres of the subsurface (Burley
398 *et al.*, 1984), and that superficial cover thicknesses locally exceed 60 m (McMillan, 2011). Whilst
399 neglecting the influences of superficial cover has not had a noticeably detrimental effect on
400 subsurface temperature predictions (e.g. Figs. 8, 11d, e and f), their admission appears to have more
401 negatively impacted the dispersion of surface heat flow density data (Fig. 11c), because these data
402 are more directly proportional to the thermal conductivity of the shallow subsurface (Eq. 1). In
403 temperate regions of the world, including northern England, transient temperature effects relating
404 to palaeoclimate are proven to also have detrimental effects on shallow heat flow density
405 predictions (e.g. Slagstad *et al.*, 2009; Majorowicz *et al.*, 2012). A steady-state subsurface
406 temperature model is, by definition, incapable of accounting for these effects; although a simplistic
407 alternation to the temperature model's top boundary condition following temperature convergence,
408 and repeated model iterations, would effectively replicate this transient effect. A surface heat flow
409 over-estimation would be anticipated had the effects of transient climate adjustment had a
410 detrimental effect on modelled heat flow data (Majorowicz *et al.*, 2012). Nonetheless, a comparison
411 between modelled and measured heat flow density data suggests no consistent over-estimation (Fig.
412 11e).

413 7. Discussion and conclusions

414 Predictive subsurface temperature and heat flow density maps can be extracted from our
415 finite difference models (Figs. 8, 9 and 10) that are more resolute and geologically realistic compared
416 to maps constructed by contouring around sparsely distributed and often unreliable data points (Fig.
417 1). Due to our averaging technique, the resolution of our geological model is scarcely compromised
418 to reduce computational intensity. Its main strengths are its robustness, simplicity, and
419 reproducibility relative to more elaborate finite element techniques (e.g. Cacace and Jacquy, 2017).
420 Compared to other finite difference techniques (e.g. Fullea *et al.*, 2009; Keller *et al.*, 2020), our
421 methodology offers more resolute, geologically more realistic, and quicker solutions for regional
422 scale (>10 km) problems such subsurface temperature and heat flow density mapping. The main
423 inaccuracies of our model in northern England relate to geological inputs, such as bedrock and
424 superficial cover. Fuchs and Balling (2016) and Fuchs *et al.* (2020) discuss the importance of
425 geological constraints and their regional variability for subsurface temperature models such as

426 these. Other inaccuracies may relate to fluid convection. When deemed necessary and where data
427 constraints are sufficient, the incorporation of fluid convection through rock units within
428 temperature calculations may comprise a simple upgrade on these methods. However, to predict
429 the influences of more complex structures, such as permeable fault zones, on subsurface
430 temperature, more elaborate methods and finer resolution models may be necessary (*cf.* Calcagno
431 *et al.*, 2014). The method presented here represents a useful tool for understanding controls on
432 subsurface temperature distribution and geothermal potential. MATLAB scripts and program files for
433 our northern England temperature model are included within the supplementary information.

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438 Fund and the National Productivity Investment Fund (NPIF) whose support is gratefully
439 acknowledged.

440 Our 3D geological model was manipulated using Petrel (Schlumberger) software. Our temperature
441 modelling technique is supported by the MATLAB (Mathworks) numerical computing environment.

442 Conflict of interest

443 The authors declare no conflict of interest.

444 Computer code availability

445 Name of code: Geothermal-Modelling

446 Developer: Louis Howell (l.p.howell@keele.ac.uk)

447 Year first available: 2020.

448 Hardware required: Our temperature modelling technique is supported by the MATLAB (Mathworks)
449 numerical computing environment.

450 Program language: MATLAB.

451 Program size: 30 MB (including geological model).

452 Source code: <https://github.com/lphowell/Geothermal-Modelling>

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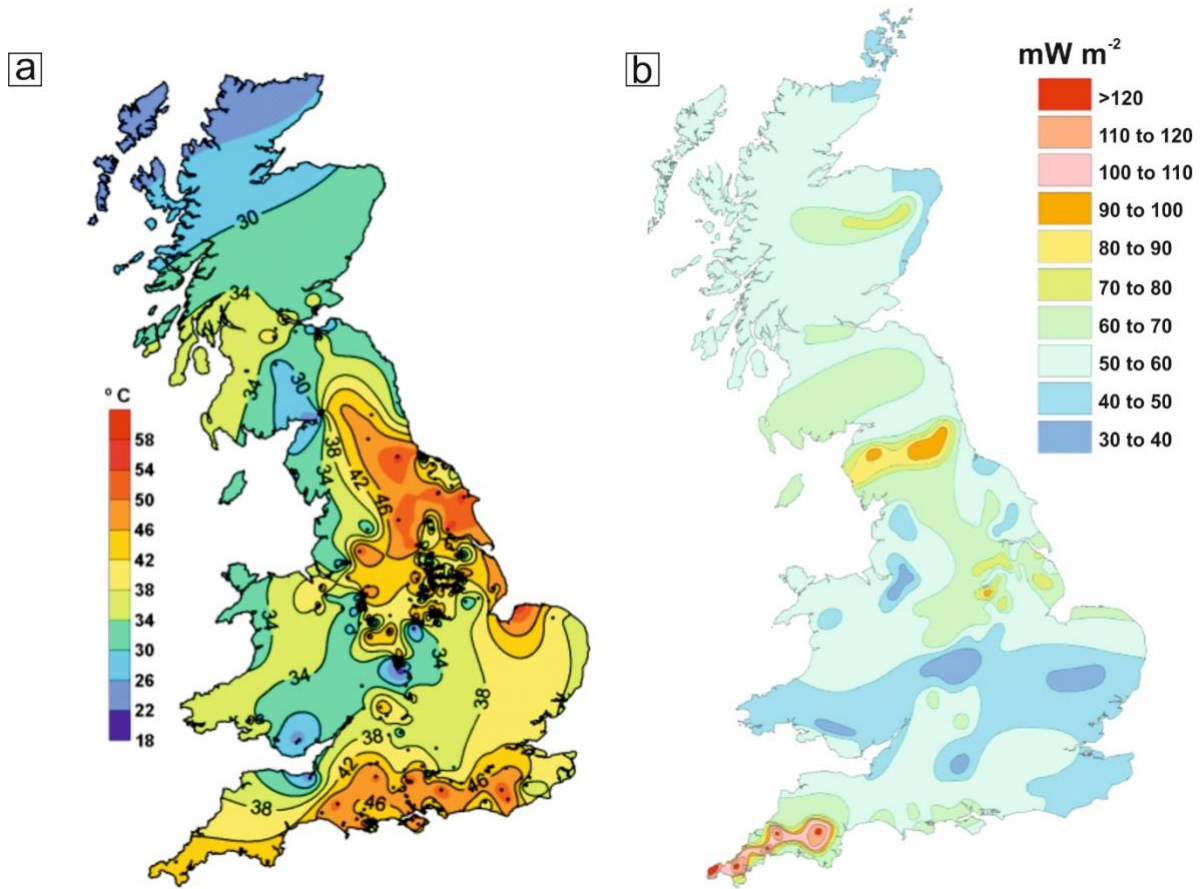
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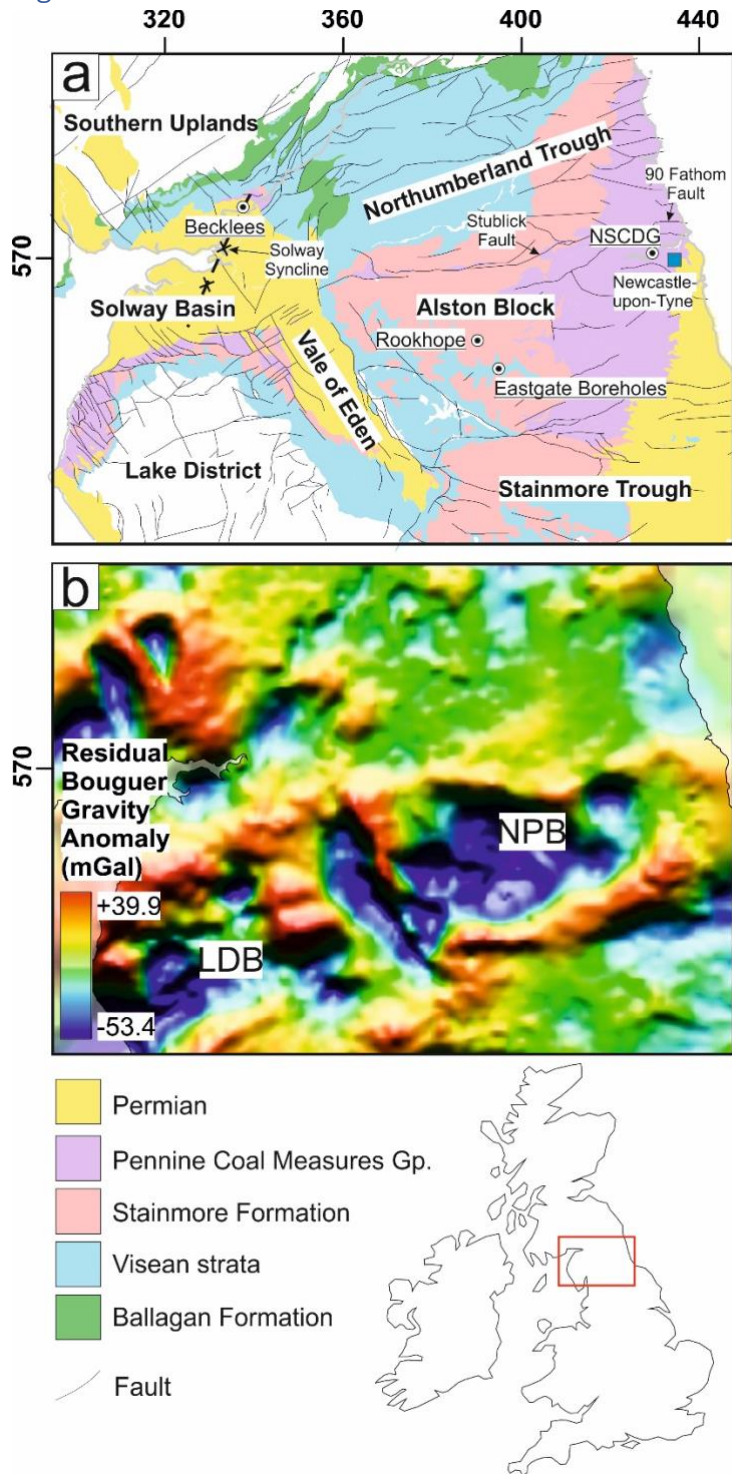
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604

605 Figures
606 Figure 1



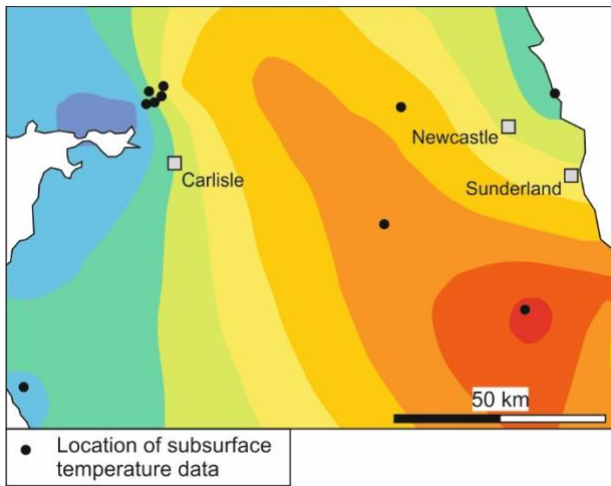
607
608 Fig. 1a: UK subsurface temperature maps for 1 km depth (after Busby *et al.*, 2011). 1b: UK heat flow
609 maps (after Downing and Gray, 1986).
610



612

613 Fig. 2a: A geological map for our case study area (British Geological Survey, 2008) with annotated
 614 structural features and borehole locations. 2b: A Bouguer gravity anomaly survey for our case study
 615 area (Kimbell and Williamson, 2015) with annotations for the negative gravitational anomalies
 616 associated with the Lake District Batholith (LDB) and the North Pennine Batholith (NPB). British
 617 National Grid coordinates are used for these and all maps in this manuscript.

618 Figure 3

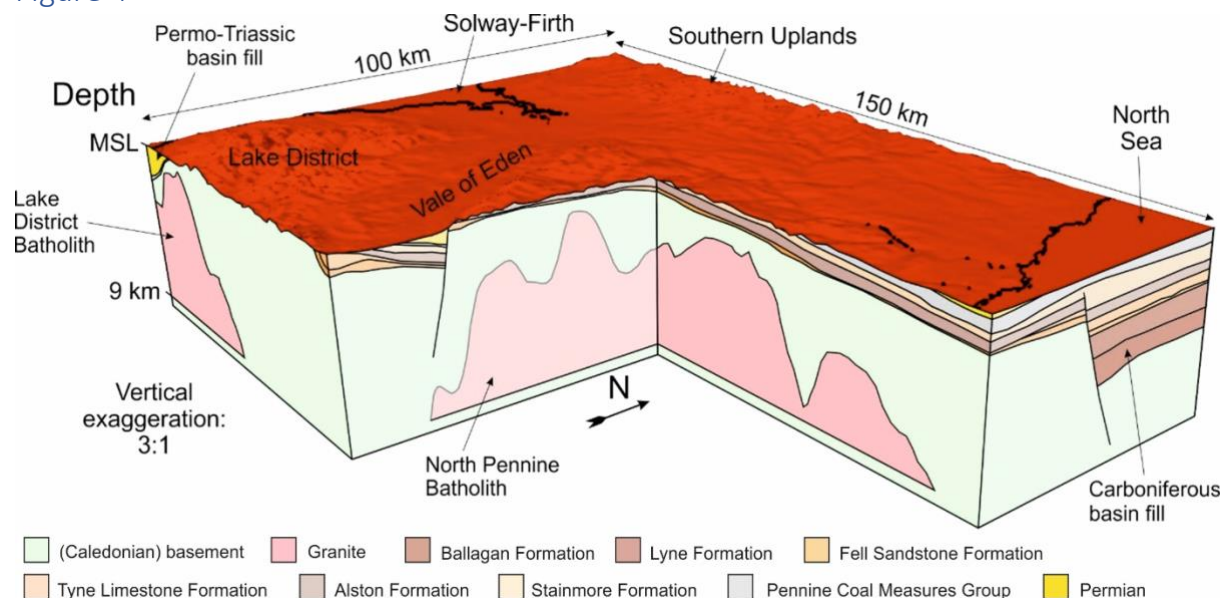


619

620 Fig. 3: Subsurface temperature contours (Fig. 1a) and locations of data points (*cf.* Burley *et al.*, 1984).

621

622 Figure 4

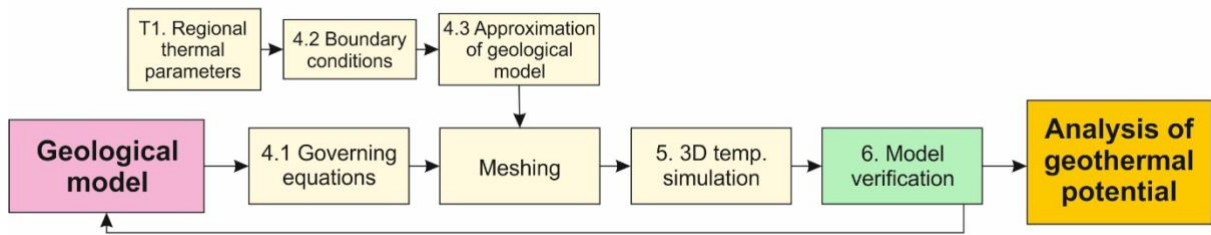


623

624 Fig. 4: A schematic illustration of our 3D geological model. Carboniferous basin structure after
625 Chadwick *et al.* (1995) and Caledonian granite thicknesses after Kimbell *et al.* (2010). MSL = mean
626 sea level. The depicted 3D model was produced using Petrel (Schlumberger) software.

627

628 Figure 5

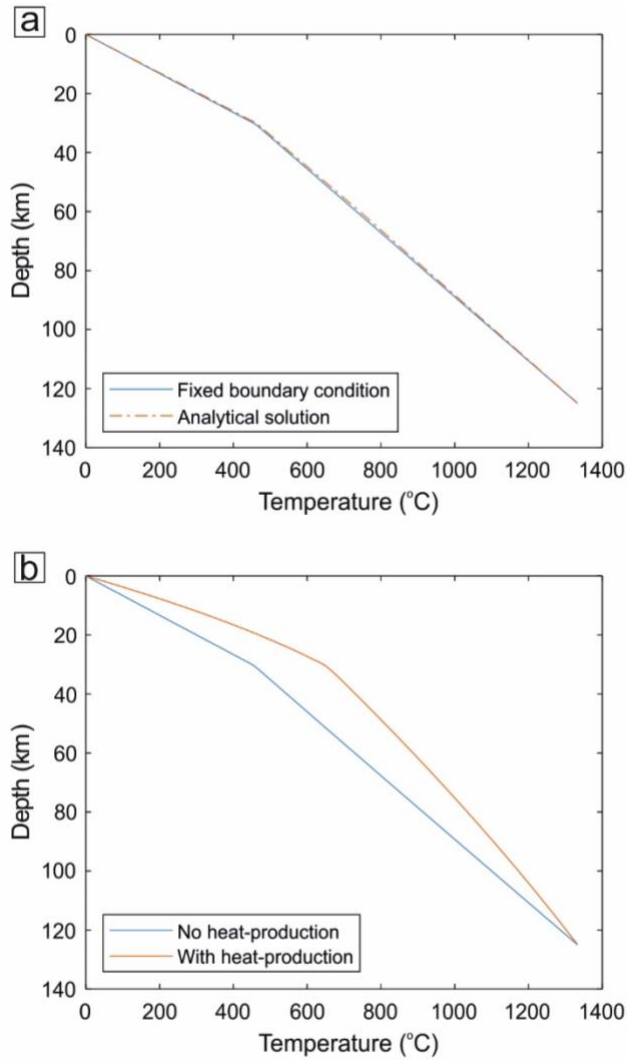


629

630 Fig. 5: An illustrated summary of our modelling approach. Numbering of method steps correspond to
631 sections or tables within this manuscript, in which these steps are described.

632

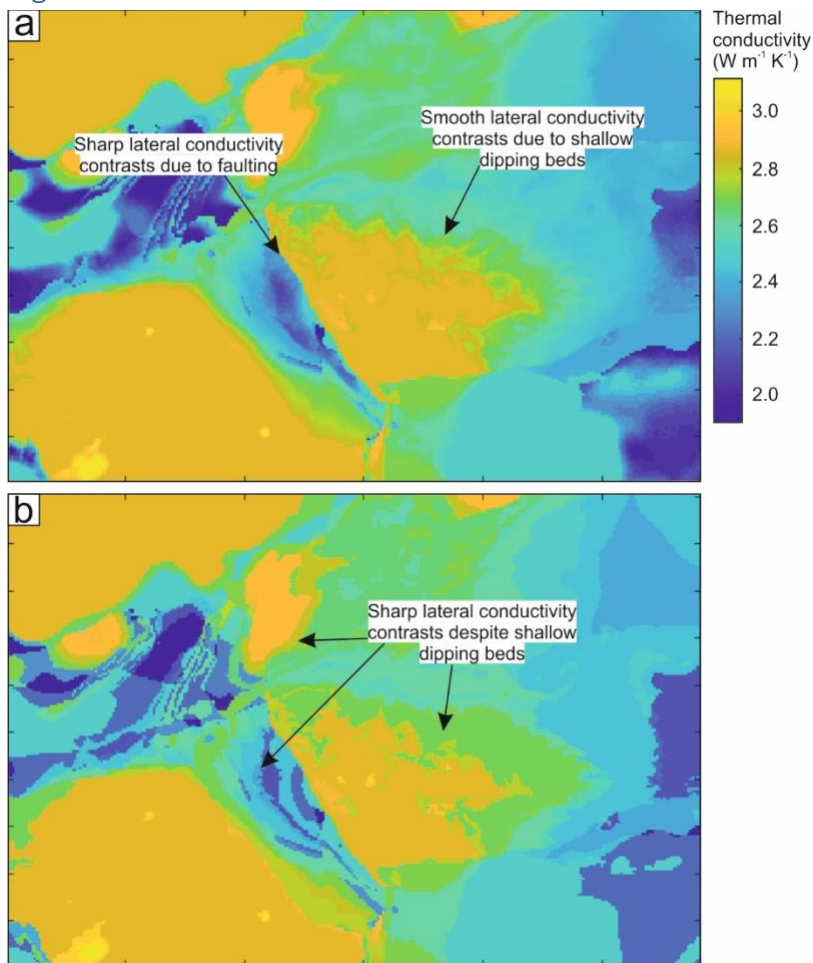
633 Figure 6



634

635 Fig. 6a: A comparison between analytical and fixed boundary condition solutions for one-
636 dimensional lithosphere-scale non-homogeneous conductive heat flow. See Table 1 for modelling
637 parameters. 6b: A comparison between fixed boundary condition solutions for one-dimensional
638 lithosphere-scale non-homogeneous conductive heat flow with no internal heat production (Q) and
639 with internal heat production.

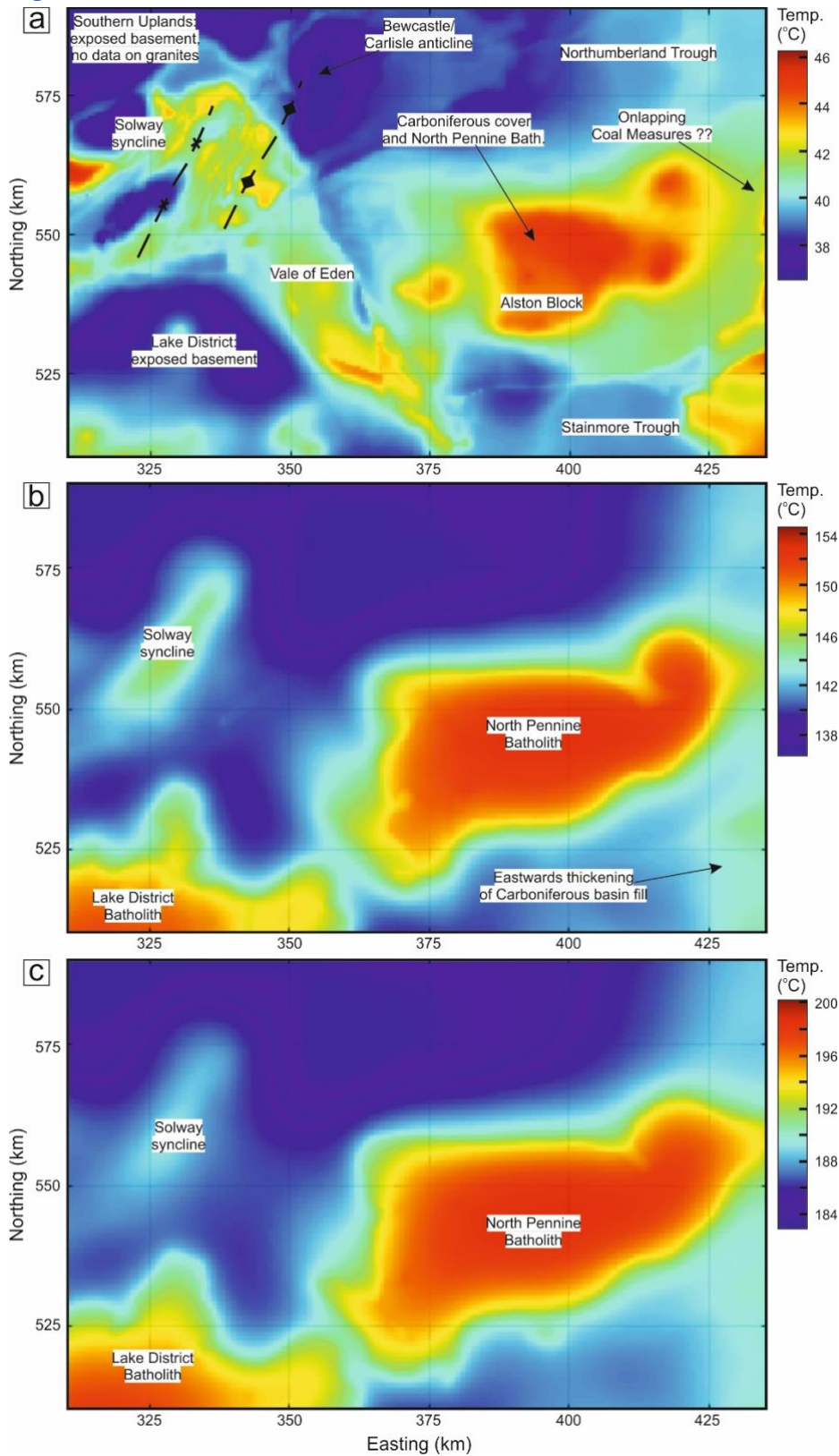
640 Figure 7



641

642 Fig. 7a: Vertical thermal conductivity tensors between 500 m and 1000 m below surface determined
643 by calculating the harmonic mean of multiple values between these two depths for northern
644 England. 7b: Vertical thermal conductivity tensors between 500 m and 1000 m below surface
645 determined by calculating the harmonic mean of just the two values at temperature nodes. For
646 thermal conductivity values of rock units see Table 1.

647

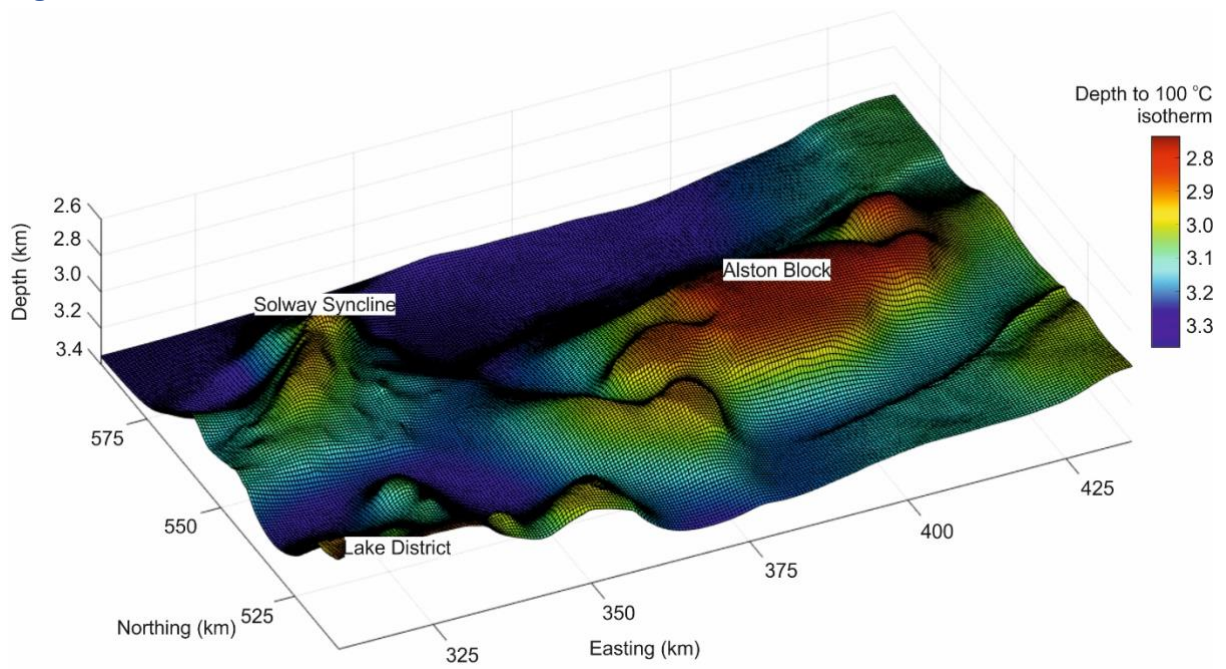


649

650 Fig. 8a: Modelled temperature at 1 km depth. Compare with Fig. 1b (Busby *et al.*, 2011). 8b:

651 Modelled temperature at 5 km depth. 8c: Modelled temperature at 7 km depth.

652 Figure 9

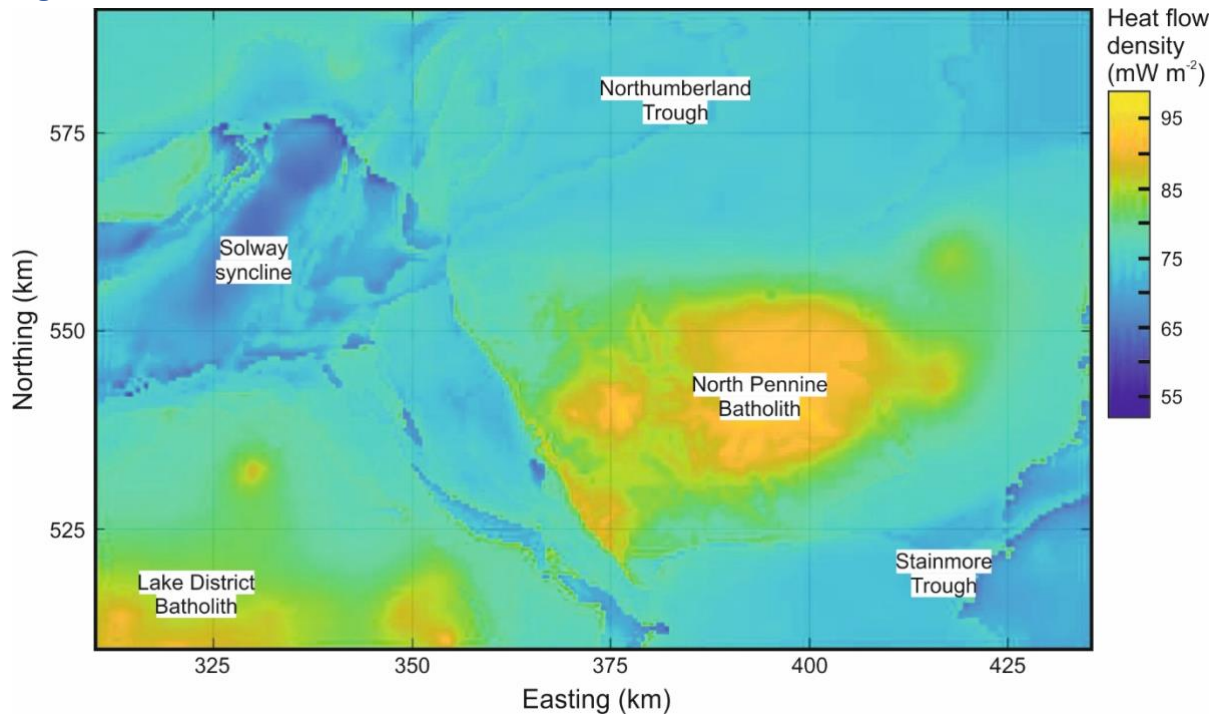


653

654 Fig. 9: Modelled depth to the 100 °C isotherm.

655

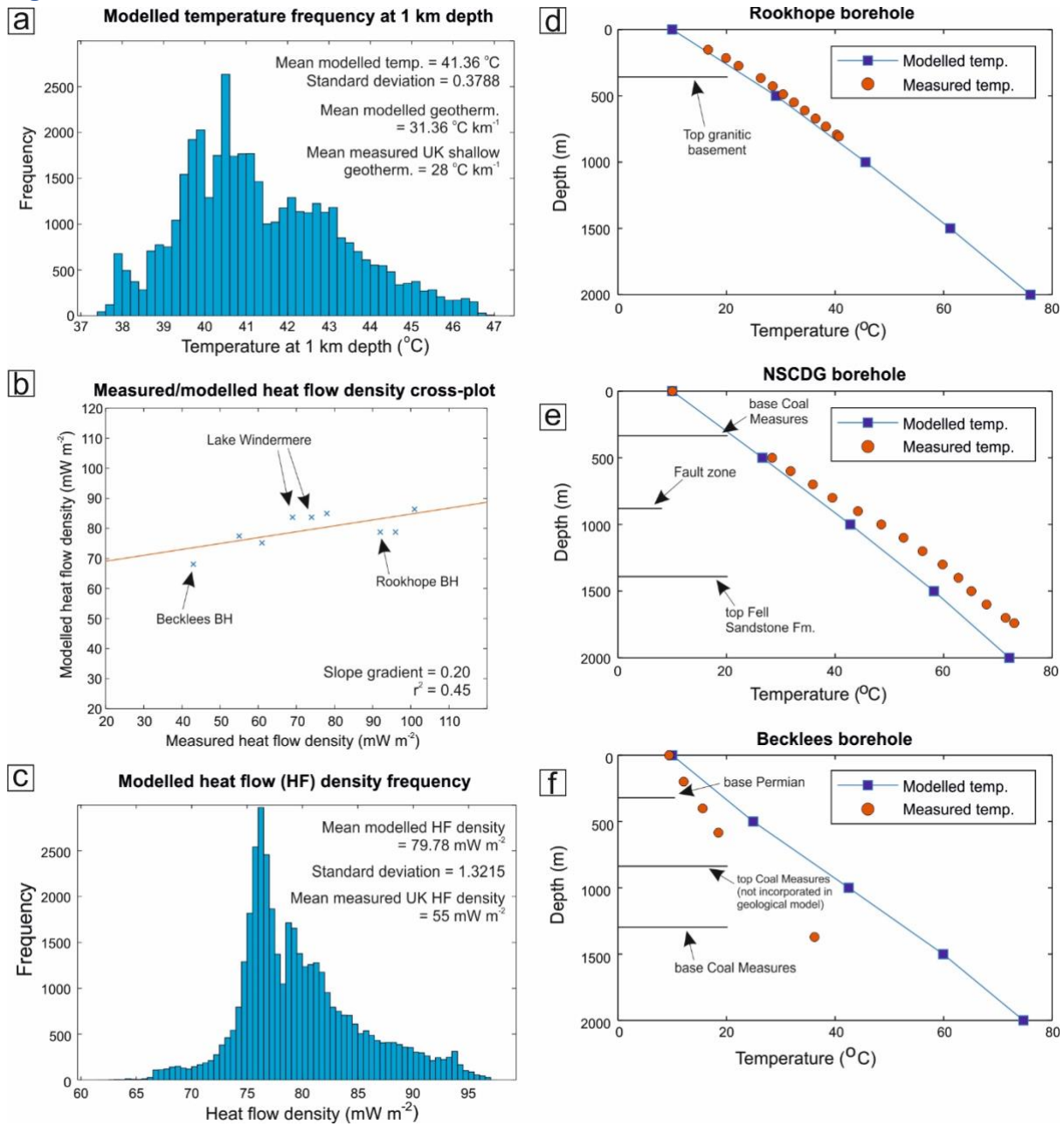
656 Figure 10



657

658 Fig. 10: Modelled surface (500 m below surface to surface) heat flow density map for northern
659 England based on predicted subsurface temperatures and vertical conductivity values. Compare with
660 Fig. 1b (Downing and Gray, 1986a).

661

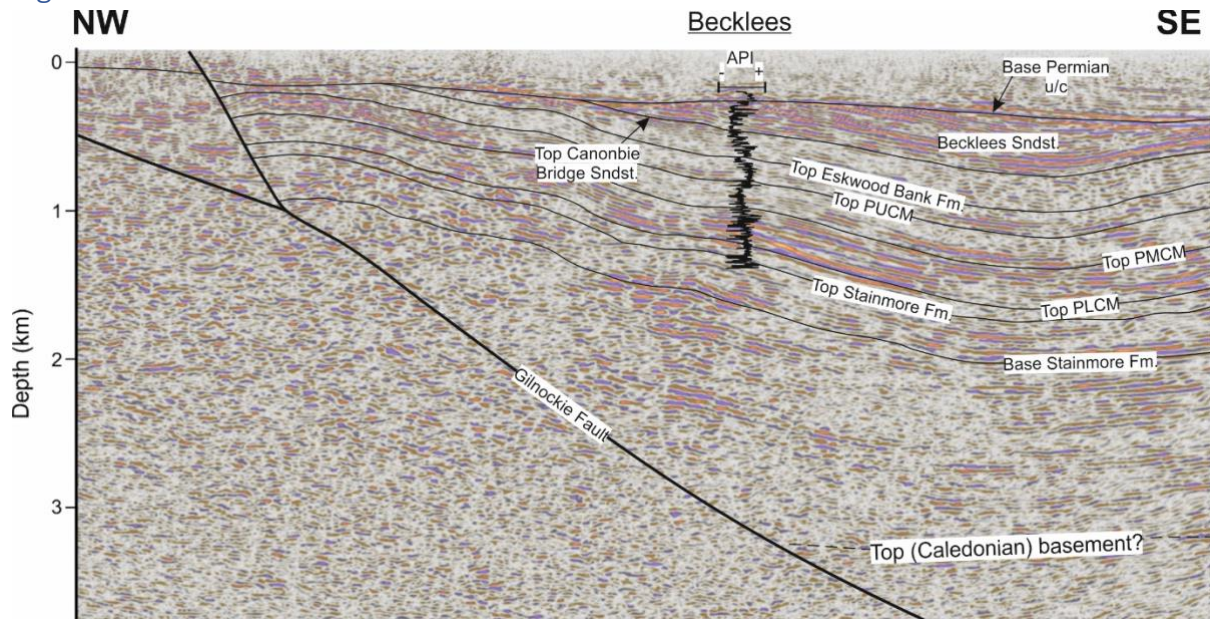


663

664 Fig. 11a: A cross-plot between measured heat flow density data and modelled data in our study
 665 area. Modelled data are taken from approximately the equivalent location as measured data. 11b
 666 and c: Frequency charts for modelled temperature values at 1 km depth, and shallow (<500 m)
 667 heat flow density values, respectively. Mean measured UK shallow (<1 km) geothermal gradient and
 668 mean measured UK heat flow density taken from Busby *et al.* (2011) and Busby (2010). 11d, e and f:
 669 Comparisons between modelled subsurface temperatures and measured equilibrium borehole
 670 temperatures for the Rookhope Borehole, the Newcastle Science Central Deep Geothermal Borehole
 671 and the Becklees Borehole, respectively. For locations of boreholes, see Figure 3a. Measured
 672 equilibrium boreholes temperatures taken from Burley *et al.* (1984) and Younger *et al.* (2016).

673

674 Figure 12



675

676 Fig. 12: A seismic reflection profile intersecting the Becklees borehole. A vertical gamma ray profile
677 for the Becklees borehole is illustrated. The Warwickshire Group comprises the Eskbank Wood,
678 Canonbie Bridge Sandstone and Becklees Sandstone formations (*cf.* Jones *et al.*, 2011). The Pennine
679 Coal Measures Group comprises the Pennine Lower Coal Measures (PLCM), Pennine Middle Coal
680 Measures (PMCM) and Pennine Upper Coal Measures (PUCM) formations. Seismic interpretation
681 based on Howell *et al.* (in press). Seismic courtesy of the UK Onshore Geophysical Library (UKOGL).

682

Geological unit	Thermal conductivity ($\text{W m}^{-1} \text{K}^{-1}$)	RHP ($\mu\text{W m}^{-3}$)	Reference
Lower Permian	2.5	1.0	Norden and Förster (2006)
Pennine Coal Measures Group	1.9	0.92	Downing and Gray (1986)
Stainmore Formation	2.38	0.88	Younger <i>et al.</i> (2016)
Alston Formation	2.5	0.88	Younger <i>et al.</i> (2016)
Tyne Limestone Formation	2.7	0.85	Younger <i>et al.</i> (2016)
Fell Sandstone Formation	2.6	0.85	Younger <i>et al.</i> (2016)
Lyne Formation	2.7	0.85	Younger <i>et al.</i> (2016)
Ballagan Formation	2.92	0.85	Downing and Gray (1986b)
Pre-Carboniferous (Caledonian) basement	2.87	1.49	Downing and Gray (1986b)
Granite Batholiths	3.1	4.1	Downing and Gray (1986b); Manning <i>et al.</i> (2007)
Middle-Lower crust	3.1-2.2	1.5	Norden and Förster (2006); Norden <i>et al.</i> (2008)
Mantle	4.1	0.1	Čermác and Rybach (1982); Vila <i>et al.</i> (2010)

684 Table 1: Regional thermal parameters for temperature simulation.

686 Supplementary information

687 MATLAB project files ([https://github.com/lphowell/Geothermal-](https://github.com/lphowell/Geothermal-Modelling/tree/master/Geothermal_NEngland)
688 [Modelling/tree/master/Geothermal_NEngland](https://github.com/lphowell/Geothermal-Modelling/tree/master/Geothermal_NEngland)).

689