The relationship between monetary growth and inflation: an application of the infinite hidden Markov model to the United Kingdom

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Abstract

The analysis is undertaken through exploration of a reduced form relationship. Two questions are central to the study. The first involves the role of structural breaks in the relationship between inflation and monetary growth. We explore this through an application of Qu (2008) SQ and DQ tests which analyse structural breaks in both the mean and quantiles of the conditional distribution. The second question involves the role of nonlinearity in the relationship between inflation and monetary growth.

The results from the SQ and DQ tests suggest the existence of multiple structural breaks in the linear relationship between inflation and monetary growth. In the thesis, we propose a modification to the critical values underlying these tests to capture the effect of various sample sizes. The results of Monte Carlo experiments suggest that this modification improves the power of test when compared to the results given by Qu (2008).

From the estimation of Markov switch model and infinite Hidden Markov model, we find that the relationship between monetary growth and inflation exhibits a maximum of five regimes over the period 1966 to 2012. However, after introduction of inflation targeting in UK in 1992, the relationship between inflation and monetary growth stayed in one regime most of the time. The financial crisis of 2008 only changed the relationship between monetary growth and inflation for a short period before returning to the pre-crisis regime. The iHMM demonstrates a range of capabilities, notably the ability to detect structural change even at the end of the sample. This feature is desirable in monitoring potential structural breaks generally and given the importance of the specific relationship between money and inflation for practical policy purposes.

Keywords: nonlinearity, structural break, SQ test, DQ test, iHMM.

Contents

List of Figures

List of Tables

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Chapter 1

Introduction

The relationship between monetary growth and inflation is one of the fundamental topics in economics. Money is the media for measuring the value of commodities and facilitating trade between people. To perform this task, money should have a standard value. The value of money depends on its quantity relative to requirement of people. When the quantity of money exceeds demand, the value of money will depreciate. We call this phenomena inflation which, to be sustained, must be accompanied by monetary growth. In the case of high inflation, especially hyper-inflation, the value of money depreciates in a very short period of time, with the result that people do not have enough time to react to the change of price[1]. For hyper-inflation, Sargent (1986) gave an explicit example. From 1970 to 2008, the price level in the US increased just over 5.5 fold; that is inflation. In Hungary, the price level increased 27 fold between 1923 to 1924; that is hyper-inflation. As result of hyper-inflation, the whole monetary system may collapse and the currency will be abandoned by people as it is no longer

[1] The hyper-inflation is defined as price increasing over 50% per month

be able to play the role as a medium for trade.

The lessons from high inflation were learned through experience from the early use of money, while the quantity of money was concerned as only the scarcer material would be chosen to be money. However, with the development of the monetary system, control of the money supply has become more and more difficult. The relationship between monetary growth and inflation became a difficult task rather than simply accounting for the quantity of money in circulation and change the material to be the money (Davies, 2002). In the 18th century, Hume (1775) introduced the quantity theory of money to explain the relationship between money and inflation and their role in the development of economy.

1.1 Summary of the quantity theory of money

In his essays “Of interest” and “Of money” (Hume,1752a,b), money is considered to only have an effect by arising the price level. The typical example is that silver is more common than gold therefore the same quantity of goods require more silver than gold to buy. However, the idea from Hume did not deny the effect of money on real economic activity. In the short run, the effect from money on real economic activity is not negligible. In “Of money”, Hume (1752a) stated that the price level will increase one by one instead of all together. Only in this intermediate situation, would real economic activity be favored by the increase of money.

Further contributions were made after David Hume first introduced the quantity theory of money. Newcomb (1885) introduced the exchange equation into the quantity theory of money. The equation is written as:

$$KP = VR,$$

where K represents total wealth, P is the price level, V is the volume of money and R denotes the velocity of money in the circulation. Newcomb explained that, other conditions being equal, increasing the amount of money will necessarily increase prices proportionally. Fisher (1896) rewrite the exchange equation in the more familiar form:

$$MV = PQ, \tag{1.1}$$

where M is the quantity of money, V is the velocity of money in the circulation, P is the price level and Q represents the total quantity of goods. Equation (1.1) clearly demonstrates that the price level varies directly as the quantity of money in circulation, given the quantity of goods and velocity of circulation. However, the exchange equation could not explain how other factors reacted to a change of quantity of money. By the way of explaining the exchange equation, Fisher (1911) concluded that changing M does not normally change V and Q but does change the price level in long term, because velocity and the quantity of goods are independent of quantity of money. In the long run, Fisher (1911) argued that velocity and production will depend on the density of population, commercial customs, natural resources and other technical conditions, none of which depend on the quantity of money.

1.2 Keynes's view on quantity theory of money

The Quantity theory of money is well accepted in describing the relationship between the quantity of money and inflation in the long run. However, in the short run, the interpretations of quantity theory of money are varied among theories. Keynes (1936) provided an alternative way of explaining economic fluctuations where spending on investment and the stability of the consumption function rather than quantity of money are key to keeping the economy growing. Keynes did not deny the validity of the quantity theory equation. However, what he did was something very different. The general price level, as he suggested in 1936, depends partly on the wage-unit and partly on the volume of employment. Hence, the effect of changes in the quantity of money on the price level can be considered as being the compound of the effect on the wage unit and the effect on employment.

The primary effect of a change in the quantity of money on the quantity of effective demand, as argued by Keynes (1936), depend upon the interest sensitivity of expenditure. If the quantity of money increased beyond what is required , the interest rate will fall, and effective demand will increase. Keeping the interest rate unchanged, the increase in the quantity of money will simply decrease velocity rather than having any effect on the right hand side of the quantity equation as either prices or output. This argument is cornerstone of Keynesian theory in explaining the relationship between quantity of money and inflation. The interest rate has been kept as the primary instrument for monetary policy and money is still excluded from consideration. Woodford (2003) stated that there is no space for the aggregate monetary base in the Keynesian model, because additional money balances beyond the optimal level provide no further liquidity services. The desirable level of money supply would be settled after the settling down of the optimal interest rate.

1.3 Criticism of Keynesian's view on money and inflation

In Keynesian theory, there is a basic assumption that the differential between monetary and non-monetary interest is constant. Friedman (1956) restated the quantity theory of money as a demand for money. Friedman's restatement in effect widened the monetary transmission mechanism from the narrow money-to-bonds channel to include goods, services and other financial assets which are different from the monetary return (nominal interest rate). Tobin (1965) argued that more money will be invested in the asset with a relative higher return than other assets. In this case, higher inflation will divert money from savings to capital. The yield on money will increase, the yield on capital will decline as a result.

Friedman (1987) contended that the difference between Monetarists and Keynesians focused on the range of assets considered. If more assets with different returns are considered rather than a single monetary return, a change in the money supply will influence the return differential between non-monetary assets which in turn cause a fluctuation in output and inflation.

1.4 Empirical studies of effect from money to inflation

Even though Monetarism criticized Keynesians for considering the interest differential as a constant, money is excluded from the mainstream Keynesians model as the transmission mechanism from money to inflation is still unclear. However, King (2001, pp17) argued:"although there is no mechanical link from monetary aggregates to inflation, the underlying relationships, in quantitative form, still hold." Much empirical evidence supported the intimate link between money and prices. Gali, Salido and Valles (2003) applied a real business cycle model to characterize the monetary response to technology shocks and its implications for US output and inflation. Their main result suggests that the effect of monetary policy on output and inflation varies over time. The volatility of the monetary change will also significantly increase the volatility of inflation.

Favara and Giordani (2009) studied the dynamic response of inflation, output and interest rates to a monetary shock based on a VAR model using US quarterly data from 1966 to 2001. Their result questioned empirically the validity of the New Keynesian model excluding money as a shock to monetary growth had a substantial and persistent effect on inflation and output. Jones and Stracca (2008) tested the effect of monetary growth on output based on an IS equation covering the period from 1994 to 2007 in the UK. Their result also suggested a significant effect of money on output. Altig, Christiano, Eichenbaum and Linde (2010) also detected the contribution of monetary policy to cyclical fluctuations in output by estimating a US business cycle model.

However, Woodford (2003) argued that if equilibrium real balances are generally very small relative to national income, a substantial percentage increase in real balances may have a "negligible" effect on output. Ireland (2004) examined the real balance effect by incorporating monetary growth into the New Keynesian model. Maximum likelihood estimates from a complete structural model using US data on money growth, inflation, output, and interest rates for the period from 1980 to 2003 found that the effect of the monetary growth on key economic variables was non-zero but negligible as the coefficient on money was not significantly different from zero. McCallum (2001) investigated the response of inflation, output and interest rate to a shock in monetary policy based on the New Keynesian model. He argued that, even though the model without money is misspecified, the error introduced is insignificant.

However, Patinkin (1965) had previously argued that approximations which neglect the real balance effect because of the smallness of this effect ignore a basic analytical factor in the theory of the determination of the price level. Thus, there can be no justification for neglecting it in monetary theory.

1.5 Empirical studies about relationship between inflation and money

Walsh (2010) argued that Keynesians interpret the transmission mechanism narrowly operating through interest rate only, whereas most monetarists take the view that changes in monetary growth lead to substitution effects over a broader range of assets than Keynesians normally considered. Despite the argument about the transmission mechanism, many empirical studies were conducted to analyse the relationship between monetary growth and inflation based on the quantity theory of money. A general form of these studies was to compare data between monetary growth and inflation over a long time period to investigate the long run relationship between money and inflation. The comparison was applied to various countries over different time periods. However, investigating the relationship between money and inflation in a fixed time period does not necessarily mean the long run in monetary theory. Therefore, research about the long run relationship based on different countries and different time periods do not have consistent results. Some results support the unitary relationship between money and inflation in the long run, for example Lucas (1980), McCandless and Weber (1995) and Grauwe and Polan (2001), while others, for example, Benati (2009) and Sargent and Surico (2010), present results that contradict the unitary relationship between money and inflation. Empirical studies about the long run relationship between money and inflation are discussed in Section 2.2.

Also, some empirical studies concern Granger causality between money and inflation. As discussed in Section 2.3, without a structural model between money and inflation, the result from Granger causality can not be used to support structural causation between money and inflation.

Friedman (1970) suggested that, on average, a change in the money supply leads the inflation by 12-18 months. In this case, other empirical studies are concerned with the linear regression of inflation on lagged monetary growth. This will be discussed in Section 2.4. However, the linear regression model usually failed to capture the change in the relationship between money supply and inflation.

A potential structural change in the relationship between monetary growth and inflation leads to the possibility of applying nonlinear models to the analysis. Since the 1980s, many kinds of nonlinear models have been introduced concerning various statistical problems. Since the structural model is uncertain, a nonlinear model applied to the relationship between money and inflation is generally conducted in a reduced form equation. The nonlinear structure is represented by changes in the coefficients of the model. The technical details of the nonlinear models and their application to the relationship between monetary growth and inflation are discussed in Section 2.5.

1.6 Structural change in modeling the relationship between monetary growth and inflation

Before modeling the relationship between monetary growth and inflation, it is first necessary to test if there exists structural break in the linear model. In Chapter 4, we discuss SQ and DQ tests for testing potential structural breaks in the linear model.

The test of structural change has been extensively studied in various applications. The general practice is to apply the Chow test, which was introduced by Chow in 1960, for detecting structural change. However, the Chow test, as argued by Bai and Perron (2009), is designed to test a single known break in the model. This designation is different from reality as structure breaks are generally unknown and could occur several times over an interval of time. To tackle this problem, Andrews (1997) considered optimal tests of structural break in a linear model with known variance, while Bai and Perron (1998) considered the theoretical issues involved in testing a linear model with multiple unknown structural changes. However, most tests of structural change focus on the conditional mean, while the structural break could also exist in the conditional quantiles. Qu (2008) introduced tests, called the SQ and DQ test, concerned with structural breaks over different quantiles. However, the critical values for the DQ and SQ test in Qu (2008) were simulated based on a single sample size. Therefore, we simulated the critical values with different sample sizes.

The linear regression model of inflation on lagged monetary growth is explored based on different definitions and growth rates of money and corresponding inflation which include: quarterly 3 month growth rate of M4 and M0 (hereafter M403 and M003) and quarterly 12 months growth rate of M4 and M0 (hereafter M412 and M012). The results of the DQ test for all datasets suggest the existence of a structural break in the linear relationship between monetary growth and inflation. Also the results of the SQ test suggests that the structural break exists in most of the quantiles and that the position of the structural breaks are varied over different quantile. These results suggest the exists of multiple structural breaks in the relationship between monetary growth and inflation.

1.7 Nonlinear relationship between inflation and money

With the development of the nonlinear models, a potential nonlinear relationship between monetary growth and inflation could be explored. However, studies of nonlinearity in the relationship between money and inflation are still limited. Existing nonlinear studies, which will be discussed in Chapter 3, focus on the regime-switching model as the actual structure between monetary growth and inflation is unknown.

However, due to technical constraints, some assumptions about nonlinear behaviour in the existing studies were made for convenience of implementation. First, the change of structure is contained within a limited number of regimes (generally two regimes). Second, the reason for structural change is assumed despite the fact that the real reason for the change of structure is unknown. Third, a structural change can not distinguish between a new structure or a reoccurrence of a previous structure. A basic nonlinear model following these constraints is Markov switch model which will be discussed in Chapter 4.

In order to tackle these problems, we apply the infinite Hidden Markov model (iHMM) where the number of regimes is unlimited, the factors for controlling structural change do not follow any specific process, and structural change can be identified as switching into a new regime or reoccurrence of a previous regime. The application of iHMM will be discussed in Chapter 5.

From estimation of iHMM, the optimal lag length for modeling inflation based on preceding monetary growth is 11 quarters for M4 and 9 quarters for M0. The relationship between monetary growth and inflation also divides into a maximum of five regimes. The regime sequence based on data of quarterly 3 month growth (M003 and M403) involves fewer regimes and structural changes when compared to the ones based on the data of the quarterly 12 months growth rate (M012 and M412).

The regime changes mainly happened in the period before the introduction of inflation targeting in UK. Also, the sum of the coefficients on monetary growth, after the introduction of inflation targeting, decreased to a lower level when compared to regimes before the introduction of inflation targeting. In addition, regime sequences based on both M4 datasets detect a structural break at the economic crisis in 2008. However, the economic crisis did not shift the relationship between money and inflation into a new regime for long before switching back to the pre-crisis regime.

We also investigate whether the iHMM can detect a structural change at the end of the sample. If so, it will be useful to monitor the change of relationship between monetary growth and inflation. For this reason, we truncate the sample size at positions of the structural break. The estimation results suggest that the iHMM can efficiently detect structural change even at the end of the sample.

However, in the absence of a structural model between monetary growth and inflation, the cause for the structural break is unknown. The reasons for the change of relationship between monetary growth and inflation are left for future research.

Chapter 2

Empirical studies of the relationship between money and inflation

2.1 Introduction

The quantity theory of money is widely accepted despite the debate relating to the role of monetary growth in the transmission mechanism. Nelson (2011) argued that it was more straightforward to establish the relationship between monetary growth and inflation than it was to establish connections between monetary policy actions and subsequent inflation movements. Similarly, Wallich (1984) stated that the impact of a given level of interest rates and GDP on inflation is far less predictable than the relationship between inflation and preceding monetary growth. In the absence of a structural model, empirical studies of the relationship between monetary growth and inflation have adopted reduced-form equations which link inflation directly to monetary growth. Generally speaking, empirical studies of the relationship between monetary growth and inflation have three strands: 1) comparing data on monetary growth and inflation over long time horizons to study the long term relationship between the two variables;2) studying the causation from money to inflation 3) employing linear regression of inflation on the prior monetary growth to study the reaction of inflation to monetary policy over time; 4) exploring nonlinear models to study the dynamic relationship between monetary growth and inflation.

In this chapter, evidence for the relationship between money and inflation is exploited as follows. Studies for the long run relationship between money and inflation are discussed in Section 2.2. Linear regression of inflation based on preceding monetary growth in Section 2.3 followed by a discussion of quantile regression model in Section 2.4. Different classes of nonlinear models together with corresponding applications in the relationship between money and inflation will be discussed in Section 2.5. This chapter is concluded in Section 2.6.

2.2 Testing the relationship over the long term

Many empirical studies have been conducted concerning the relationship between monetary growth and inflation in the long term across different countries and time horizons. McCandless and Weber (1995) examined the relationship between monetary growth and inflation for 110 countries from 1960 to 1990 using different definition of money, namely, M0, M1 or M2. For each country and definition of money, the correlation coefficient between monetary growth and inflation was 0.925 or higher which, as argued by

McCandless and Weber (1995), supported the unitary relationship between monetary growth and inflation in the long term. A similar result can be found in Grauwe and Polan (2001), who tested the quantity theory relationship between money and inflation in 160 countries from 1970 to 2000.

Lucas (1980) compared moving averages of M2 growth and inflation using quarterly US data from 1953 to 1977. The result suggested that monetary growth rate induced an equi-proportional change in inflation in the long term. However, Sargent and Surico (2010) argued that the result from Lucas (1980) depends largely on the data sample chosen. Sargent and Surico (2010) extended the sample period in Lucus (1980) from 1900 to 2005 and divided the whole sample into six sub-periods. Their findings suggested that the result of a one-to-one relationship between monetary growth and inflation could only be obtained from two subperiods, including the period from Lucas (1980). Sargent and Surico (2010) also suggested that the periods which support Lucas, emerged when the monetary authority allowed persistent increases in monetary growth despite the inflation pressure. Benati (2009) also extended the sample period in Lucas (1980) to test the relationship between both narrow and broad monetary growth and inflation in the US from 1875 to 2008 and in the UK from 1871 to 2007. The result also suggested that inflation moved less than one for one with monetary growth in the long run. However, Benati (2009) found evidence of a relationship between monetary growth and inflation close to one in periods of high inflation, such as World War I.

However, in monetary analysis, the long run does not necessary mean a very long calender time. Nelson (2008,pp12) defined the long run in monetary analysis as:" the economic conditions prevailing after prices have fully adjusted to monetary policy actions." Therefore, simply averaging the data over a fixed long period need not represent the long run for monetary policy. As discussed above, the results for the long run relationship between money and inflation vary with the period of calender time. However, the period required for inflation to fully reflect the effect of a monetary policy is unknown in advance.

2.3 Linear regressions of inflation on money

From existing empirical studies, monetary growth has the property of leading with respect to inflation. Haug and Willam (2004) examined the correlation between preceding monetary growth and inflation for the period 1880 to 2001 for 11 different countries. The results suggested that monetary growth leads inflation by 1 to 3 years. By calculating the correlation between inflation and preceding monetary growth from 1975 to 2005, Rua (2012) found that monetary growth leads inflation by up to 2 years in the Euro area.

McCallum and Nelson (2010) suggested that it is more sensible to analyse the relationship between monetary growth and inflation by using non-averaged time series data to allow for lags of monetary growth. Holden and Peel (1979) examined the relationship between inflation and monetary growth in 18 Latin American countries by regressing inflation on lagged monetary growth. The results suggested that the adjustment process between inflation to monetary growth takes around three years. The result of the estimation also supported monetarist explanations of inflation in Latin American.

McCallum and Nelson (2010) also regressed inflation on money growth for G7 countries from 1958 to 2008. The results showed that the coefficients on monetary growth change over time as the model performed better when an intercept dummy variable was introduced for the period after 1973. Also, introducing a lagged monetary growth term improved the performance of the model.

These findings are consistent with King (2001) who suggested that there is no reason to expect a simple relationship between monetary growth and inflation in a reduced form as the coefficients will be complex functions of the true underlying economic structural parameters. Lucas (1976, pp.126) also argued that "if optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models." Since there exists no clear transmission mechanism from monetary growth to the inflation, the specific function of the coefficients will be unknown. This suggests that a linear regression model may have difficulty representing a complicated underlying structural model between money and inflation.

2.4 Alternative estimator for the linear model

The estimation of a linear regression model, as discussed previously, focused on the sample mean of a normal distribution. However, it is often observed that errors follow a non-Guassian distribution which make estimators such as least squares estimator inferior to the case of a normal distribution, especially for distributions with longer tails than a normal distribution. To counter this problem, Koenker and Bassett (1978) argued that it appeared desirable to choose an estimator which modified the sample mean by putting reduced weight on extreme observations. Therefore, they introduced a quantile regression in order to improve the efficiency of estimators based on the sample mean.

Let $y_t : t = 1, ..., T$ denotes a random sample of variable Y following an random distribution; $x_t, t = 1, .., T$ denotes a sequence of K-vectors of a matrix. Then, $u_t = y_t - x_t\beta$ is a sequence of errors following an random distribution. The θth regression quantile, $0 < \theta < 1$, is defined as a solution to minimize:

$$min_{\beta \in R_k} [\sum_{t \in \{t : y_t \geq x_t \beta\}}]$$

The quantile regression is simply a modification of least squares estimation to taking account of the effect from outlying observations. This modification is capable of describing the full picture of the relationship between the dependent variable and sample observations, and is especially robust against outliers of te distribution. The application of quantile regression in the relationship between inflation and money will be discussed in the next chapter.

2.5 Nonlinear relationship between monetary growth and inflation

In recent years, nonlinear models have been used to analyse the relationship between monetary growth and inflation. Although the research is limited, the results suggest that nonlinear models can improve the findings traditionally ascribed to linear regression techniques. In this section, we review the development of nonlinear models and the associated studies of the relationship between monetary growth and inflation.

Nonlinear models include a very wide-range of models with varied applications in economics. The concept of nonlinearity is a broad idea. Terasvirta, Tjostheim and Granger (2010) defined nonlinearity as anything other than linearity which is then itself actually a small sub-class of nonlinearity. In the case of linearity, Lee,White and Granger (1993) consider a model with the general form as:

$$y_t = \alpha z_t + g_(z_t).$$

The model is said to be linear if $g(z_t) \equiv 0$. Nonlinearity, on the other hand, covers a wide-range of models which can be classified by type according to various criteria, such as the type of data, method of estimation, etc. Here, we focus the discussion of nonlinear regression models based on time series data as this is suitable to the inflation rate and money growth rate.

First, we consider a model with the general form:

$$y_t = g(x_t, \theta_t, \varepsilon_t)$$

where g is a known function, x_t is a vector of explanatory variables, θ_t is an unknown parameter vector, and ε_t is an error term. A nonlinear model can be classified by describing the change in the parameter, θ_t, and the error term, ε_t. In practice, the study of relationship between money and inflation focused on the potential for structural breaks, in other words, a change in the coefficients. In the absence of a structural model, the study is generally conducted by assuming a direct relationship between money and inflation. The change in monetary growth will cause lagged changes in inflation. However, with the assumed conditions changed, the coefficients in model, which represents the effect from money growth to inflation, will change correspondingly. A typical model deployed to this study is the smooth transition model.

2.5.1 Dynamics in the coefficients of model

Linear regression model provide an important benchmark for our discussion. The linear regression model can be used as a good approximation for many situations where the statistical structure is fairly straightforward. However, the linear model is unsuitable in the case where the model's coefficients change over time.In this section, we outline the development of the smooth transition model and its applications for studying the money and inflation relationship

In introducing the smooth transition model, we begin with the standard switch regression (SR) model which is piecewise linear and can be defined as:

$$y_t = \sum_{j=1}^{r} (\theta_j x_t + \varepsilon_{jt}) I(c_{j-1} < g(s_t) < c_j)$$

where $\varepsilon_t \overset{iid}{\sim} N(0, \sigma^2)$, $I(A)$ is an indicator function: $I(A) = 1$ when event A occurs, zero otherwise, c_j is the threshold parameter defining the range of $g(\cdot)$ for each regime. The linear model is a special case for the model when $r = 1$. Alternatively, the SR model breaks the whole sample into r regimes. Each regime is represented by a linear model with coefficient θ_j and the change of regime is governed by the function $g(s_t)$, where s_t is an observable variable. If $g(s_t)$ in contained in the range between c_{j-1} and c_j, then the coefficient θ_j will be assigned to x_t in regime j.

Based on the standard switch regression model, Tong (1980) introduced the Transition Autoregressive (TAR) model to consider SR model that switches between two regmies. The x_t in SR model is replaced by $w_t = (y_{t-1}, y_{t-2}, ..., y_{t-n})$. The value of y_{t-1}, a threshold variable, determines the regime switch. Therefore, the TAR model can be written as:

$$y_t = \theta_1 w_t I(y_{t-1} \leq c) + \theta_2 w_t I(y_{t-1} > c) + \varepsilon_t.$$

Whenever the value of y_{t-1} exceeds the constant c, the regime will switch between 1 and 2. However, in some situations, the switch between regimes is smooth rather than changing regime instantly. In response, Terasvirta (1994) introduced the Smooth Transition Autoregressive (STAR) model based on the TAR model by introducing a continuous transition function $g(s_t; \gamma, c)$ that is bounded between 0 to 1:

$$y_t = \theta_1 w_t + \theta_2 w_t g(s_t; \gamma, c) + \varepsilon_t. \tag{2.3}$$

The regime that occurs at time t is determined by the observable variable s_t and the associated value of $g(s_t; \gamma, c)$. Different choices for the transition function $g(s_t; \gamma, c)$ give rise to different types of regime-switching behaviour.

One option for the transition function is the first-order logistic function:

$$g(s_t; \gamma, c) = (1 + exp\{-\gamma(s_t - c)\})^{-1}$$

with positive γ. The resultant model is called the logistic STAR [LSTAR] model. The parameter γ determines the smoothness of the change in the value of the logistic function and, thus, the smoothness of the transition from one regime to the other. As γ becomes very large, the regime transition becomes almost instantaneous at $s_t = c$ and, consequently, the logistic function $g(s_t; \gamma, c)$ approaches the indicator function. When $\gamma \to \infty$, the logistic function equals a constant (equal to 0.5) and when $\gamma = 0$, the LSTAR model reduces to a linear model.

Alternatively, an exponential function $g(s_t; \gamma, c) = 1 - exp\{-\gamma(s_t - c)^2\})$ with positive γ can be used as transition function. The resultant model is called the exponential STAR [ESTAR] model. The exponential function has the property that $g(s_t; \gamma, c) \to 1$ both as $s_t \to \infty$ and$s_t \to -\infty$ whereas $g(s_t; \gamma, c) = 0$ at $s_t = c$.

Based on the specification of LSTAR model, Milas (2007) tested the effect of M4 money growth on inflation in UK from 1992Q4 to 2007Q1 by considering a Phillips curve equation augmented by M4 monetary growth. The first underlying model takes the form:

$$\begin{aligned} \pi_t = \beta_0 + \beta_{low}(\pi_{t-1}, gap_{t-4}, M4_{t-1}, R_{t-4})\theta_{M4_{t-1}} \\ + \beta_{high}(\pi_{t-1}, gap_{t-1}, M4_{t-1}, R_{t-4})(1 - \theta_{M4_{t-1}}) + u_t \end{aligned} \tag{2.4}$$

where π_t is the inflation, $m4_t$ is the M4 monetary growth, gap_t is the output gap given by the residuals from regressing log real output on a quadratic trend. R_t is the interest rate, and θ is the transition function where $M4$ is chosen to control the regime switch between low level and high level. The low level suggests that M4 growth is below certain threshold and vice versa. The threshold is endogenously determined by the model. The result of equation (2.4) suggested that the threshold is 10 percent for annual M4 growth. Whenever, the M4 growth is less than 10 percent, its effect on inflation is equal to 0.05 which suggests that a 1 percentage increase in money growth will induce 0.05 percentage increase in inflation. On the contrast, if the M4 growth exceeds 10 percent, then its effect on inflation is equal to 0.09 which suggests that a 1 percentage increase in money growth will induce 0.09 percentage increase in inflation. Both coefficients on M4 growth are statistically significant. The result of model suggestes that the effect from monetary growth to inflation change with underlying conditions.

Kulaksizoglu and Kulaksizoglu (2009) also used LSTAR model to test the effect of M1 money supply on inflation in US from 1959q2 to 2007q3. The equation of inflation on money is written as:

$$\pi_t = (\alpha_0 + \alpha_1 \pi_{t-1} + \cdots + \alpha_p \pi_{t-p} + \beta_1 M1_{t-1} + \cdots + \beta_p M1_{t-p})$$
$$(\theta_0 + \theta_1 \pi_{t-1} + \cdots + \theta_p \pi_{t-p} + \phi_1 M1_{t-1} + \cdots + \phi_p M1_{t-p}) g(s_t; \gamma, c) + \varepsilon_t$$

where π_t is the inflation, $\mathrm{M1}_t$ is the M1 growth, and $g(\cdot)$ is a logistic function where c is equal to 1. Development of inflation is split into two regimes controlled by the second lag of inflation. When $g(\cdot)$ is 1 the inflation is in the high inflation regime, and vice versa. The result of the estimation by Kulaksizoglu and Kulaksizoglu (2009) suggested that M1 has a significant effect on inflation in US. The effect of M1 on inflation changes when second lag of inflation change over threshold at 1.4 percentage for second lag of inflation. This means when inflation increase decrease over 1.4 percentage, the effect of money on inflation will increase or decrease.

Results from Milas (2007) and Kulaksizoglu and Kulaksizoglu (2009) also suggested that the reason for the change of effect from money to inflation is uncertain or mixed.

2.5.2 Extensions of STAR model

The representation of the STAR model in equation (2.3) can be considered as a weighted average of two AR models, where the weights for two models are determined by the value taken by the transition function $g(s_t; \gamma, c)$. Hence, Dijk, Terasvirta and Franses (2002) argued that the STAR model cannot accommodate more than two regimes irrespective of what form the transition function takes. Even though two regimes might be sufficient in many applications, it can be desirable on occasion to allow for multiple (more than two) regimes. Dijk and Franses (1999) introduced the Multiple Regime STAR [MRSTAR] model to account for this. The MRSTAR model takes the form:

$$\begin{aligned} y_t &= \theta_1 w_t + (\theta_2 - \theta_1) w_t g_1(s_t) \\ &\quad + (\theta_3 - \theta_2) w_t g_{\mathbf{2}}(s_t) + \cdots + (\theta_m - \theta_{m-1}) w_t g_{m-1}(s_t) + \varepsilon_t \end{aligned}$$

where the $g_j(s_t) = g_j(s_t; \gamma_j, c_j), j = 1, ..., m-1$, are logistic function as in LSTAR model. Modeling more than two regimes is achieved by introducing other threshold values c_j to combine more linear AR models. For example, a four regime model can be obtained by combining two different two-regime LSTAR models as follows:

$$\begin{aligned} y_t &= [\theta_1 w_t (1 - g_1(s_t; \gamma_1, c_1)) + \theta_2 w_t (g_1(s_t; \gamma_1, c_1))][1 - g_2(s_t; \gamma_2, c_2)] \\ &\quad + [\theta_3 w_t (1 - g_1(s_t; \gamma_1, c_1)) + \theta_4 w_t (g_1(s_t; \gamma_1, c_1))][g_2(s_t; \gamma_2, c_2)] + \varepsilon_t. \end{aligned}$$

Each linear AR model is associated with a particular combination of $g(s_t; \gamma_1, c_1)$ and $g(s_t; \gamma_2, c_2)$ being equal to 0 or 1.

A special case of the MRSTAR model which considers both nonlinear dynamics of the STAR-type and time-varying characteristics was introduced by Lundbergh, Terasvirta and Dijk (2000). The time-varying STAR (TV-STAR) model with smoothly time-varying parameters is obtained by setting $s_t = t$ in the transition function. The resultant model can be written as:

$$y_t = \theta_1(t) w_t (1 - g_1(s_t; \gamma_1, c_1)) + \theta_2(t) w_t g_1(s_t; \gamma_1, c_1) + \varepsilon_t$$

where

$$\theta_1(t) = \theta_1[1 - g_2(t; \gamma_2, c_2)] + \theta_3 g_2(t; \gamma_2, c_2),$$

and

$$\theta_2(t) = \theta_2[1 - g_2(t; \gamma_2, c_2)] + \theta_4 g_2(t; \gamma_2, c_2).$$

The TV-STAR model involves regime transition in two dimensions: thresholds on s_t and t drive the change of parameters. The threshold on time t divides the whole sample period into two sub-periods. Within each sub-period, the threshold on the transition variable s_t will also drive the parameter switch between states.

However, there are several limitations for the STAR-type models in modeling the nonlinear relationship between variables. First, the number of regimes is limited, generally contains only two regimes. However, this setting may cause misspecfication as the number of regime is hardly to known in advance. Second, the factor, which controls regime-switch behaviour, is not necessarily a threshold which is static over time. Alternatively, the state space model is desire to tackle the problems faced by STAR-type model.

2.5.3 State Space Model and its estimation

Kim and Nelson (1999) defined the state space model as an observed variable being the sum of a linear function of state variables which in turn evolve according to a stochastic equation depending on unknown parameters. This set-up leads to the so-called state space processes which can be written as:

$$y_t = h\alpha_t + \theta x_t + \varepsilon_t, \tag{2.8}$$

where

$$\alpha_t = \phi\alpha_{t-1} + \upsilon_t, \tag{2.9}$$

$$\varepsilon_t \overset{iid}{\sim} N(0, R),$$

$$\upsilon_t \overset{iid}{\sim} N(0, Q),$$

and α_t is the unobservable state variable, which can be written as a function of α_{t-1}, driving the dynamics of the model. Equation (2.8) together with (2.9) generates the standard state space model where equation (2.8) is the measurement equation describing the relation between data and state variable, and equation (2.9) is the transition equation describing the dynamics of the state variable. Compared to the STAR-type model, the state space model does not have a limit on the number of regimes as coefficients evolve over time. Also, the reason for regime-switch is not static but follows a stochastic process. In recent years, a state space model, the Markov switch model, has been deployed to analyse the nonlinear relationship between money and inflation. The application of Markov switch model will be discussed in Chapter 4.

In discussing estimation of the parameters, we first assume that the value of all hyper-parameters h, θ, ϕ, R and Q are known with certainty[1]. A Kalman filter, named for the contributions of Kalman (1960), can then be used for calculating the value of α_t based on information observed through to date $t-1$.

The Kalman filter is an algorithm for calculating the conditional mean $\alpha_{t+1|t}$ and its mean squared error, $P_{t+1|t}$, based on information I_t through the iteration based on the result from previous calculations. Starting values for the Kalman filter are obtained by assuming the initial value of state variable α_1 is drawn from a normal distribution with mean $\alpha_{1|0}$ and variance $P_{1|0} = E(\alpha\alpha^{'})$. The conditional mean and variance of the state vector α_t is then obtained by the standard update equation of the Kalman filter[2].

The Kalman filter can be written as:

$$\alpha_{t+1|t} = \phi\alpha_{t|t-1} + \phi P_{t|t-1} h(h^{'} P_{t|t-1} h + R)^{-1}(y_t - h\alpha_{t|-1} + \theta x_t),$$

$$P_{t+1|t} = \phi P_{t|t-1}\phi^{'} - \phi P_{t|t-1} h(h^{'} P_{t|t-1} h + R)^{-1} h^{'} P_{t|t-1} h^{'} + Q.$$

[1]Hyper-parameters are parameters of a prior distribution for the underlying variable in Bayesian statistics.

[2]For details of the standard Kalman filter, see Hamilton (1994,pp.3047-3051). Terasvirta, Tjostheim and Granger (2010) provide a review of extensions to the standard Kalman filter.

The update equation for the Kalman filter obtains the value for state vector based on the known hyper-parameters. However, in practice, some of these parameters are usually unknown.

In this case, we need to estimate the parameters first with the estimation of the state vector, α_t, conditional upon these estimated parameters. This can be achieved by estimating the hyper-parameters through maximum likelihood. Estimation of α_t is then based on the estimated hyper-parameters. The log likelihood function for this, as suggested by Hamilton (1994), is given by:

$$\begin{aligned}\ln L = & -\frac{Tn}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{T}\log|\Sigma_t| \\ & - \frac{1}{2}\sum_{t=1}^{T}[y_t - (h\alpha_t + \theta x_t)]^{-1}[\Sigma_t][y_t - (h\alpha_t + \theta x_t)],\end{aligned}$$

where $\Sigma_t = \theta' P_{t|t-1}\theta + \varepsilon_t$. The log likelihood function can be maximized with respect to the unknown parameters of the model. However, Kim and Nelson (1999) argued that the maximum point achieved though maximizing the log likelihood function may not be unique. This is a potential disadvantage of the classical approach which treats the Maximum Likelihood estimates as if they were the true values for the model's hyper-parameters.

Within an alternative Bayesian approach, both the model's hyper-parameters and the state variable α_t are treated as random variables. In contrast to the classical approach, inference on α_t is based on the joint distribution of α_t and the hyper-parameters, not the conditional distribution. Gibbs sampling,which was introduced by Geman and Geman (1984), makes Bayesian inference in the state space model easy to implement.

For Gibbs sampling, Kim and Nelson (1999) considered the following iterative procedure, with initial values for the hyper-parameters:

- Step one: simulate the state variable, α_t, from a distribution, conditional on the model's hyper-parameters and the observed data;
- Step two: simulate the hyper-parameters from a distribution conditional on estimated α_t from step one and the observed data.

For the generation of the state vector, $\bar{\alpha}_T$, given hyper-parameter H_T and observed data sequences $\bar{y}_T$ and $\bar{x}_T$, there are generally two ways to apply Gibbs sampling: single-move Gibbs-sampling and multi-move Gibbs-sampling. The single-move Gibbs sampling, originally suggested by Carlin,Polson and Stoffer (1992), generates the state vector α one element at a time. In this method, the state vector is generated from the following conditional distribution:

$$p(\alpha_t|\bar{\alpha}_{\neq t}, h, \theta, \phi, y_T, x_T), t = 1, 2, ..., T$$

where $\bar{\alpha}_{\neq t}$ is the state vector excluding α_t. Single-move Gibbs sampling is usually considered inefficient in computation and for achieving convergence. As a result, Carter and Kohn (1994) introduced an alternative multi-move Gibbs sampling technique where the state vector is generated as a whole from the joint distribution, given by:

$$
\begin{aligned}
&p(\bar{\alpha}_T|H, \bar{y}_T, \bar{x}_T) \\
=&p(\alpha_T|H, \bar{y}_T, \bar{x}_T)p(\bar{\alpha}_{T-1}|\alpha_T, H, \bar{y}_{T-1}, \bar{x}_{T-1}) \\
=&... \\
=&p(\alpha_T|H, y_T, x_T)p(\alpha_{T-1}|\alpha_T, H, \bar{y}_{T-1}, \bar{x}_{T-1})...p(\alpha_1|\alpha_2, H, \bar{y}_1, \bar{x}_1) \\
=&p(\alpha_T|H, y_T, x_T)\prod_{t=1}^{T-1} p(\alpha_t|\alpha_{t+1}, H, \bar{y}_t, \bar{x}_t).
\end{aligned}
\tag{2.10}
$$

The validity of (2.10) is established by the Markov property of α_t which contains only information about α_{t-1}. Equation (2.10) suggests that the whole state vector for $\bar{\alpha}_T$ can be generated by first generating α_T given observed data and hyper-parameters, and then, for $t = T-1, ..., 1$, generating α_t from $p(\alpha_t|\alpha_{t+1}, H_T, y_T, x_T)$, given the generated values for α_{t+1}. After estimating the state vector, the hyper-parameter in the state space model can be calculated based on the estimated state vector and observed data.

The Gibbs sampler therefore proceeds as follows:

$$\alpha^i \sim p(\bar{\alpha}_T|H^i, \bar{y}_T, \bar{x}_T),$$

$$H^{i+1} \sim p(H|\bar{\alpha}^i_T, \bar{y}_T, \bar{x}_T)$$

where $i = 1, 2, ..., k$. Repeating this process k-times generates the Gibbs sequence. A key issue in the successful implementation of Gibbs sampling is the number of runs required until the sequence approaches convergence (burn-in period). Typically, as suggested by Kim and Nelson (1999), the first 1000 to 5000 elements of a Gibbs sequence are discarded as they are considered to be correlated with initial values. By excluding the burn-in period, the estimation of hyper-parameters and state vector are approximated through taking the average of the simulation results of sampler. In Chapter 5, we will extend the discussion of Gibbs sampling as it is critical in the estimation of infinite Hidden Markov switch model.

2.6 Conclusion

In this chapter, we have discussed a variety of empirical studies related to the relationship between monetary growth and inflation. The result of these studies suggest an intimate link between money and inflation. Many studies claimed that there is a unitary relationship between money and inflation in the long run as suggested by the quantity theory of money. However, this conclusion is challenged by Benati (2009) and Sargent and Surico (2010) through testing the relationship between money and inflation across different time periods and countries. Another type of study concerns Granger causality between money and inflation. However, in the absence of a structural model, the result of Granger causality test can not suggest direct causality between money and inflation.

Otherwise, the linear regression model is used to study the reaction of inflation to preceding monetary growth as monetary growth rate being suggested to lead the inflation rate. Existing studies suggest that the linear model performs better when lagged monetary growth are included.

Various nonlinear models nonlinear models are reviewed in this chapter. Studies based on the nonlinear model claim that the nonlinear model supports the assumption of a nonlinear relationship between monetary growth and inflation. In the next chapter, we will discussed the linear regression model between inflation and lagged monetary growth based on UK data. We also test potential structural breaks so as to check the suitability of the linear model in describing the relationship between monetary growth and inflation.

Chapter 3

Testing the relationship between the money supply and inflation

3.1 Introduction

The linear regression model has been a primary model in analysing the relationship between inflation and money. However, as discussed in Chapter 2, the relationship between inflation and money was unstable in the short run. Empirical studies based on nonlinear models, as discussed in Chapter 2, also support this contention. We start our study by investigating underlying linear model of regressing inflation on preceding monetary growth. Then, we test if there exist structural break in the linear model. If so, the relationship between inflation and money should not be expressed by fixed parameters.

In the absence of a structural model between monetary growth and inflation, the linear model is investigated by regressing inflation on preceding monetary growth. Then, the test of potential structural breaks in the linear model will be discussed based on the DQ and SQ test which were introduced by Qu (2008). The remaining sections of this chapter are organized as follows. Section 3.2 discusses the data used to model the reduced form between inflation and preceding monetary growth. There are four different monetary series are considered in the study: quarterly 3 month growth rate of M0 (hereafter M003), the quarterly 3 month growth rate of M4 (hereafter M403), quarterly 12 month growth rate of M0 (hereafter M012), quarterly 12 month growth rate of M4 (hereafter M412). The linear regression models based on the different datasets are discussed in section 3.3. The quantile regression model, which is alternative to the linear model, is discussed in section 3.4. The SQ and DQ tests, which are designed to test structural breaks based on the quantile regression model, are discussed in section 3.5. The subsequent section discussed the results of SQ and DQ tests. The Chapter is concluded in section 3.7.

3.2 Dataset

Both a broad (M4) definition and a narrow (M0) definition of the money supply are considered in our study. In both cases, data are taken from the Bank of England. Data for M4 covers the period from 1966Q2 to 2012Q4; data for M0 covers the period from 1973Q2 to 2006Q1 [1]. Inflation is measured as RPI inflation from Office for National Statistics (ONS)[2]. The quarterly data series are examined as both 3 months growth

[1] The data of M0 from Bank of England discontinued in April 2006.

[2] Currently, the UK monetary policy target the CPI instead of RPI.There are two major differences between CPI and RPI. Frist, RPI includes house price in the basket. Second, RPI is calculated by

rate and 12 months growth rate. For the 3 month growth rate, it reflects the effect of growth in the previous quarter. However, the seasonal effect is expected to have effect on the movement of data. For 12 months growth rate, it suggests the growth rate change over a year instead of one quarter. Therefore, 12 months growth rate intends to have larger variability compared to 3 months growth rate. This expectation is proved in the our data as shown in Table 3.1.

Table 3.1 presents summary statistics for the data. In general, monetary growth has a lower minimum value than inflation, but inflation has a higher maximum value than monetary growth except in the comparison between inflation and M003. Except for M012, the variances of monetary growth are higher than inflation. Otherwise, inflation has a larger skewness and kurtosis than monetary growth in all cases. This suggests that the distribution of inflation has a shaper peak and is more skewed to the right. This suggests that a larger proportion of inflation stay closer and above the mean value when compared to monetary growth. Also, the fatter tail of the distribution of inflation suggest estimation in mean from linear regression model as being inefficient discussed in Chapter 2. To tackle this problem, we implement quantile regressions to explore the relationship between money and inflation, and further to detect potential structural breaks in the relationship between money and inflation. However, for the intention of comparison, we will firstly conduct linear regression to explore the underlying relationship between money and inflation followed by analysis of the drawback resulting from linear regression and potential structural breaks from quantile regression.

using arithmetical mean between old price and new, while CPI uses geometric mean. However, the data of CPI in UK started at 1997. For the continuous of dataset, we adopt RPI as a measurement for inflation in our study.

As shown in Figures 3.1 and 3.2, peaks and troughs in the movement of M412 and M012 exhibit a leading property over the movement of inflation. In contrast, movements of M403 and M003 do not appear to lead the movement of inflation as shown in Figure 3.3 and 3.4. Otherwise, as shown in Figure 3.1, the seasonal effect seems to dominate the movement of M003 despite the movement of inflation over the same period. In that case, the inflation rate evolves independently from the movement of monetary growth. Therefore, we exclude the M003 from all research as it is suspicious .

Table 3.1: Summary statistics of dataset

	$\pi_3^{(1)}$	M403	$\pi_{12}^{(2)}$	M412	$\pi_3^{133(3)}$	M003	$\pi_{12}^{133(4)}$	M012
Mean	1.514	2.598	6.261	10.942	1.651	1.769	6.937	6.991
Min	-2.518	-3.077	-1.566	-4.326	-0.682	-8.544	0.706	-0.729
Max	10.297	8.937	26.576	23.371	10.297	11.164	26.576	17.094
Var	2.238	3.449	27.145	31.074	2.711	23.859	34.0312	12.379
Skewness	1.932	0.155	1.723	-0.241	1.965	-0.383	1.432	0.688
Kurtosis	9.847	3.942	5.906	3.042	8.400	2.244	4.392	3.292

(1)π_3 denotes quarterly 3 months growth rate of RPI
(2)π_{12} denotes quarterly 12 months growth rate of RPI
(3)π_3^{133} denotes quarterly 3 months growth rate of RPI 1973-2006
(4)π_{12}^{133} denotes quarterly 3 months growth rate of RPI 1973-2006

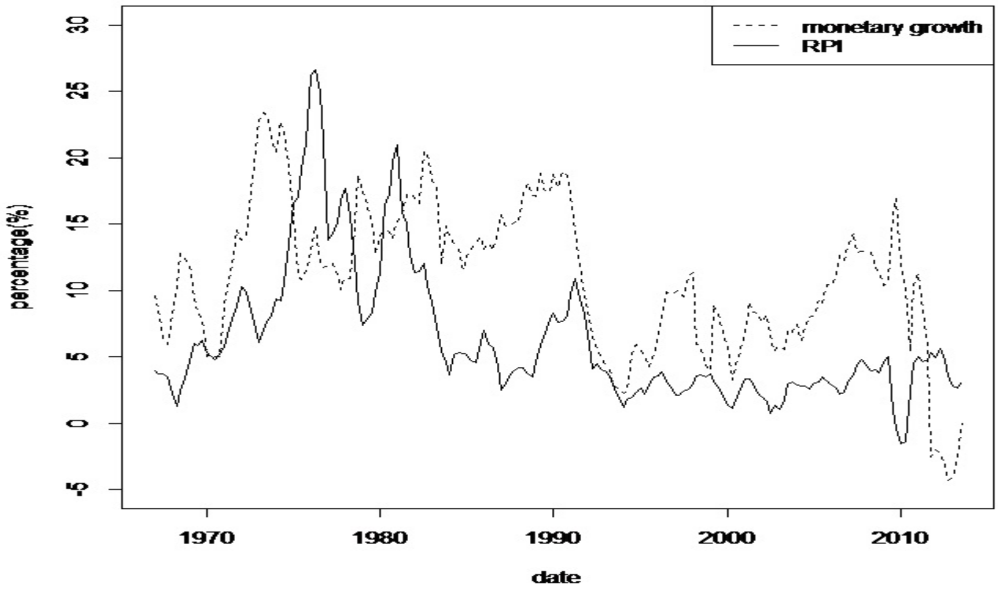

Figure 3.1: Comparison of movements between inflation and M412

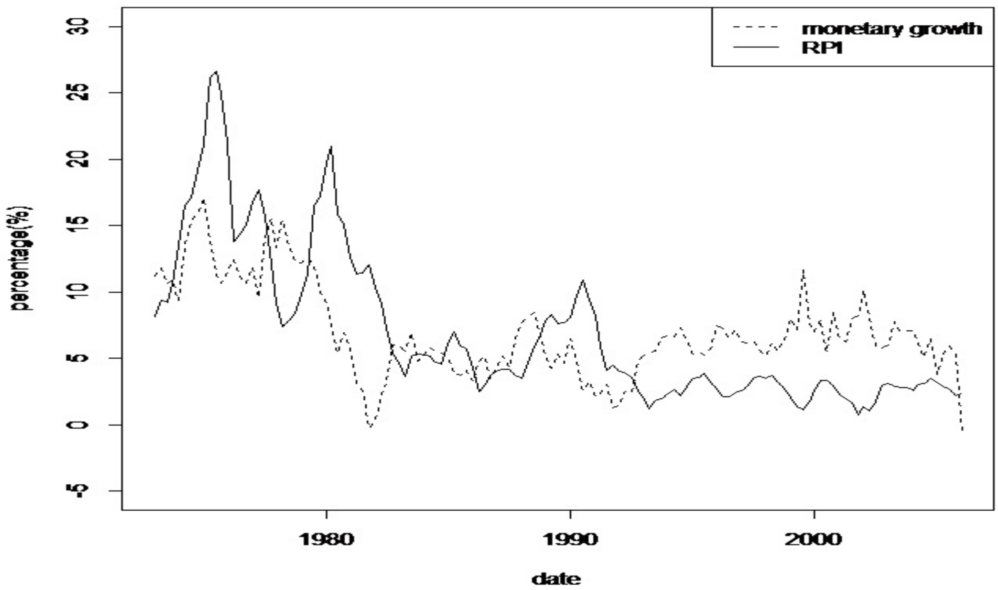

Figure 3.2: Comparison of movements between inflation and M012

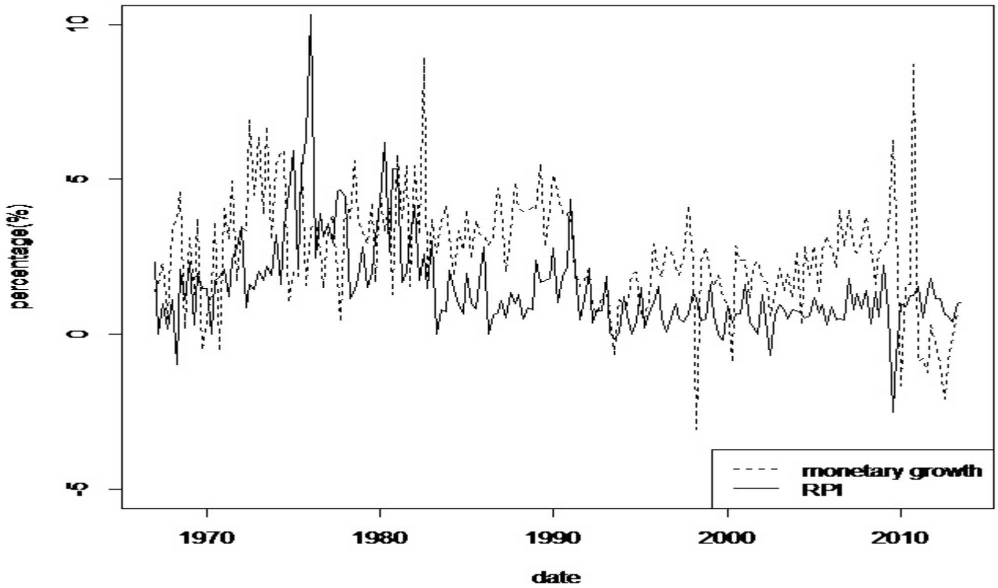

Figure 3.3: Comparison of movements between inflation and M403

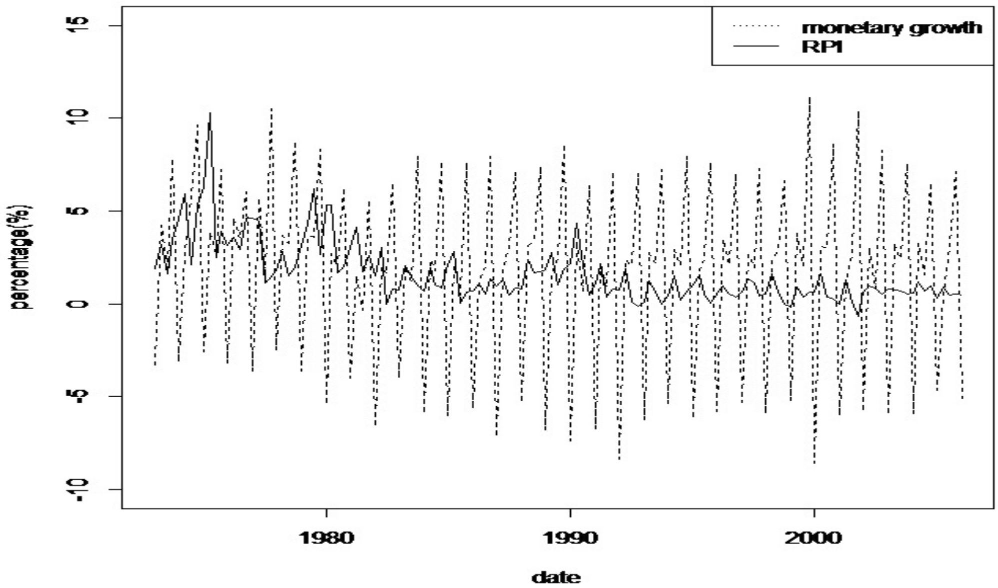

Figure 3.4: Comparison of movements between inflation and M003

3.3 The linear relationship between the money supply and inflation

Despite the debate in transmission mechanism, the quantity theory of money has been widely accepted as the basis for a relationship between money and inflation. In the long run, the quantity theory suggest that inflation varies directly with money. In the short run, the relationship between money and inflation is unstable as inflation is also affected by other economic factors.

As discussed in Chapter 2, McCallum and Nelson (2010) suggested that the introduction of lagged monetary growth would improve the performance of a linear regression model between inflation and monetary growth. A linear regression model of inflation on monetary growth, as suggested by McCallum and Nelson (2010), can be written as:

$$\pi_t = \beta_0 + \beta_1 m_t + \beta_2 m_{t-1} + \beta_3 m_{t-2} + \beta_4 m_{t-3} + \varepsilon_t$$

However, the number of lags for monetary growth in the model is not necessarily three. Friedman (1972) found that the highest correlation with the inflation rate was for money leading twenty months for M1 and twenty-three months for M2 in the US for the period, 1966 to 1979. This result is consistent with the finding of Batini and Nelson (2002), who claimed that the lead between monetary growth rate over inflation is relatively unstable from one to three years. Friedman (1961,pp.476) also stated that "the lag of monetary policy may be long because the effects are distributed over an extended period rather than being concentrated in time." Otherwise, the number of lags could also be varied over time with the change of economic conditions even though they are unknown.

First, we investigate the optimal lag length in a linear model describing the relationship between monetary growth and inflation. We write the general regression for inflation on lagged money following McCallum and Nelson (2010) as:

$$\pi_t = \alpha + \sum_{k=0}^{q} \beta_k m_{t-k} + \varepsilon_t, q \in [0, t) \tag{3.1}$$

where ε_t is an error term, α is a constant and k indicates the lag length. For a comparison of model fitness between different lag-settings, we examined the mean squared error (MSE) and mean abosolute error (MAE). The MSE is given by:

$$MSE = \frac{1}{n} \sum_{n=1}^{n} (\tilde{y} - y)^2$$

where $\tilde{y}$ is the estimated value and y is the observed value. MSE uses the average of squared errors to measure the fitness of the estimation to the data. However, like the variance, the MSE weights large errors more heavily than small ones by taking the square. Models with smaller MSE values outperform models with higher MSE.

The Mean absolute error (MAE) is given by:

$$MAE = \frac{1}{n} \sum_{n=1}^{n} |\tilde{y} - y|.$$

Compared to the MSE, MAE measures the average of the absolute errors applying a symmetric linear penalty irrespective of sign rather than focusing on large errors.

Based on our data, the MSE and MAE have been calculated as a way of comparing the model in the form of (3.1) with different lag-order. As shown in Table 3.1, with increasing lags, the fitness of model improves in terms of having lower MSE and MAE.

Table 3.2: The value of MSE and MAE for linear regression model

	m403		m412		m012	
lags	MSE	MAE	MSE	MAE	MSE	MAE
k=1	2.090	0.965	23.377	3.356	25.934	4.064
k=2	1.900	0.948	22.769	3.289	23.730	3.955
k=3	1.899	0.951	22.070	3.216	21.146	3.818
k=4	1.852	0.922	21.155	3.156	18.852	3.605
k=5	1.830	0.925	20.252	3.138	17.389	3.451
k=6	1.716	0.911	18.862	3.108	16.544	3.349
k=7	1.700	0.904	17.886	3.091	15.730	3.263
k=8	1.641	0.890	17.341	3.036	14.703	3.161
k=9	1.610	0.877	17.182	3.004	14.048	3.099
k=10	1.563	0.853	17.026	2.978	13.755	3.053
k=11	1.561	0.854	16.740	2.987	13.590	3.036
k=12	1.550	0.844	16.553	2.968	13.586	3.035

In addition to the MAE and MSE, we also apply the Akaike information criterion (AIC), which was introduced by Akaike (1974). The AIC not only measures the goodness of fit for the model, but also includes a penalty for model complexity. The AIC can be written as:

$$AIC = 2k - \ln(L),$$

where L is the log likelihood of the selected model and k is the number of model parameters. Burnham and Anderson (2002) argue that using the AIC would possibly face a problem of overfitting with increased probability of selecting a model with more parameters than desired when the sample size is not many times larger than the number of parameters. In our case, the number of observations for both quarterly monetary growth and inflation are less than 200. In order to offset the overfitting problem, Hurvich and Tsai (1989) introduced the AICc test which incorporated an additional penalty $2k(k+1)/(n-k-1)$ into the AIC test. When the sample size is large, the AICc value converges to the AIC. However, when the sample size is close to k^2, the AICc test should select the correct model more efficiently than the AIC test. The result for both the AIC and the AICc based on various lag lengths listed in Table 3.3, where the model with lowest AIC and AICc occur at a lag length less than 12. However, sample size does not seem to cause overfitting problem as both AIC and AICc suggesting same result. Otherwise, the lag length suggested by both AIC and AICc is less than results from MSE and MAE. This result suggests that the linear regression model with more lags of monetary growth is not necessarily efficient in the estimation of inflation. In next Chapter, we will investigate that nonlinear model will also improving the fitness of linear model with less lags of monetary growth in estimating inflation. Nevertheless, all the results suggest that the lag length in equation (3.1) should be larger than three quarters suggested by MacCullum and Nelson (2010).

Table 3.3: Value of AIC and AICc

	m403		m412		m012	
lags	AIC	AICc	AIC	AICc	AIC	AICc
k=1	-1045.771	-1045.551	-571.7316	-571.503	-397.1972	-396.8772
k=2	-1061.621	-1061.29	-574.473	-574.128	-406.7463	-406.2625
k=3	-1059.750	-1059.284	-578.086	-577.600	-419.7318	-419.0488
k=4	-1062.390	-1061.764	-583.703	-583.052	-432.6578	-431.7398
k=5	-1062.672	-1061.863	-589.554	-588.712	-441.16411	-439.974
k=6	-1072.623	-1071.606	-600.361	-599.302	-445.6366	-444.1366
k=7	-1072.370	-1071.12	-607.919	-606.618	-450.1927	-448.344
k=8	-1077.008	-1075.5	-611.494	-609.274	-456.9721	-454.7348
k=9	-1078.581	-1076.787	-611.866*	-609.922*	-460.9017	-458.235
k=10	-1082.140*	-1080.036*	-611.143	-609.291	-461.6421*	-458.5042*
k=11	-1080.364	-1077.922	-610.788	-608.596	-461.2061	-457.554
k=12	-1079.716	-1076.909	-610.864	-608.937	-459.2460	-455.0355

Otherwise, the results of those tests are based on the linear regression model which, as discussed in Chapter2, focuses on the conditional mean of distribution. However, the relationship between money and inflation may not be stable over time. In different quantiles of the distribution, the relationship between inflation and money may vary. In order to study changes in the relationship between inflation and money. we implement quantile regression in the next section as a comparison for the results from the linear regression.

3.4 Quantile regression for the relationship between inflation and money

As discussed in previous Chapter, a linear regression is the result of the conditional mean value which can not draw a full picture of the relationship between inflation and money. The linear regression is particularly inefficient in describing outliers in the distribution of inflation.

The quantile regression, as discussed in section 2.5, is capable of describing the full inflation distribution, conditional on money to a range of quantiles. In this section, we will compare the estimated value from a quantile regression with the estimated value from a linear regression of inflation on monetary growth.

The estimated function of inflation on money is the same as equation 3.1. The lag length of money chosen for each dataset is taken from the results of AIC and AICc based on the linear regression as discussed in section 3.3. The range of quantiles is set from 0.1 to 0.9. For three different dataset, we compare estimation from the quantile regression, the linear regression and the actual inflation movements. The results are listed from Figure 3.5 to Figure 3.10 where the red line is the movement of inflation, the blue line is the estimation from linear regression, the dark line represents the estimation of the quantile regression range from 0.1 to 0.9 quantile.

The estimations based on M403 from the quantile regression and the linear regression are listed in Figure 3.5 and 3.6. The estimated values from linear regression failed to capture the peaks and troughs of movement in inflation. On the contrary, the estimated values from quantile regression cover a range of results based on quantiles from 0.1 to 0.9. In Figure 3.5, estimations of quantile regression cover results based on lower quantiles from $q = 0.1$ to $q = 0.4$. As shown in Figure 3.5, the quantile regression outperform the linear regression in capturing troughs in the movement of inflation. In Figure 3.6, the estimated values from quantile regression cover results based on higher quantiles from $q = 0.5$ to $q = 0.9$. As shown in Figure 3.6, the quantile regression outperform the linear regression in terms of fitting the peaks of movement in inflation. The same phenomena can be found in case of quantile regression based on M412 and M012 as shown from Figure 3.7 to 3.10.

The results from quantile regression also suggest that the relationship between inflation and money is not stable over time as the development of inflation switches between the estimated values of quantile regression based on different quantiles.

However, the way quantile regression describe the relationship between inflation and money also has its disadvantages. First, it is difficult to check the date when the relationship between money and inflation changes. Second, it is hard to decide which quantile is more suitable for describing the relationship between inflation and money at a certain time. In order to detect the time of change, Qu (2008) introduced the SQ and DQ test which described in the next section.

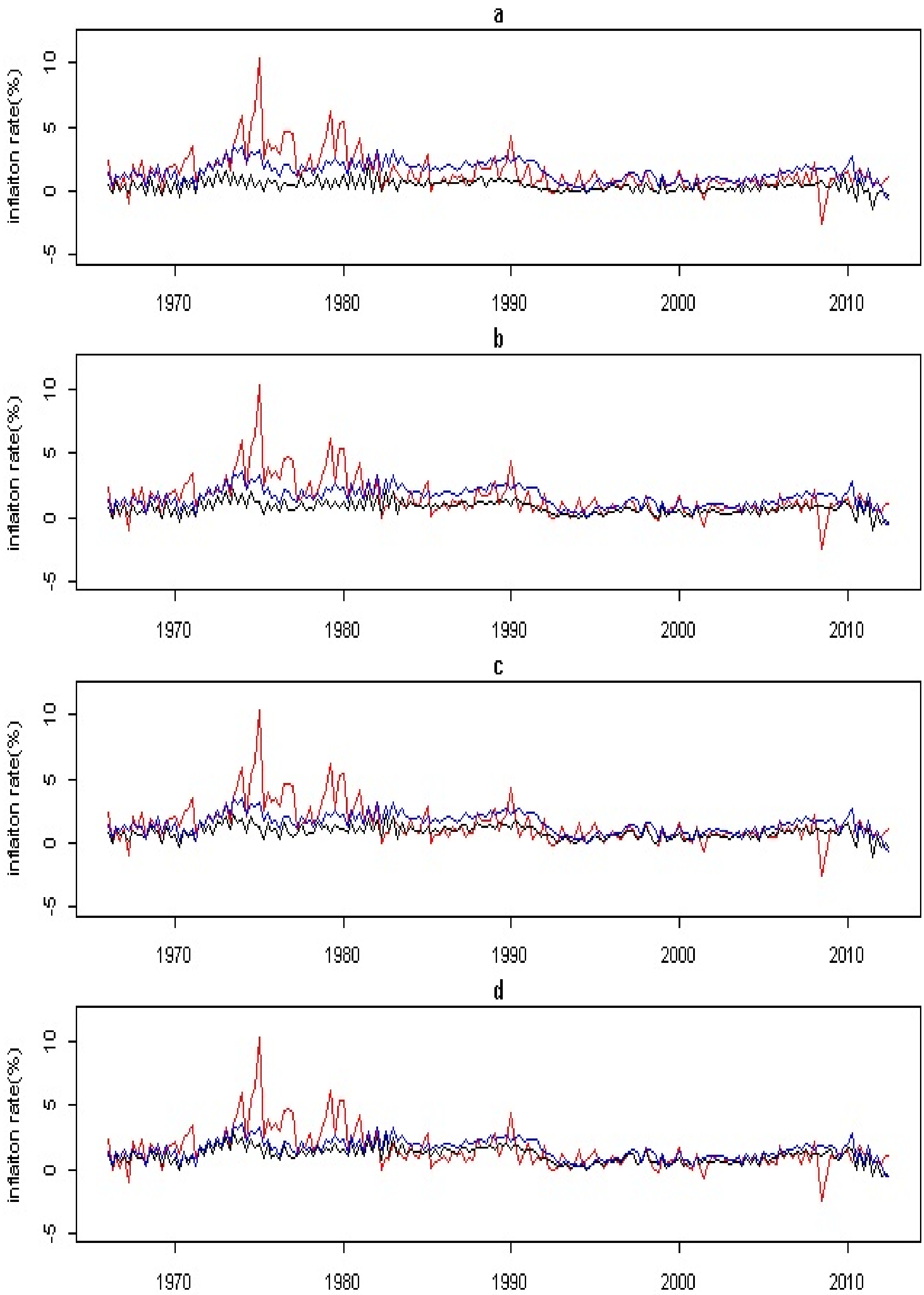

Figure 3.5: The estimation of quantile regression based on M403; (a) quantile=0.1; (b) quantile=0.2; (c) quantile=0.3; (d) quantile=0.4

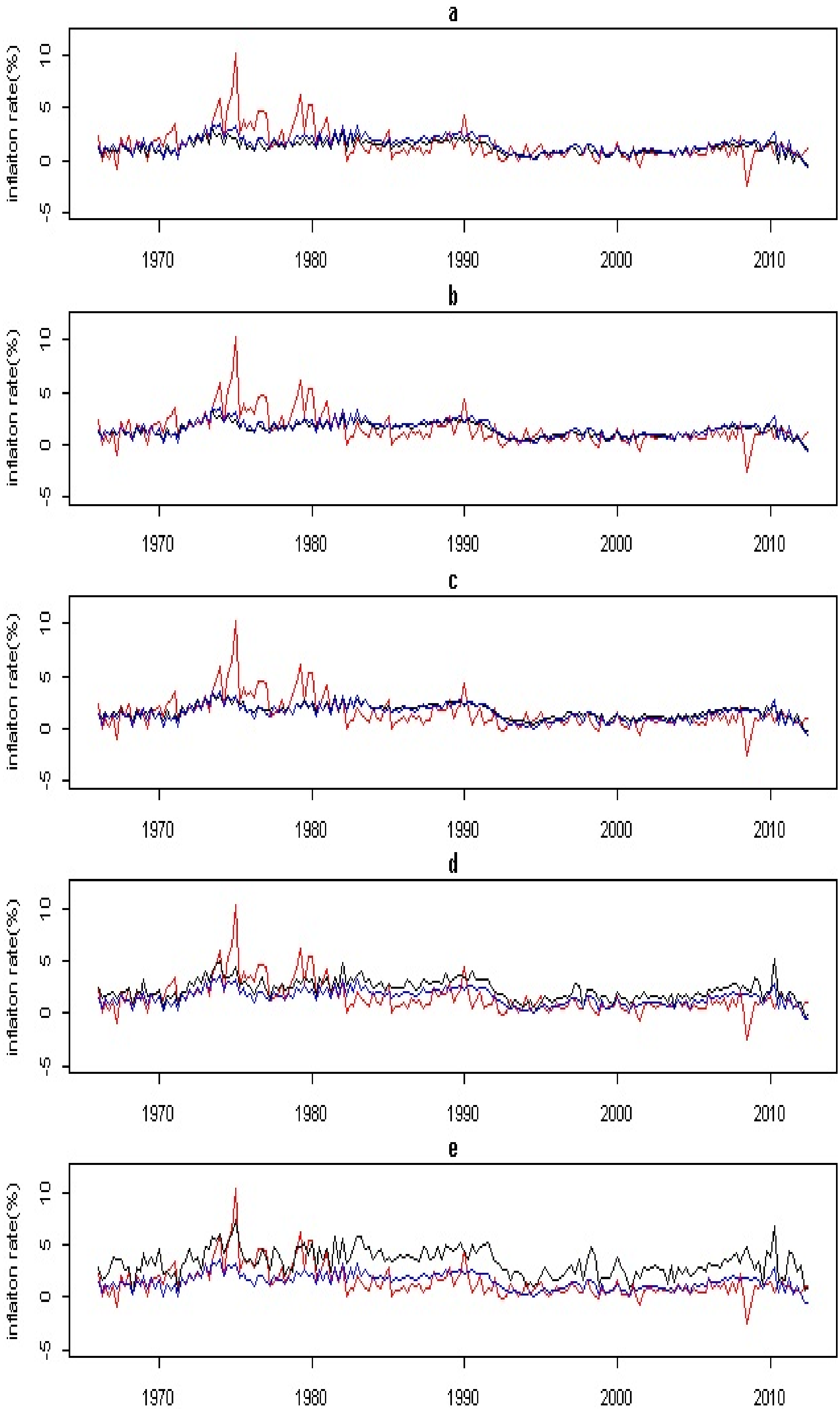

Figure 3.6: (a) The estimation of quantile regression based on M403;quantile=0.5; (b) quantile=0.6; (c) quantile=0.7; (d) quantile=0.8; (e) quantile=0.9

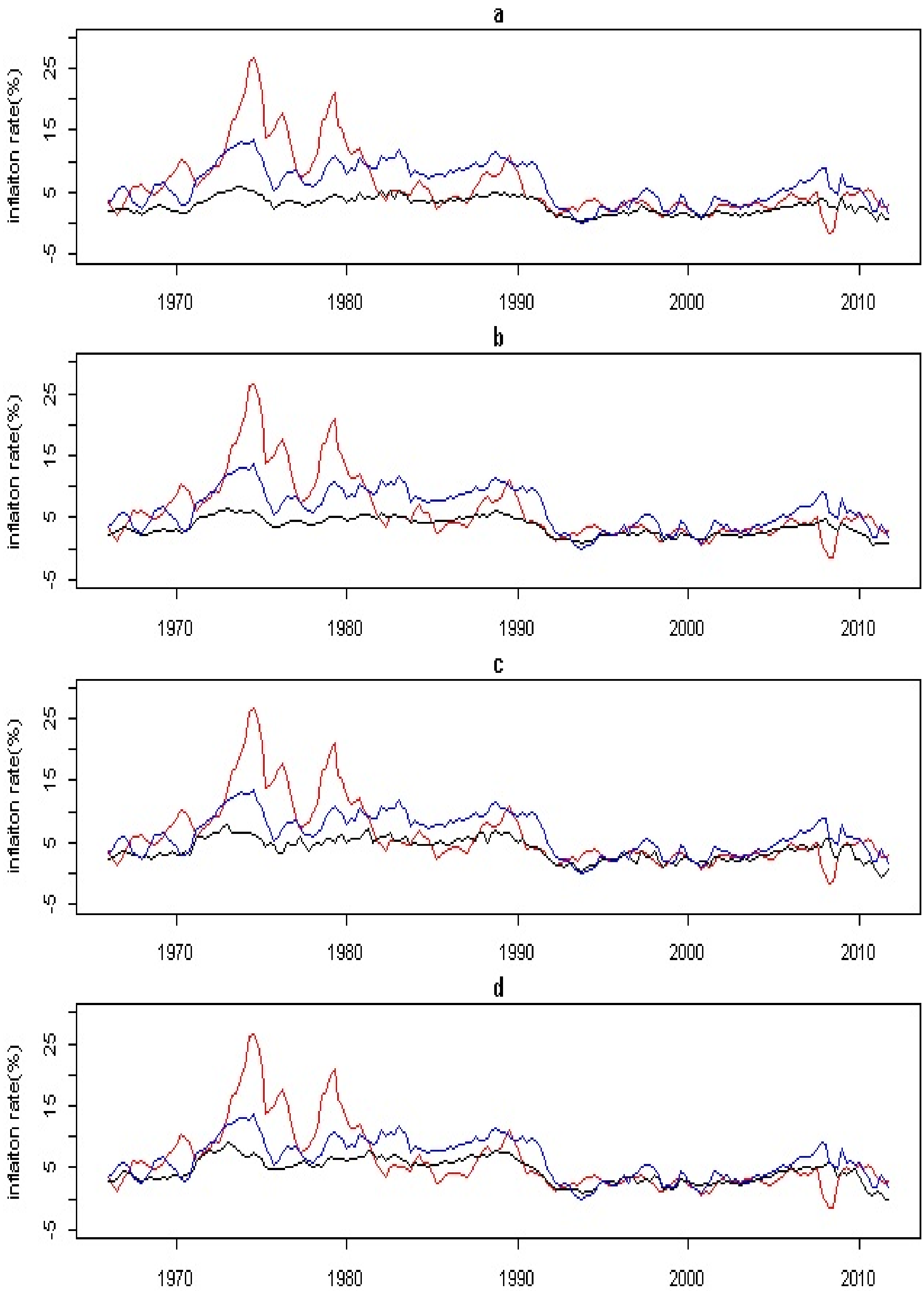

Figure 3.7: The estimation of quantile regression based on M412;(a) quantile=0.1; (b) quantile=0.2 ; (c) quantile=0.3 ; (d) quantile=0.4

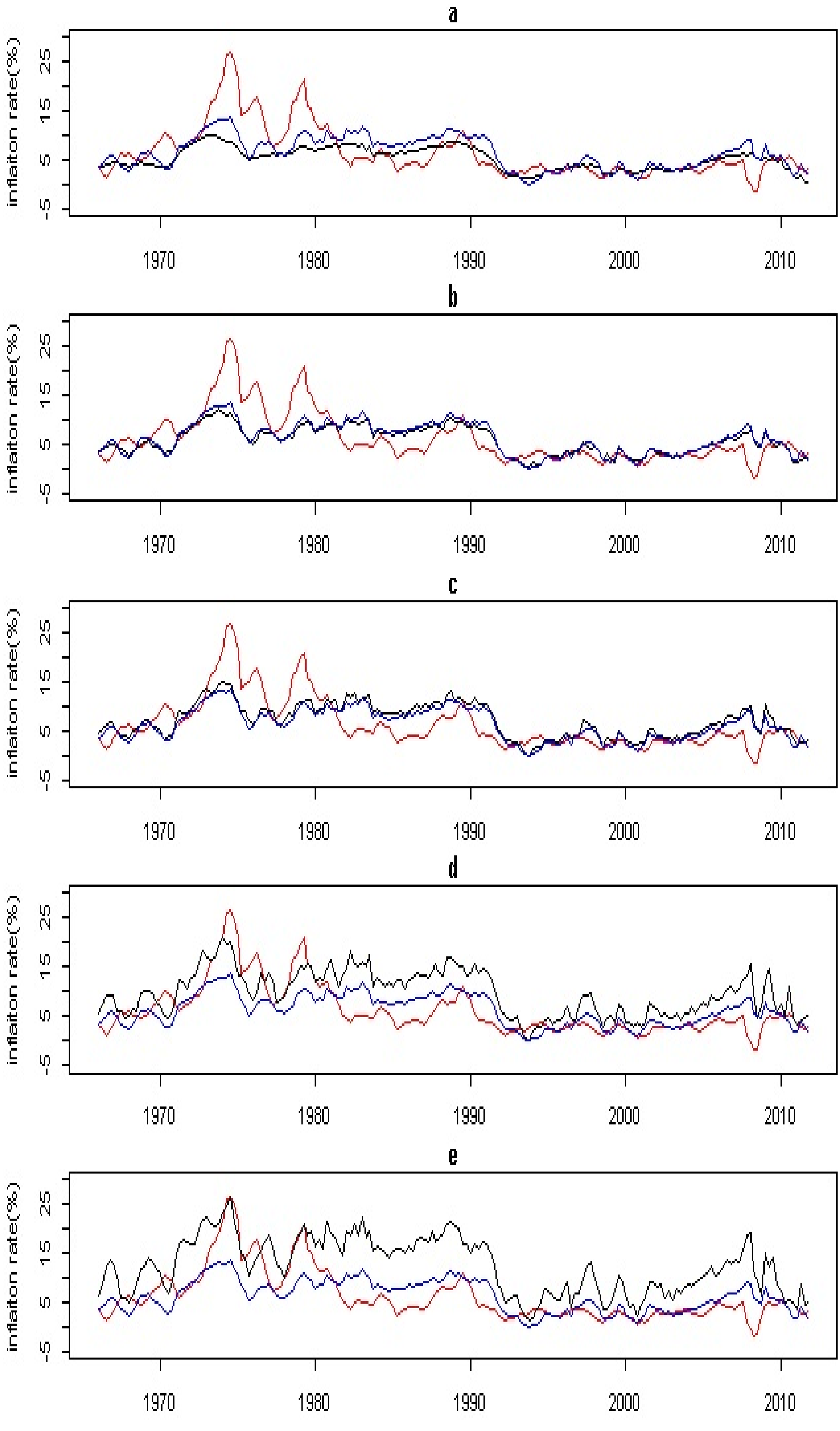

Figure 3.8: The estimation of quantile regression based on M412;(a) quantile=0.5; (b) quantile=0.6 ; (c) quantile=0.7 ; (d) quantile=0.8 ; (e) quantile=0.9

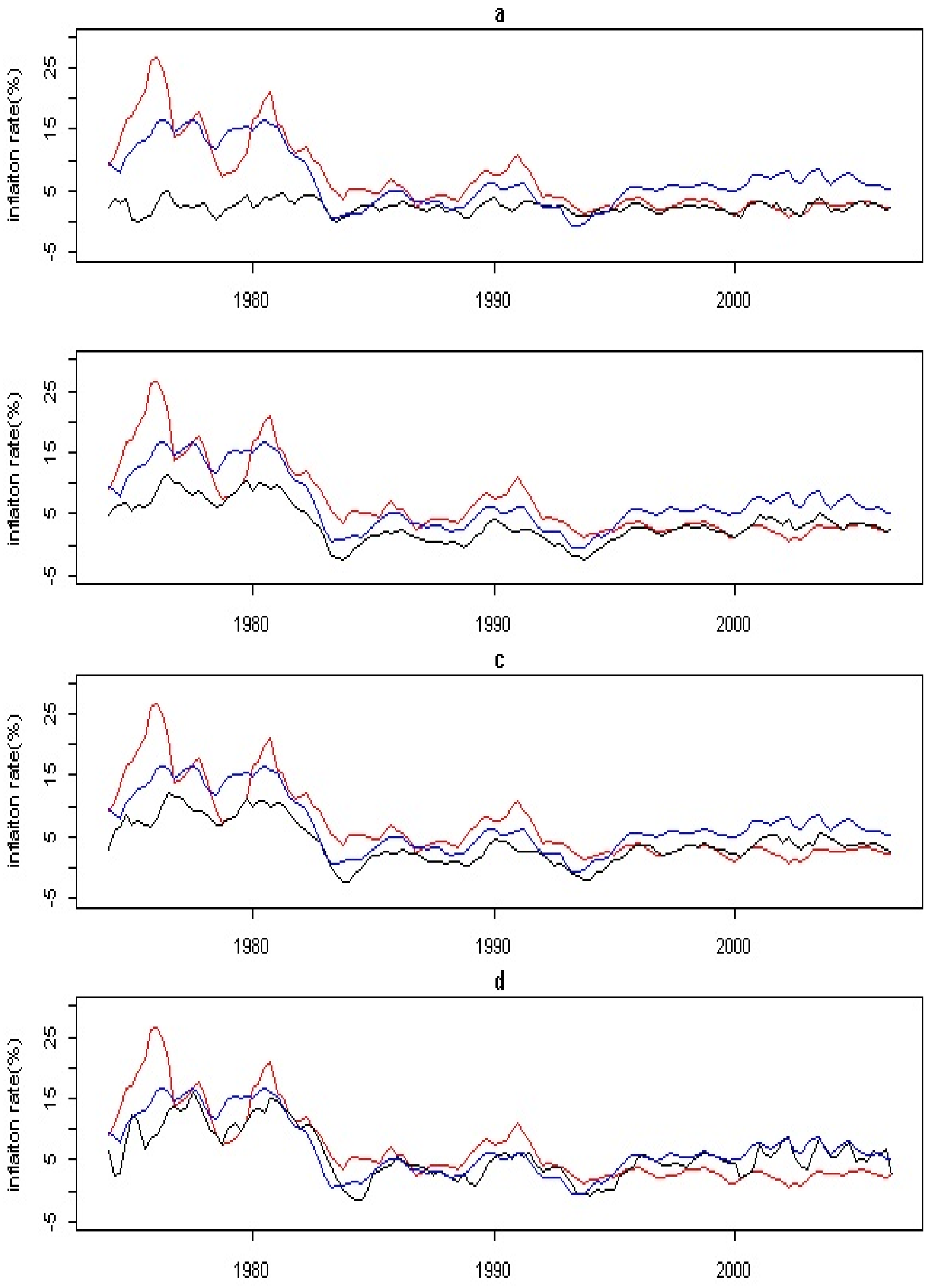

Figure 3.9: The estimation of quantile regression based on M012 (a) quantile=0.1; (b) quantile=0.2 ; (c) quantile=0.3 ; (d) quantile=0.4

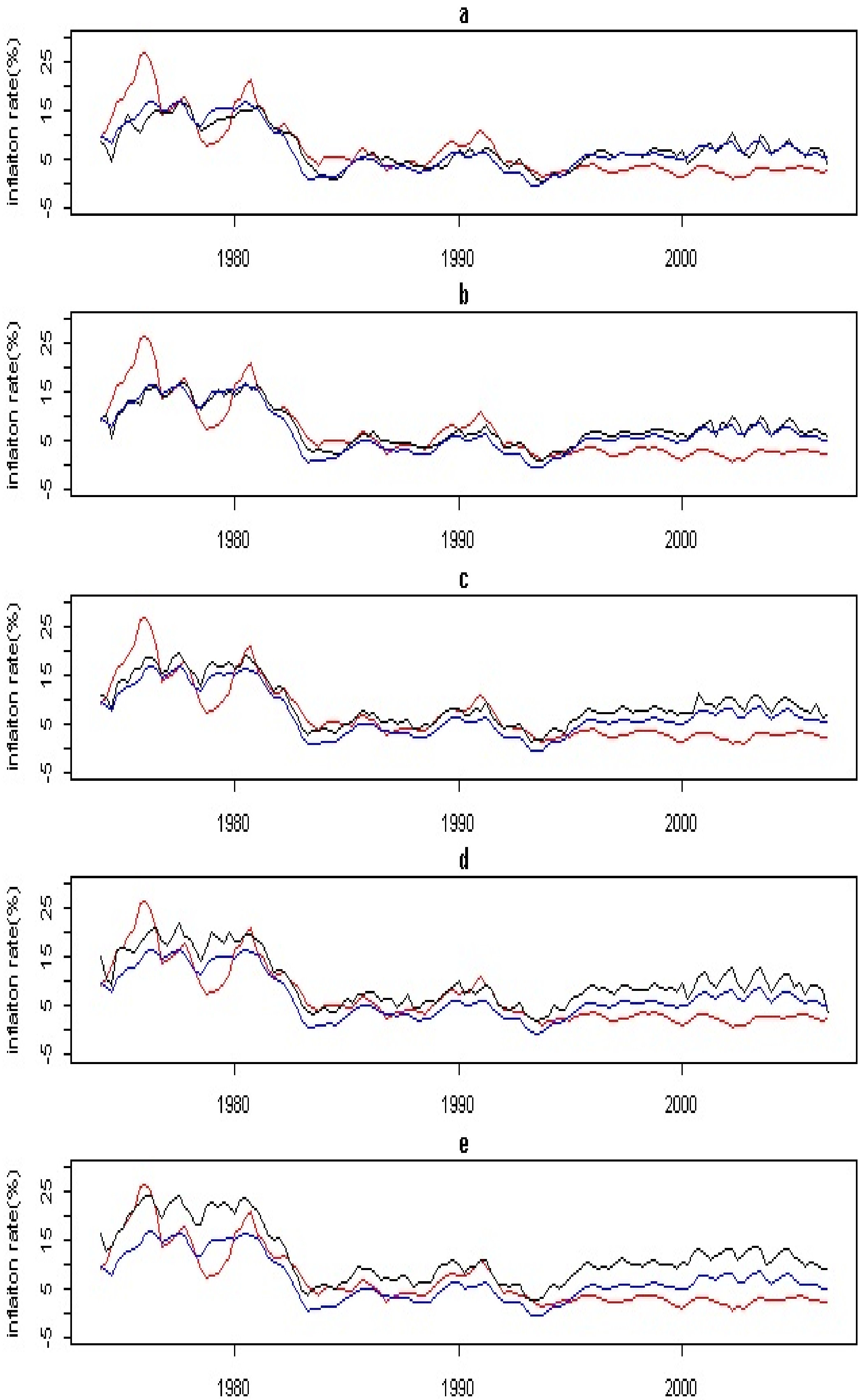

Figure 3.10: The estimation of quantile regression based on M412 (a) quantile=0.5; (b) quantile=0.6 ; (c) quantile=0.7 ; (d) quantile=0.8 ; (e) quantile=0.9

3.5 Test of structural breaks

The diagnosis of parameter instability or structure change is a much studied subject. Various test statistics are proposed in the literature. Chow (1960) suggested the Chow test which is a widely applied method for detecting a structure break. Taking equation (3.1) as an example, if we split our dataset into two groups, we then have:

$$\pi_t = \alpha_1 + \sum_{k=0}^{q} \beta_{1k} m_{t-k} + \varepsilon_{1t}, q \in [0, t) \tag{3.2}$$

and

$$\pi_t = \alpha_2 + \sum_{k=0}^{q} \beta_{2k} m_{t-k} + \varepsilon_{2t}.q \in [0, t) \tag{3.3}$$

The null hypothesis of the Chow test assumes that $\alpha_1 = \alpha_2$ and $\beta_{1k} = \beta_{2k}$. Then, the statistic test for Chow test is given by:

$$s = \frac{(s_c - (s_1 + s_2))/k}{(s_1 + s_2)/(n_1 + n_2 - 2k)}$$

where s_c is the sum of squared residuals for model (3.1), s_1 and s_2 are the sum of squared residuals for group 1 and group 2 separately. k is the number of parameters. n_1 and n_2 are number of observations in each group.

For an application of the Chow test, Hendry and Ericsson (1990) test the consistency of money demand in the UK and found inconsistency in money demand in 1973 and 1979. Hansen (2001) applied the Chow test to find out whether a structural break existed in labor productivity in US in 1970.

However, the Chow test, as discussed above, should assume a position for parameter change in advance and then test it. This make Chow test inefficient in testing structural change for the relationship between money and inflation in the light of unclear transition mechanism from money to inflation.

3.5.1 CUSUM test

In previous section, we applied quantile regression to estimate the movement of inflation and suggested that the relationship between inflation and money may change over time. In this section, we will test for potential structural change based on the quantile regression. Qu (2008) introduced the SQ and DQ test which are designed to detect structural change based on quantile regression. The SQ and DQ tests are based on the technique of CUSUM (cumulative sum) test which represents a well-developed class of test for detecting structural break in an economic model. Dumbgen (1991) and Carlstein (1998) proposed to estimate a break point under the regression model based on these test statistics. Bai and Perron (1998) extended this class of test to regression models without trending regressors and with an unknown number of changes. Other applications of this class of test include: Csorgo and Horvath (1987), Deshayes and Picard (1986), and Szyszkowicz (1994). To introduce the idea of CUSUM test, we first consider the empirical distributions:

$$P_n := \frac{1}{nt} \sum_{i=1}^{nt} f_1(x_i)$$

and

$$P_{n-t} := \frac{1}{n(1-t)} \sum_{i=nt+1}^{n} f_2(x_i).$$

t is a hypothetical change point in $T_n := 1/n, 2/n, ..., (n-1)/n$. x_i is a random variable where $i \in (1, n)$. The expectation of these empirical distribution is denoted by E_n and E_{n-t} respectively. The difference $P_{n-t} - P_n$ estimates the signed measure

$$E_{n-t} - E_n = [(\theta/t) \wedge ((1-\theta)/(1-t))](f_2 - f_1)$$

where θ is a unknown change point to be estimated. The difference $E_n - E_{n-t}$ can be rewritten as:

$$E_{n-t} - E_n = D(t)(f_2 - f_1)$$

where

$$D(t) = (1-\theta)[t/(1-t)]^{1/2} \wedge \theta[(1-t)/t]^{1/2}$$

On the interval $[0, \theta]$ the function $D(t)$ is strictly increasing, and on $[\theta, 1]$ it is strictly decreasing. Therefore, the change point θ is obtained through estimator:

$$\hat{\theta} := argmax[D(t) \cdot (f_2 - f_1)]$$

which corresponds to the maximum likelihood estimator in parametric models.

3.5.2 DQ and SQ test

In the recent literature, Qu (2008) and Oka and Qu (2011) extend basic CUSUM test to the conditional distribution by incorporating quantile regression to obtained the so called SQ and DQ test. Here, we apply the SQ and DQ test to analyse parameter change in the model of the relationship between inflation and monetary growth. We start by briefly reviewing the methodology of the SQ and DQ test.

Let $(y_i, x_i), i = 1, ..., n$ denote a sample of size n, where x_i is a $p \times 1$ vector and i corresponds to a time index. A conditional quantile regression function is then given by

$$Q_{y_i}(\tau|x_i) = x_i'\beta_i(\tau)$$

where τ is the τth quantile, and $\beta_i(\tau)$ are the components of a vector $\beta(\tau)$ that is allowed to be quantile dependent. Suppose that the τ conditional quantile of y_i is a linear function in which a structural change exists if and only if the response of y_i to x_i is different from that of y_j to x_j, that is,

$$\beta_i(\tau) \neq \beta_j(\tau)$$

for some $\tau \in [0, 1]$.

The SQ test is concerned with a structural change in a pre-specified quantile, and it use subsamples up to $[\lambda n]$ with $0 \leq \lambda \leq 1$:

$$S_n(\lambda, \tau, b) = n^{-1/2} \sum_{i=1}^{[\lambda n]} x_i \psi_\tau(y_i - x_i'b)$$

where b is a vector representing an estimate of $\beta(\tau)$ and

$$\psi_\tau = 1(u \leq 0) - \tau,$$

$$where u = y_i - x_i'b$$

Let $X = (x_1', \cdots, x_n')$ and define

$$H_{\lambda,n}(\beta(\tau)) = (n^{-1}X'X)^{-1/2}S_n(\lambda, \tau, \beta(\tau)),$$

which is a weighted empirical process, and is asymptotically distribution-free even if the mean regressor is zero, as discussed in Bai (1996).

The SQ test statistic can be written as:

$$SQ_\tau = \sup_{\lambda\in[0,1]}\|(\tau(1-\tau)^{-1/2}[H_{\lambda,n}(\hat{\beta}(\tau)) - \lambda H_{[1,n]}(\hat{\beta}(\tau))]\|_\infty.$$

The DQ test is also a subgradient-based test, but it is concerned with structural changes across multiple quantiles. The test statistic for the DQ test can be written as:

$$DQ = \sup_{\tau\in\Upsilon_\omega} sup_{\lambda\in[0,1]}\|H_{\lambda,n}(\hat{\beta}(\tau)) - \lambda H_{1,n}(\hat{\beta}(\tau))\|_\infty$$

where $\mathcal{T}_\omega = [\omega, 1-\omega]$ with $0 < \omega < 1$ is a closed set consisting of the quantiles of interest. Both the SQ_τ and the DQ tests resemble the prototypical Kolmogorov-Smirnov two-sample test, and weakly converge to the Brownian bridge which can be written as a Gaussian process[3].

Qu (2008) simulated the critical values for the SQ_τ and DQ tests by using a sample size, $n = 500$, and evaluated the size and power of both tests at the 5% significance level α for $n = 300, 200$ and 100 using critical values based on n=500. The results of size and power from Qu (2008) show that the empirical rejection frequencies for both tests decrease with sample size. In particular, size drops to 0.027 for the DQ test with $n = 100$. Given this, it is interesting to ask whether the critical values would be better estimated for the SQ_τ and DQ tests with respect to each sample size separately. In order to illustrate this point, we re-estimated the critical values for both tests with a sample size set respectively at $n = 100, 200$ and 300.

The simulation of critical values for SQ_τ and DQ tests follows the method in Qu (2008). The SQ_τ test statistics are approximated by $\|n^{-1/2}(\sum_{i=1}^{[\lambda n]} e_i - \lambda \sum_{i=1}^{n} e_i)\|_\infty$, searching over the set $\lambda \in [0, 1]$ in steps of 1/500, where each e_i follows the standard normal distribution. The DQ test statistics are generated by $\|n^{-1/2}(\sum_{i=1}^{[\lambda n]} 1(e_{ji} \leq \tau) - \lambda \sum_{i=1}^{n} 1(e_{ji} \leq \tau))\|$, with e_{ji} independently and uniformly distributed on $[0, 1]$. The number of replications used for estimating critical values of both tests are 50000, and cover cases with up to 10 regressors. The methods of simulating critical values for both tests are set to be consistent with the critical values simulation methods in Qu (2008).

The new critical values based on various sample sizes are listed in Table 4.5 to 4.8, where the critical value decrease with the sample size in all the cases. Furthermore, the difference with critical values between sample size are enlarged with the increase

[3]The proof of the existence of Gaussian processes satisfying the requirements of Brownian bridge which can be found in Polland (1984), pp. 100–103)

of regressors. For example, the difference in DQ's critical values between $n = 100$ and $n = 500$ in the case of 10 regressors would be equivalent to the difference between the case of 4 regressors and 10 regressors for $n = 500$. This difference would be important in the case of small sample size which is quite common in the study of low frequency data.

Table 3.4: Asymptotic critical values for SQ_τ test

		Number of regressors									
	α	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10
	10%	1.165	1.292	1.367	1.414	1.451	1.483	1.503	1.528	1.547	1.561
n=100	5%	1.300	1.417	1.489	1.532	1.567	1.596	1.611	1.638	1.654	1.669
	1%	1.569	1.672	1.727	1.776	1.8	1.825	1.839	1.864	1.883	1.891
	10%	1.184	1.314	1.382	1.429	1.472	1.497	1.525	1.544	1.561	1.579
n=200	5%	1.316	1.441	1.504	1.546	1.586	1.614	1.633	1.652	1.673	1.687
	1%	1.586	1.679	1.750	1.786	1.827	1.836	1.861	1.875	1.900	1.909
	10%	1.190	1.320	1.391	1.437	1.477	1.503	1.534	1.552	1.572	1.585
n=300	5%	1.325	1.443	1.510	1.554	1.591	1.619	1.646	1.661	1.677	1.691
	1%	1.585	1.689	1.750	1.790	1.814	1.845	1.871	1.882	1.891	1.913

Table 3.5: Asymptotic critical values for DQ test when $n = 300$

		Number of regressors									
ω	α	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10
	10%	0.750	0.810	0.842	0.863	0.880	0.893	0.905	0.916	0.923	0.931
0.2	5%	0.808	0.865	0.894	0.916	0.932	0.943	0.955	0.966	0.973	0.980
	1%	0.933	0.981	1.009	1.026	1.039	1.049	1.061	1.068	1.074	1.079
	10%	0.750	0.810	0.842	0.863	0.880	0.893	0.905	0.915	0.923	0.930
0.15	5%	0.808	0.865	0.894	0.916	0.931	0.945	0.956	0.966	0.973	0.978
	1%	0.933	0.981	1.009	1.026	1.039	1.049	1.061	1.069	1.073	1.079
	10%	0.750	0.810	0.842	0.863	0.881	0.894	0.905	0.916	0.923	0.931
0.1	5%	0.808	0.865	0.894	0.916	0.932	0.944	0.956	0.965	0.972	0.980
	1%	0.933	0.981	1.009	1.026	1.040	1.049	1.059	1.068	1.073	1.081

3.5.3 Monte Carlo Experiments

In order to compare the performance of our new critical values with those from Qu (2008), we evaluate the size and power of both the DQ and SQ test at the 5% signifi-

Table 3.6: Asymptotic critical values for DQ test when $n = 200$

		Number of regressors									
ω	α	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10
0.2	10%	0.745	0.802	0.834	0.858	0.874	0.888	0.899	0.909	0.917	0.923
	5%	0.805	0.861	0.888	0.911	0.927	0.937	0.948	0.956	0.964	0.972
	1%	0.925	0.972	0.994	1.019	1.027	1.043	1.053	1.060	1.068	1.072
0.15	10%	0.746	0.807	0.834	0.858	0.876	0.888	0.899	0.908	0.918	0.924
	5%	0.806	0.862	0.890	0.911	0.929	0.938	0.948	0.959	0.967	0.973
	1%	0.925	0.972	0.994	1.019	1.032	1.044	1.052	1.060	1.067	1.074
0.1	10%	0.746	0.806	0.834	0.858	0.876	0.888	0.898	0.909	0.918	0.923
	5%	0.806	0.861	0.888	0.909	0.927	0.938	0.948	0.959	0.967	0.973
	1%	0.925	0.973	0.993	1.019	1.030	1.043	1.052	1.059	1.069	1.072

Table 3.7: Asymptotic critical values for DQ test when $n = 100$

		Number of regressors									
ω	α	p=1	p=2	p=3	p=4	p=5	p=6	p=7	p=8	p=9	p=10
0.2	10%	0.732	0.790	0.820	0.844	0.860	0.875	0.885	0.893	0.900	0.909
	5%	0.791	0.846	0.876	0.896	0.909	0.924	0.933	0.942	0.950	0.955
	1%	0.914	0.957	0.986	1.000	1.010	1.025	1.035	1.044	1.050	1.054
0.15	10%	0.733	0.790	0.821	0.845	0.860	0.874	0.885	0.894	0.900	0.908
	5%	0.792	0.846	0.876	0.896	0.910	0.924	0.934	0.940	0.950	0.956
	1%	0.914	0.955	0.986	1.000	1.008	1.024	1.035	1.046	1.050	1.053
0.1	10%	0.734	0.790	0.821	0.845	0.858	0.874	0.885	0.894	0.900	0.908
	5%	0.792	0.846	0.876	0.896	0.909	0.921	0.934	0.940	0.950	0.955
	1%	0.914	0.955	0.986	1.000	1.010	1.025	1.035	1.044	1.052	1.054

cance level. The data generating process (DGP) is set to be consistent with Qu (2008) based on sample size at $n = 300, 200$ and 100. The DGP for dependent variable y_i is given by:

$$y_i = \alpha_i + \beta_i x_i + (1 + \gamma_i x_i) u_i$$

where $u_i \overset{iid}{\sim} N(0, 1)$ and x_i is assumed to follow a χ^2 distribution with 3 degree of freedom, $x_i \overset{iid}{\sim} \chi^2_3/3$. Although the data generating process is quite simple, it allows us to obtain useful insights into the performance of the model.

Table 3.8: Size of SQ and DQ test at 5% significant level

n	$SQ_{0.5}$	$SQ_{0.7}$	$SQ_{0.85}$	DQ
300	0.049(0.049)	0.046(0.042)	0.039(0.041)	0.045(0.040)
200	0.049(0.044)	0.045(0.041)	0.039(0.033)	0.043(0.037)
100	0.048(0.040)	0.044(0.037)	0.038(0.025)	0.041(0.027)

$*$ the data in the bracket are based on critical values estimated by n=500

For the size test, we set $\alpha_i = \beta_i = \gamma_i = 1$ for each i. Then the DGP is given as:

$$y_i = 1 + x_i + (1 + x_i)u_i.$$

According to the DGP, there exists no structural break in the relationship between y_t and x_t. The size test is carried out by conducting SQ and DQ test based on critical values at 5% significance level. The rejection frequencies for the null hypothesis of no structural break based on our critical values and Qu's are compared in Table 3.9.

In Table 3.9, we compare the rejection frequencies for our three cases with the corresponding result in Qu (2008). With larger differences between sample size, the difference between rejection frequencies is more significant. Qu (2008) claimed that the size for both tests is close to the nominal level 5% even with $n = 100$. However, in the case of $n = 100$, the size is only 0.027 for the DQ test, and slightly smaller still (0.025) for the SQ test at quantile 0.85. However, the size for each case, based on the corresponding critical value is clearly closer to the nominal level than the result in Qu(2008). This is simply because the convergence rate of the asymptotic distribution varies with sample size. However, if the convergence rate is indifferent to the sample size, then the test can be conducted based on a single set of critical values. Obviously, this is not the true for DQ and SQ tests.

Table 3.9: Finite sample power for SQ test at 5% significant level: location change ($n = 200$)

		$n_s = n/2$		
δ	$SQ_{0.5}$	$SQ_{0.7}$	$SQ_{0.85}$	DQ
0.5	0.188(0.147)	0.162(0.119)	0.112(0.078)	0.210(0.161)
1	0.595(0.496)	0.529(0.457)	0.362(0.297)	0.689(0.551)
1.5	0.915(0.864)	0.872(0.819)	0.695(0.590)	0.957(0.909)
2	0.994(0.987)	0.985(0.973)	0.905(0.867)	0.999(0.999)
		$b_s = 3n/4$		
δ	$SQ_{0.5}$	$SQ_{0.7}$	$SQ_{0.85}$	DQ
0.5	0.119(0.102)	0.112(0.084)	0.096(0.067)	0.122(0.087)
1	0.357(0.285)	0.362(0.290)	0.285(0.213)	0.380(0.275)
1.5	0.703(0.587)	0.723(0.625)	0.613(0.510)	0.744(0.620)
2	0.915(0.852)	0.929(0.893)	0.874(0.820)	0.951(0.897)

$*$ the data in the bracket are based on critical values estimated by n=500

We also test sample power in the change of location and scale. For consistent with in Qu (2008), all cases are tested at $n = 200$. For the location change, the following DGP is used;

$$y_i = 1 + x_i + \eta_i 1(i > n_c) + (1 + x_i)u_i$$

where the model suggests that the structural change occurred at n_c. Two cases are considered: $n_c = n/2$ and $n = 3n/4$. For each case, η ranges from 0.5 to 2. The power of the test in terms of location is reported in Table 3.10, which shows the improvement in the power of the test for change at all the positions.

For scale change, the DGP considered is:

$$y_i = 1 + x_i + (1 + x_i + \eta(1 > n_c))u_i.$$

Again, n_c is considered to appear at $n/2$ and $3n/4$ with η varying between 1 and 4. For all cases as shown in Table 3.11, the rejection frequencies of the null hypothesis,

Table 3.10: Finite sample power for SQ test at 5% significant level: scale change ($n = 200$)

	$n_s = n/2$			
δ	$SQ_{0.5}$	$SQ_{0.7}$	$SQ_{0.85}$	DQ
1	0.052(0.043)	0.131(0.106)	0.261(0.224)	0.129(0.089)
2	0.062(0.055)	0.284(0.240)	0.633(0.560)	0.396(0.308)
3	0.071(0.066)	0.438(0.399)	0.846(0.840)	0.689(0.628)
4	0.073(0.069)	0.547(0.521)	0.945(0.935)	0.875(0.850)
	$b_s = 3n/4$			
δ	$SQ_{0.5}$	$SQ_{0.7}$	$SQ_{0.85}$	DQ
1	0.050(0.045)	0.089(0.085)	0.172(0.134)	0.076(0.054)
2	0.052(0.043)	0.151(0.115)	0.394(0.368)	0.159(0.102)
3	0.058(0.046)	0.196(0.187)	0.574(0.562)	0.268(0.176)
4	0.059(0.048)	0.254(0.231)	0.679(0.693)	0.396(0.332)

$*$ the data in the bracket are based on critical values estimated by n=500

that there exist no structure break, is higher than the one based on the critical values corresponding to $n = 500$. This result suggests that the DQ and SQ test would be more sensitive to the structural break compared to the result based on $n = 500$. The results of sample power in the change are to be expected as the critical value decreases with the sample size, more structural breaks can be detected with lower critical values for a sample size smaller than 500.

3.6 Detection of a structural break

Based on the critical values estimated according to each specific sample size, we test the relationship between inflation and monetary growth in the form (3.1). For the convenience of the reader, we re-write (3.1) here

$$\pi_t = \alpha + \sum_{k=0}^{q} \beta_k m_{t-k} + \varepsilon_t, q \in [0, t). \tag{3.4}$$

The number of lags is varied from $k = 1$ to $k = 12$ in our study. In Table 3.11, we present the results of DQ test for the whole sample of three dataset. By comparing the test statistics with critical values listed in Table 3.6, we found that the null hypothesis of no structural break is rejected in every case shown in Table 3.11.

We then conduct SQ test to investigate if the structural break only exist in a certain quantile. The result of SQ test are listed from Table 3.12 to 3.14. The structural break is detected in all quantiles for each lags-setting and dataset. However, in each lags-setting, the date of structural breaks are varied over quantiles. This suggests that more than one structural break may exist in the relationship between monetary growth and inflation. Table 3.12 lists the result of SQ test based on M403. The change point is varied from 1980 to 1985. A similar finding can be observed in Table 3.14 where the change point is varied from 1979 to 1982 for different quantiles on M412. In the case of lower quantiles, as shown in Table 3.12 and 3.13, the change point is suggested at 1982 and 1985 where British government raise the M3 target in 1982 as it was always overshoot and suspended the monetary target in 1985. On the contrary, in the case of higher quantiles, the change point is varied from 1979 to 1980. This matched the onset of the Medium Term Financial Strategy (March 1980) which targeted £M3. The novelty was that the strategy set permissible bands of monetary growth into the future with these bands progressively reduced over time.

For the case of SQ test based on M012 as shown in Table 3.14, the higher quantiles also suggest a break at date around 1982. However, the lower quantiles of SQ test suggests a break in the period from 1993 to 1999 in which the inflation was stabilized after introduction of inflation target in UK.

Table 3.11: The result of DQ test for the whole sample

	m403		m412		m012	
lags	DQ value	break date	DQ value	break date	DQ value	break date
k=1	1.850	1982Q2	2.738	1982Q2	2.621	1993Q1
k=2	2.401	1982Q2	3.103	1982Q2	3.402	1993Q1
k=3	2.675	1982Q3	3.646	1982Q4	4.408	1993Q2
k=4	3.054	1982Q1	4.253	1982Q3	4.831	1993Q2
k=5	3.348	1981Q4	3.348	1982Q2	4.878	1993Q2
k=6	3.414	1982Q2	3.414	1982Q2	5.271	1993Q3
k=7	3.791	1982Q2	3.791	1982Q2	5.309	1993Q4
k=8	4.391	1982Q2	4.391	1982Q2	5.436	1993Q4
k=9	4.736	1982Q2	6.780	1982Q2	5.854	1994Q4
k=10	5.227	1982Q2	7.051	1982Q2	5.899	1993Q4
k=11	5.318	1982Q2	7.239	1982Q3	6.391	1993Q4
k=12	5.495	1981Q4	7.767	1982Q3	6.338	1993Q4

If the null hypothesis of no break against alternative hypothesis of one break being rejected in DQ and SQ test, then following the procedure in Qu (2008), the sequential step is to build a model which includes one break. Testing the null hypothesis of one break against the alternative hypothesis of two breaks, if the null hypothesis being rejected, build a model involve two breaks. This procedure continues until the null hypothesis of n breaks is accepted. However, this procedure is inefficient in the case where many structural breaks exist.

Otherwise, in the case where two or more breaks exist as suggested by results of SQ and DQ tests in our case, a potential pitfall with this procedure is that the SQ and DQ tests can not tell whether the structural break introduced a new structure or an old structure reappeared. In general, if the estimated model introduces a new structure or regime which is actually the reoccurrence of a old structure, the model would then mislead with a misspecified type of structural break. This phenomena will also be discussed in Chapter 5.

Table 3.12: SQ test statistic for the whole sample based on on M403

	τ=0.1	τ=0.2	τ=0.3	τ=0.4	τ=0.5	τ=0.6	τ=0.7	τ=0.8	τ=0.9
k=1	2.155	2.974	3.761	3.614	3.846	3.339	3.331	3.781	2.922
break	2001Q1	1985Q2	1982Q2	1981Q4	1981Q4	1981Q4	1981Q2	1981Q2	1980Q2
k=2	2.761	3.684	4.223	4.189	4.578	5.123	4.911	4.127	3.982
break	1985Q2	1982Q2	1982Q2	1981Q3	1982Q1	1982Q3	1982Q3	1982Q1	1980Q2
k=3	3.211	4.689	4.836	4.998	5.084	5.785	5.821	4.751	3.883
break	1985Q2	1982Q2	1982Q2	1981Q3	1982Q1	1982Q3	1982Q3	1981Q2	1980Q2
k=4	3.618	4.768	5.741	5.189	5.991	6.574	6.941	5.495	4.685
break	1982Q3	1982Q2	1982Q2	1982Q1	1982Q1	1982Q3	1981Q3	1981Q2	1980Q2
k=5	3.547	5.298	6.132	5.239	6.442	5.862	7.184	5.347	4.613
break	1982Q2	1982Q2	1982Q2	1982Q2	1981Q4	1981Q3	1981Q4	1981Q2	1980Q2
k=6	3.698	5.681	6.535	6.817	6.427	7.098	7.549	6.342	5.044
break	1982Q2	1982Q2	1982Q1	1982Q2	1981Q4	1981Q3	1981Q2	1981Q2	1980Q2
k=7	4.222	5.247	7.382	6.574	6.745	7.975	8.759	6.664	4.875
break	1982Q2	1982Q2	1982Q2	1982Q1	1982Q2	1981Q3	1981Q2	1981Q2	1980Q2
k=8	5.248	6.715	8.048	8.640	9.035	8.995	7.367	6.965	5.325
break	1982Q2	1982Q2	1982Q2	1982Q2	1982Q2	1981Q3	1981Q2	1981Q2	1980Q2
k=9	6.641	6.662	7.287	8.839	9.731	8.685	8.675	8.395	8.129
break	1982Q2	1982Q2	1982Q2	1982Q2	1981Q4	1981Q3	1981Q1	1981Q1	1980Q2
k=10	3.532	6.023	8.432	8.613	10.431	9.294	8.922	8.089	6.651
break	1982Q3	1982Q2	1982Q2	1982Q2	1981Q4	1981Q4	1981Q4	1981Q1	1980Q2
k=11	5.625	5.906	8.746	8.637	8.631	10.421	9.456	9.453	5.964
break	1982Q2	1982Q2	1982Q2	1982Q2	1981Q4	1981Q4	1981Q4	1981Q3	1980Q2
k=12	4.386	6.213	8.231	9.452	11.648	11.026	7.962	9.549	9.546
break	1985Q2	1982Q2	1982Q2	1982Q2	1982Q2	1981Q4	1981Q3	1981Q3	1981Q3

Furthermore, the date of structural break varied cross lags-settings and quantiles as shown from Table 3.12 to 3.15. On the one hand, this result may indicates that several structural breaks exist in the relationship between money and inflation. On the other hand, without certain structural model, it is difficult to distinguish between structural breaks in the model and the misspecification in the setting of model.

Table 3.13: SQ test statistic for the whole sample based on M412

	τ=0.1	τ=0.2	τ=0.3	τ=0.4	τ=0.5	τ=0.6	τ=0.7	τ=0.8	τ=0.9
k=1	3.964	5.397	7.178	9.218	10.166	11.432	11.164	11.041	9.328
break	1985Q2	1985Q2	1985Q1	1982Q3	1982Q2	1982Q2	1982Q1	1981Q4	1979Q2
k=2	3.388	6.632	6.825	7.952	10.901	11.851	14.078	12.801	9.276
break	1985Q2	1985Q2	1985Q1	1982Q3	1982Q2	1982Q2	1982Q1	1981Q3	1979Q1
k=3	3.533	5.715	8.273	10.147	11.665	12.396	1.377	10.981	9.767
break	1985Q2	1985Q2	1985Q1	1982Q2	1982Q2	1982Q2	1982Q1	1981Q3	1979Q1
k=4	3.095	6.324	7.883	9.369	10.591	12.374	11.513	11.208	7.773
break	1985Q2	1982Q3	1982Q3	1982Q2	1982Q2	1982Q2	1982Q1	1981Q3	1979Q3
k=5	4.404	5.907	7.793	8.871	9.001	10.131	10.431	8.606	5.803
break	1985Q2	1982Q3	1982Q3	1982Q3	1982Q2	1982Q2	1982Q1	1981Q3	1979Q1
k=6	3.699	5.736	7.958	9.966	10.697	11.242	11.228	8.742	8.767
break	1982Q4	1982Q3	1982Q3	1982Q3	1982Q2	1982Q2	1982Q1	1981Q3	1979Q2
k=7	3.439	6.224	8.098	9.728	11.724	11.559	12.603	9.971	8.753
break	1985Q2	1982Q4	1982Q3	1982Q2	1982Q2	1982Q2	1982Q2	1981Q1	1979Q1
k=8	4.636	6.138	8.526	11.078	13.553	13.436	12.861	12.249	5.814
break	1985Q2	1985Q2	1982Q3	1982Q3	1982Q2	1982Q2	1982Q1	1981Q1	1979Q1
k=9	4.862	6.884	8.942	10.227	12.338	13.821	12.653	11.216	10.138
break	1985Q2	1982Q2	1982Q3	1982Q2	1982Q2	1982Q2	1982Q1	1980Q4	1978Q4
k=10	6.298	7.185	9.335	10.726	13.514	13.049	11.664	11.363	11.598
break	1985Q2	1982Q4	1982Q3	1982Q2	1982Q2	1982Q2	1981Q4	1979Q2	1979Q1
k=11	6.665	7.468	9.714	11.218	13.921	14.103	12.834	9.126	5.726
break	1985Q2	1982Q4	1982Q3	1982Q2	1982Q2	1982Q1	1981Q4	1979Q2	1979Q1
k=12	5.874	9.061	11.152	11.853	14.763	15.237	14.723	9.684	5.751
break	1985Q2	1982Q4	1982Q3	1982Q3	1982Q2	1982Q1	1981Q1	1979Q2	1979Q2

3.7 Conclusion

Rejection of a linear relationship between monetary growth and inflation is compatible with many possible structural relationships. However, no clear structural model for the relationship between inflation and monetary growth is suggested by the theoretical literature. One way to overcome this is to approximate this relationship using a combination of linear models with each one representing a separate period of time. Then, the real structure for the whole sample could be represented by the combination of linear models for each period.

Table 3.14: SQ test statistic for the whole sample based on M012

	τ=0.1	τ=0.2	τ=0.3	τ=0.4	τ=0.5	τ=0.6	τ=0.7	τ=0.8	τ=0.9
k=1	3.201	4.567	5.552	5.443	5.349	4.297	2.929	2.486	3.146
break	1993Q1	1993Q1	1993Q1	1993Q1	1993Q1	1993Q1	1983Q1	1982Q4	1982Q3
k=2	5.734	6.067	7.024	7.416	6.478	5.158	4.381	4.826	3.261
break	1993Q1	1993Q2	1993Q2	1993Q1	1993Q1	1993Q2	1983Q2	1983Q2	1983Q2
k=3	5.149	6.914	7.853	8.581	7.301	5.862	5.014	5.084	5.121
break	1993Q2	1993Q3	1993Q3	1993Q3	1993Q2	1993Q1	1983Q2	1983Q2	1983Q2
k=4	7.238	8.125	8.347	9.362	8.113	6.652	6.021	5.313	3.044
break	1999Q2	1996Q3	1993Q4	1993Q4	1993Q2	1993Q3	1983Q3	1983Q3	1983Q2
k=5	6.262	7.821	10.265	10.919	7.874	8.591	8.576	3.763	5.298
break	1999Q2	1996Q4	1994Q3	1994Q1	1993Q2	1993Q2	1983Q3	1983Q3	1983Q2
k=6	7.156	8.933	9.561	10.081	7.421	6.248	5.091	3.957	8.036
break	1999Q2	1996Q4	1994Q1	1994Q2	1994Q1	1993Q3	1983Q4	1983Q4	1983Q3
k=7	7.642	8.521	9.441	9.689	9.114	6.137	5.717	4.096	4.191
break	1999Q2	1996Q4	1993Q3	1994Q3	1994Q3	1993Q4	1983Q3	1975Q3	1983Q2
k=8	8.161	11.074	13.165	11.298	9.061	10.273	5.066	8.055	6.874
break	1999Q2	1996Q4	1993Q3	1994Q3	1994Q3	1993Q4	1983Q3	1975Q3	1975Q3
k=9	8.541	12.811	13.874	9.531	11.224	7.731	10.777	4.431	3.221
break	1999Q2	1996Q4	1993Q3	1994Q4	1994Q4	1993Q4	1983Q1	1975Q3	1983Q2
k=10	8.954	12.284	9.991	15.084	14.059	5.678	5.552	10.473	6.442
break	1999Q2	1999Q1	1994Q3	1994Q2	1993Q4	1992Q4	1975Q4	1975Q3	1983Q2
k=11	7.767	13.991	11.752	12.835	13.846	5.894	5.601	5.772	4.453
break	1999Q2	1999Q1	1994Q4	1995Q1	1993Q4	1992Q2	1975Q1	1975Q3	1983Q2
k=12	5.276	11.238	13.314	14.296	10.782	8.711	9.937	8.812	7.218
break	1999Q2	1999Q1	1994Q4	1995Q1	1993Q4	1992Q2	1975Q1	1975Q3	1975Q3

However, as we have discussed in chapter 2, many exist nonlinear models are unable to overcome the difficulty to approximate the relationship between money and inflation. In general, there are three requirements to fulfill. First, without a clear transmission mechanician, the factor causing the structural break would be unknown. Thus, a model should be able to control the structural change while remaining agnostic to the source of the change itself. Second,it must be recognized that a structural break does not necessarily indicate a new structure. Some old structures mean re-occur after their first occurrence. Therefore, a model should be able to distinguish between the introduction of a new structure and the reoccurrence of an old structure. Third, the number of structure should be unlimited in order to explore the relationship between monetary growth and inflation.

Chapter 4

Application of a Markov switch model to the relationship between monetary growth and inflation

4.1 Introduction

The STAR-type models, as discussed in chapter 2, can be thought of as modeling smooth changes between regimes[1]. Even though the models' specification are different, they share some important characteristics. First, a regime variable or a stochastic process is selected to drive the regime change. Second, the regime sequence follows a specified function, such as stochastic process or smooth transition process. Third, the number of regimes is typically chosen and is not determined from the data. However,

[1]The regime hereafter suggests a linear relationship between variables. Therefore, switch between regimes refers to the change of coefficient in the linear function.

in some circumstances, these characteristics are not suitable as the determination of the regime is unclear.

These circumstances arise because researchers may have little information on the date at which parameters change and thus need to make inference about the switch points as well as the significance of parameter shifts. Hamilton (1989) introduced the regime-dependent Markov-switching model to capture the unobservable regime variable controlling the regime switch. His model can be viewed as an extension of the Goldfeld and Quandt's (1973) model to the important case of structural changes in the parameters arising from an autoregressive process.

The sections in this chapter is arranged as follows. The key features of a Markov switch model are introduced in section 4.2. We apply the Markov switch model to analyse the relationship between inflation and money in section 4.3. This chapter is concluded in section 4.4 with a discussion of disadvantages in the Markov switch model.

4.2 Introduction of Markov switch model

First, considering a linear regression model without any switch as follow:

$$y_t = \theta x_t + e_t, e_t \overset{iid}{\sim} N(0, \sigma^2).$$

To estimate the parameters of the model in this simple case, the log likelihood function can be written as:

$$\ln L = \sum_{t=1}^{T} ln(f(y_t))$$

where

$$f(y_t) = \frac{1}{\sqrt{2\pi\sigma^2}} exp(-\frac{(y_t - x_t\theta)^2}{2\sigma^2}),$$

which can be maximized with respect to θ and σ^2.

For a model with two regimes, we have :

$$y_t = x_t\theta_{s_t} + e_t,$$

$$e_t \sim N(0, \sigma^2_{s_t}),$$

$$\theta_{s_t} = \theta_0(1 - s_t) + \theta_1 s_t,$$

$$\sigma^2_{s_t} = \sigma^2_0(1 - s_t) + \sigma^2_1 s_t,$$

$$s_t = 0 \; or \; 1, \;\; (Regime \; 0 \; or \; 1)$$

where, under regime 1, the parameters are given by θ_1 and σ_1^2 and, under regime 0, the parameters are given by θ_0 and σ_0^2. In this case, the log likelihood function, as discussed by Hamilton (1989), is given by:

$$\ln L = \sum_{t=1}^{T} ln(f(y_t|s_t)) \tag{4.1}$$

where

$$f(y_t|s_t) = \frac{1}{\sqrt{2\pi\sigma^2_{s_t}}} exp(-\frac{(y_t - x_t\theta_{s_t})^2}{\sigma^2_{s_t}}).$$

Equation (4.1) can be maximized with respect to $\theta_0, \theta_1, \sigma_0^2$ and σ_1^2.

A major problem for solving such log likelihood functions arises when s_t is unobservable.

Kim and Nelson (1999) suggest a two-steps solution for this problem:

- Step one: consider the joint density of y_t and the unobserved variables s_t, which is the product of the conditional and marginal densities:

$$f(y_t, s_t|I_{t-1}) = f(y_t|s_t, I_{t-1})f(s_t|I_{t-1}),$$

where I_{t-1} refers to information up to time t-1.

- Step two: obtain the marginal density for y_t, integrating the s_t variable out of the joint density by summing over all possible regimes s_t:

$$\begin{aligned} f(y_t|I_t) &= \sum_{s_0=0}^{1} f(y_t, s_t|I_t) \\ &= \sum_{s_0=0}^{1} f(y_t|s_t, I_t)f(s_t|I_{t-1}) \\ &= \frac{1}{\sqrt{2\pi\sigma_0^2}} exp(-\frac{(y_t|x_t\theta_0)^2}{2\sigma_0^2})P[s_t = 0|I_{t-1}] \\ &+ \frac{1}{\sqrt{2\pi\sigma_1^2}} exp(-\frac{(y_t|x_t\theta_1)^2}{2\sigma_1^2})P[s_t = 1|I_{t-1}]. \end{aligned}$$

The log likelihood function is then given by:

$$\ln L = \sum_{t=1}^{T} \ln(\sum_{s_t=0}^{1} f(f_t|s_t, I_{t-1})P[s_t|I_t]).$$

The marginal density above can be interpreted as a weighted-average of the conditional densities given $s_t = 0$ and $s_t = 1$ respectively. Thus, the calculation of the log likelihood function requires the calculation of weighting factors, $P[s_t = 0|I_{t-1}]$ and $P[s_t = 1|I_{t-1}]$ without initial assumptions about the stochastic behavior of the regime variables.

The evolution of s_t can follow different forms. For example, the variable s_t can evolve independently. In this case, the probability, as suggested by Kim and Nelson (1999), for each regime is defined as:

$$P[s_t = 1] = \frac{exp(p_0)}{1 + exp(p_0)},$$

$$P[s_t = 0] = 1 - \frac{exp(p_0)}{1 + exp(p_0)} \tag{4.2}$$

where p_0 is an unconstrained parameter. The stochastic behavior of s_t is not dependent upon any other exogenous or predetermined variables. The maximum log likelihood function in this case can be maximized with respect to $\theta_0, \theta_1, \sigma_0^2, \sigma_1^2$ and p_0.

In a more complicated case, s_t might evolve independently of its own value but dependent upon some exogenous variables, z_t. Then the transition probability can be written as:

$$P[s_t = 1 | z_{t-m}] = \frac{exp(p_0 + z_{t-1}p_1)}{1 + exp(p_0 + z_{t-1}p_1)},$$

$$P[s_t = 0 | z_{t-m}] = 1 - \frac{exp(p_0 + z_{t-1}p_1)}{1 + exp(p_0 + z_{t-1}p_1)},$$

where $m =\in [1, T]$. The solution in this case is achieved by maximizing the corresponding log likelihood function with respect to $\theta_0, \theta_1, \sigma_0^2, \sigma_1^2, p_0$ and p_1.

Assuming s_t follows a Markov process makes the probability of s_t conditional on s_{t-1}. Then, the probability of the regime at each time t is determined by the transition probability, which, as discussed by Hamilton(1994), is given by:

$$P(s_t = 1 | s_{t-1} = 1) \frac{exp(p)}{1 + exp(p)},$$

$$P(s_t = 0|s_{t-1} = 0)\frac{exp(q)}{1 + exp(q)}.$$

The solution for unobserved s_t is the same as shown above. However, the calculation of the weighting terms in (4.2), $P[s_t = i|I_{t-1}], j = 0, 1$, is not straightforward through the log likelihood function. Hamilton (1989) suggested a two steps procedure to calculate these weighting terms:

- Step one: given $P[s_{t-1} = j|I_{t-1}], j = 0, 1$ at the beginning of the time t, the weight $P[s_{t-1} = j|I_{t-1}], j = 0, 1$ is calculated as:

$$P[s_{t-1} = j|I_{t-1}] = \sum_{i=0}^{1} P[s_t = i, s_{t-1} = j|I_{t-1}]$$
$$= \sum_{i=0}^{1} P[s_t = i|s_{t-1} = j]P[s_{t-1} = j|I_{t-1}].$$

- Step two: the weighting term is then updated in the following way:

$$p[s_t = i|I_t] = p[s_t = i|I_t, y_t] = \frac{f(s_t = i, y_t|I_{t-1})}{f(y_t|I_{t-1})} \times \frac{f(y_t|s_t = i, I_{t-1})P[s_t = i|I_{t-1}]}{\sum_{i=0}^{1} f(y_t|s_t = i, I_t)P[s_t = i|I_{t-1}]}.$$

Repeating the two-step iteration yields $P[s_t = i|I_{t-1}]$. However, unconditional probabilities of s_t are required to initialize the filter at time $t = 1$,

$$P[s_0 = 0|I_0] = \frac{1 - p}{2 - p - q}$$
$$P[s_0 = 1|I_0] = \frac{1 - p}{2 - p - q}.$$

Thus, the parameters are solved by maximizing the log likelihood function which is a function of $\theta_0, \theta_1, \sigma_0^2, \sigma_1^2, p$ and q.

Based on the Markov switch model, Anglingkusumo (2005) analyses the relationship between narrow money (M1) and inflation in Indonesia. His result suggests that there is a structure break in the relationship during the Asian crisis in 1997. Amisano and Colavecchio (2013) applied a bayesian Markov switch model to analyse the relationship between money and inflation in UK from 1960 to 2012. Their results suggests that the relationship between money and inflation can be divided into two regimes: a low inflation regime and a high inflation regime.

4.3 Estimation of Markov switch model

Based on the Markov switch model introduced by Hamilton (1989), we carried out the estimation of markov switch model for the relationship between monetary growth and inflation. The relationship between inflation and preceding monetary growth is taken in the form of:

$$\pi_t = \alpha + \sum_{k=0}^{q} \beta_k m_{t-k} + \varepsilon_t, q \in [0, t), k = 1, ..., 12 \tag{4.3}$$

The dataset for both inflation and monetary growth are consistent with the one in chapter 3. For a comparison between different lag-settings, we examined the mean squared error (MSE) and mean abosolute error (MAE). The result of MSE and MAE based on each dataset and different lag-setting are presented in Table 4.1, where all models based on different dataset suggest that the result of Markov switch model improved with increasing lags. In the case of lag length equal to 12, The estimation of Markov switch model has lower MAE and MSE compared to other lags-setting for all three dataset. Therefore, the model with 12 lags-setting can be used to study the positions of structural break instead of SQ and DQ test where test result is uncertain as positions of structural breaks varied with different settings.

Furthermore, Markov switch model outperform linear regression model, which was discussed in Chapter 3, in terms of having lower MSE and MAE for each case. This result supports the existence of nonlinear relationship between inflation and money.

Table 4.1: The value of MSE and MAE for Markov switch model

	m403		m412		m012	
lags	MSE	MAE	MSE	MAE	MSE	MAE
k=1	0.994	0.664	12.357	2.048	8.671	1.845
k=2	0.904	0.633	11.454	2.007	4.371	1.584
k=3	0.885	0.631	10.496	1.928	3.773	1.516
k=4	0.869	0.633	9.267	1.840	3.266	1.526
k=5	0.866	0.620	7.128	1.597	3.067	1.508
k=6	0.771	0.608	5.973	1.519	3.926	1.507
k=7	0.687	0.586	5.003	1.461	3.527	1.387
k=8	0.650	0.568	4.908	1.468	3.226	1.324
k=9	0.649	0.567	4.853	1.458	3.045	1.327
k=10	0.648	0.565	4.854	1.459	2.702	1.277
k=11	0.652	0.557	3.891	1.411	3.601	1.316
k=12	0.641	0.553	3.431	1.354	2.682	1.230

The regime sequence based on different dataset are listed in Figure 4.1 where structural breaks are consistent with the result from SQ and DQ test in Chapter 3 that several structural breaks exist in the relationship between inflation and money. Furthermore, the Markov switch model provides positions of structural breaks without the need to repeat the exercise.

For the case of M4, both M403 and M412 shared a similar pattern of regime sequences. As with the result from the DQ and SQ tests, both dataset suggest that the relationship between inflation and monetary growth was stabilized after 1992 when the inflation targeting was formally introduced in UK. Also, a structural break is found in 1982 when the UK modified the monetary target it constantly had overshot.

The Markov switch model also detected three other major structural breaks based on M403 and M412. First occurred around 1970. In 1971, the US unilaterally suspended convertibility of the US dollar into gold resulting in the end of the Bretton Woods fixed exchange rate system. By 1973, sterling started to float. Meanwhile, in early 1970, the UK government held the idea that stimulation of the economy could be achieved through monetary expansion, while the control of inflation is through fiscal policy, for example, statuary income policy announced in 1972 and extension of food subsidies in order to reduce retail price. Also, in 1973, OPEC raises the price of oil following Yom Keppur war. As the result of these movement, the inflation soared up to its record high level in 1976.

The second switch is in 1989. From 1987 to 1990, the UK informally linked sterling to the German's Mark. During this period, UK monetary policy closely followed Bundesbank's monetary policy. For example, in October 1989 the UK increase short-term interest rate by 100 base point immediately after the Bundesbank increased by the same amount. During the same period, we observed the monetary growth rate soaring up with the inflation rate.

The third switch happened in 2008 during the financial crisis. However, the relationship between money and inflation returned to its pre-crisis regime within two quarters. During the period of financial crisis, both inflation and M4 plummeted to their record low level, but bounced back to their pre-crisis level after a short time of fluctuation.

In the case of M012, as shown in Figure 4.1, there are more structural breaks detected compared to Markov switch model based on M403 and M412. In addition to the one in 1973, we observe fluctuations of relationship between inflation and money from 1976 to 1982.

The announcement of monetary targeting £M3 in 1976. Also the inflation is observed to soar up to over 20 percentage in 1980. As a result of growing concern with high inflation, the UK government in July 1976 adopted a monetary targeting policy in order to curtail inflationary pressure by informally targeted a broad aggregate, £M3. The formal introduction of a target for £M3 was published by the UK government in late 1976. However, the targets were frequently overshot in this period. As a result, MTFS was heavily revised rising the £M3 targets in 1982.

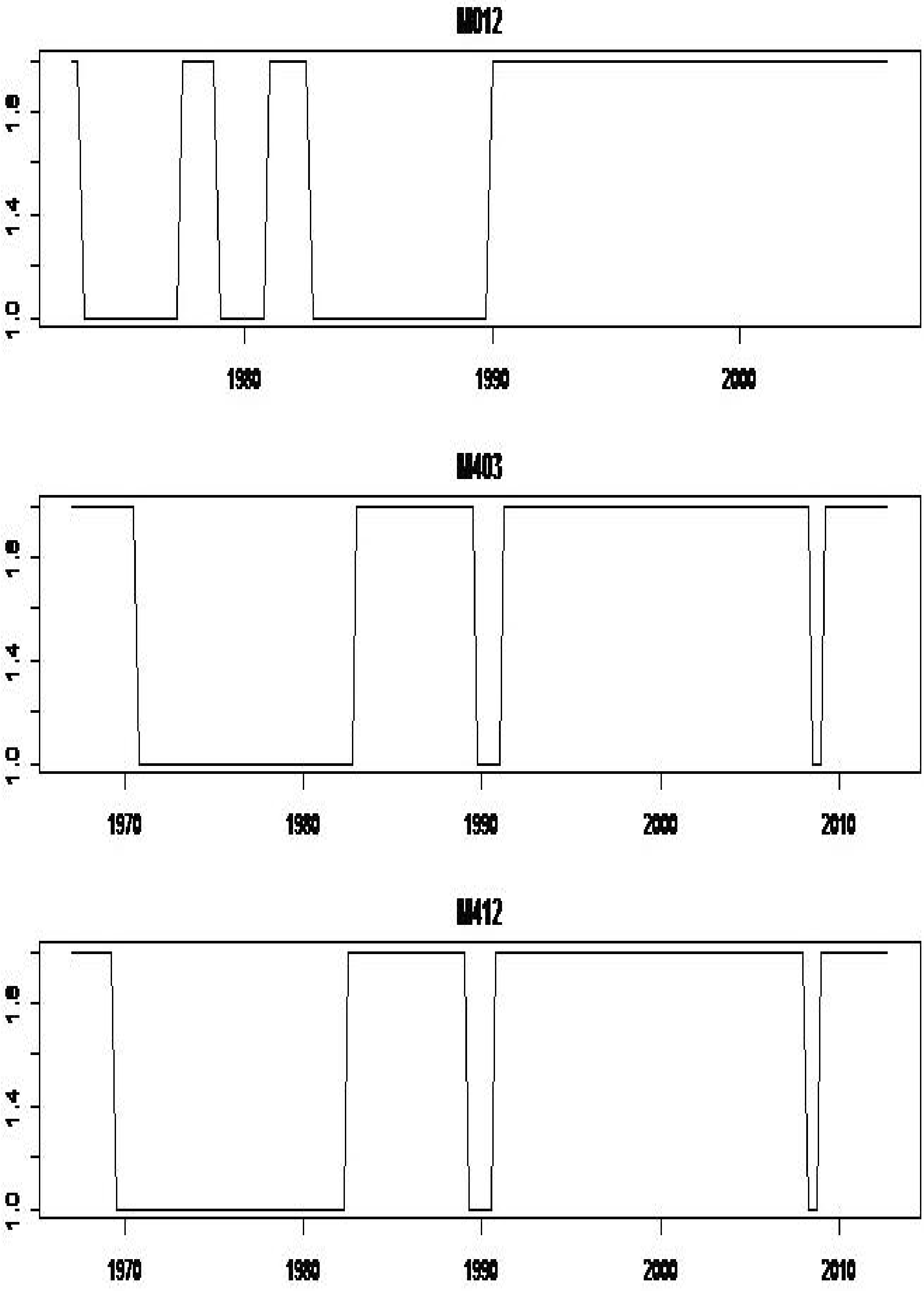

Figure 4.1: regime sequence for preferred model in each dataset

Table 4.2: Estimation of Markov switch model with 12 lags of M403

	regime 1		regime 2	
parameters	regime 1	Std.error	regime 2	Std.error
α_0	0.2785	0.1530	4.3553	1.4075
ϕ_0	0.0045	0.0406	-0.6017	0.1696
ϕ_1	0.0571	0.0406	-0.4015	0.1474
ϕ_2	0.0795	0.0371	0.0908	0.1680
ϕ_3	-0.0734	0.0421	-0.1563	0.1718
ϕ_4	0.0350	0.0412	-0.0747	0.1613
ϕ_5	0.0275	0.0412	-0.0473	0.1548
ϕ_6	0.0424	0.0417	0.3570	0.1366
ϕ_7	-0.0681	0.0405	0.3756	0.1613
ϕ_8	0.0781	0.0408	0.1626	0.1673
ϕ_9	-0.0041	0.0366	0.0788	0.1436
ϕ_{10}	0.0684	0.0368	-0.0022	0.1800
ϕ_{11}	0.0176	0.0379	-0.2416	0.1488
ϕ_{12}	0.0257	0.0449	0.1691	0.1632
Sum of Coef.	0.2902		0.2914	

Table 4.3: Estimation of Markov switch model with 12 lags of M012

	regime 1		regime 2	
parameters	regime 1	Std.error	regime 2	Std.error
α_0	-0.0560	0.0098	-0.0050	0.0052
ϕ_0	-1.4304	0.2409	0.9913	0.1687
ϕ_1	0.6978	0.3628	-0.5372	0.1809
ϕ_2	1.2927	0.3534	-1.0051	0.2027
ϕ_3	0.7909	0.3342	-0.6320	0.2009
ϕ_4	-0.8517	0.3463	1.4699	0.1806
ϕ_5	0.6824	0.4151	0.6272	0.1890
ϕ_6	0.5732	0.3340	-0.5822	0.2300
ϕ_7	-0.3340	0.4089	-0.3292	0.2230
ϕ_8	-0.7113	0.3809	0.9595	0.1669
ϕ_9	0.3369	0.2355	0.2523	0.2259
ϕ_{10}	0.9571	0.2669	-1.1436	0.2059
ϕ_{11}	0.4800	0.3066	-0.0097	0.2192
ϕ_{12}	-0.7109	0.2296	0.7121	0.1548
Sum of Coef.	1.7727		0.7733	

Table 4.4: Estimation of Markov switch model with 12 lags of M412

	regime 1		regime 2	
parameters	regime 1	Std.error	regime 2	Std.error
α_0	1.7952	3.9354	1.2298	0.3396
ϕ_0	0.8114	0.3437	-0.1270	0.0778
ϕ_1	-0.4734	0.5532	-0.0058	0.1108
ϕ_2	-1.1662	0.5024	0.2246	0.1159
ϕ_3	-0.1096	0.5032	-0.0749	0.1140
ϕ_4	1.1600	0.5595	-0.0432	0.1150
ϕ_5	-0.2979	0.5907	0.1536	0.1203
ϕ_6	-0.6271	0.6310	0.1218	0.1251
ϕ_7	0.3404	0.6837	-0.1391	0.1335
ϕ_8	1.3089	0.5818	0.0772	0.1175
ϕ_9	0.1023	0.5512	0.0801	0.1159
ϕ_{10}	-0.8013	0.6453	0.0086	0.1048
ϕ_{11}	-0.4237	0.6820	-0.0468	0.1229
ϕ_{12}	0.9945	0.3678	0.0216	0.0853
Sum of Coef.	0.8183		0.2507	

Diebold,Lee and Weinbach (1994) extended the basic Markov switch model to allow the time varying transition probabilities to depend on some underlying economic fundamentals, z_t. The transition probabilities have the following specification:

$$P[s_t = 0] = P[S_t < 0]$$

$$P[s_t = 1] = P[S_t \geq 0]$$

where S_t is defined by:

$$S_t = g_1 S_{0,t-1} + g_2 S_{1,t-1} + z_t \zeta + e_t, e_t \overset{iid}{\sim} N(0, 1).$$

Here $S_{0,t-1} = 0, S_{1,t-1} = 1$, and ζ is a constant which measures the sensitivity of transition probability with respect to z_t. The transition probabilities are then given by:

$$P_{1i,t} = P[s_t = 1|s_t = i, z_{t-1}] = P[e_t < -(g_i + z_t \zeta)] = \Phi(-(g_i + z_t \zeta)),$$

$$P_{2i,t} = P[s_t = 2|s_t = i, z_{t-1}] = P[e_t < -(g_i + z_t \zeta)] = 1 - \Phi(-(g_i + z_t \zeta))$$

where $\Phi(\cdot)$ is the c.d.f. of the standard normal distribution. The estimation of parameters follow the EM algorithm, which is an iterative procedure method introduced by Dempster,Laird and Rubin (1977). The EM algorithm general consists of two steps:

- Step one: given the parameter estimated from iterating $t - 1$ times, the expectation of unobservable regime is formed.
- Step two: conditional on the expectation of the regime variable s_t, the likelihood

function is maximized with respect to the parameters of model.

Based on the time-varying transition probability Markov switch model, Amisano and Fagan (2010) divided the relationship between inflation and monetary growth into two regimes in Canada, Euro area, US and UK from 1960 to 2010. The results suggest that the quarterly monetary growth rate provides an early warning signal about regime shift in the development of the inflation rate. However, Amisano and Fagan (2010) also admitted that the signal from monetary growth to inflation is noisy given the limited number of regimes.

Like the other nonlinear models discussed before, the time-varying transition probability Markov switch model faces the same constraints in dealing with potential nonlinear structures. First, the number of regimes is still predetermined. In this case, a fixed number of regimes will be assigned to the model. However, the true number of regimes is unlikely to be known in advance, in which case the model will be misspecified if the assumption about the number of regimes is invalid. Second, the transition probability follows the standard normal distribution where the value is jointly determined by the regime variable at $t-1$ and the underlying variable z_t. However, without the support of a clear structure, neither the distribution of transition probability nor the underlying variable driving the regime-switch is known in advance.

Beal (2002) introduced the infinite Hidden Markov switch model (iHMM) which relaxs the constraint on the number of regimes and on the assumption of regime-switch. As discussed in chapter 2, there is considerable uncertainty surrounding the nature of the structural model as the transmission mechanism from money to inflation is not clear. Under these circumstances, the variable for changing regime and the number of regimes will be unknown. Therefore, the application of iHMM model to the relationship between monetary growth and inflation will be useful. The details of iHMM model will be discussed in Chapter 5.

Chapter 5

The relationship between monetary growth and inflation: an application of iHMM

5.1 Introduction

As discussed in Chapter 3, testing for structural breaks in a linear model does not suggest an appropriate structural model for the relationship between monetary growth and inflation as there exist many potential nonlinear models. In this respect, describing the relationship between monetary growth and inflation faces many difficulties as factors determining this relationship are unclear. The constraints on many models limit their ability to approximate the relationship between money and inflation as discussed in Chapter 3 and Chapter 4. In this Chapter, we discuss the infinite Hidden Markov

Model (iHMM) which helps to relax some of the constraints forced by other nonlinear models.

There are several characteristics for iHMM. First, the number of regimes can be infinite in theory. In practice, we assign a number which is considered to be larger than the actual number of regimes. If the maximum number of regimes is reached, then the limit on the number of regimes will be increased until it exceeds the actual number of regimes. Second, changing regime can be classified as a structural break or a regime switch. If a structural change switches the regime into one that has previously occurred, the break is called a regime switch, otherwise, it is called a structural break. Third, the factor, which controls the structure change, does not follow any specific distribution. In what follows, we introduce the iHMM in detail in section 5.2. section 5.3 tests the convergence of Bayesian estimation underlying the iHMM. The estimation results for iHMM based on various definitions of money in our dataset are discussed in section 5.4. section 5.5 discusses the application of iHMM in data requirement for identifying the structural break. The chapter will be concluded in section 5.6 with further discussion.

5.2 The iHMM model

In order to build a model which satisfies the requirements introduced in the previous chapters, we begin by discussing the basic requirements for modeling relationship between monetary growth and inflation in more detail. First, the structure of relationship between monetary growth and inflation is unclear. This feature requires the model to incorporate a wide range of possible relationships between variables. It will be difficult to directly build a model to describe such a complex relationship. One way is to divide the whole sample into several different regimes, each representing a simple linear relationship. Then, the combination of linear models based on each regime will give us the resultant model, which could be achieved by many piecewise linear models. However, as discussed in Chapter 2 and Chapter 4, most piecewise linear models assume a limited number of regimes, which could lead to a misspecification of the model as the real number of regimes is unknown in advance.

Second, as the transmission mechanism between money and inflation is still unclear, the reason for structural change is unknown. In most nonlinear models, the factor controlling structural change is specified, such as a constant threshold or normal distribution which could cause misspecification in estimation. In this case, an unknown factor is needed to control the structural change.

In addition, the model should be able to distinguish between a regime switch and a structural break. In this case, the model should be able to gather the observations in the sample sharing the same state and estimate their coefficients rather than limit the number of regimes or introduce a new regime as if there is a change in the data dynamics. Based on these requirements, we apply the infinite Hidden Markov Model (iHMM) to the problem of modeling the relationship between monetary growth and inflation.

The infinite Hidden Markov model was first introduced by Beal et al (2002) and successfully applied to inferential problems such as genetics and visual scene recognition by Teh et al.(2006). Fox et al.(2011) introduced a 'sticky' variant to the basic iHMM model where the unrealistic high dynamics could effectively be ruled out. Song (2011), for example, applied the model to the problem of analysing US interest rate. For other applications, Jochmann (2013) used the sticky iHMM to explore inflation persistence and identify the period of greatest persistence historically.

Jochmann (2013) argued that a Bayesian non-parametric model is able to automatically infer an adequate model size without a need to explicitly conduct model comparisons. This means that the iHMM does not fix the number of underlying regimes initially, but infers them from the data. The features of the iHMM model allowing this to happen are discussed below.

5.2.1 The Dirichlet Process

In order to achieve the structural change without a specified controlling factor, iHMM involves use of a Dirichlet process. A Dirichlet process (DP) is a probability measure on probability measures first introduced by Ferguson (1973) as an extension of the Dirichlet distribution from a finite dimension to an infinite dimension.

Given a Dirichlet process $DP(\alpha, G_0)$, where the shape parameter G_0 is a base probability measure drawn from the Dirichlet distribution, and α is a positive scalar concentration parameter. Ferguson (1973) has shown that any draw $G \sim DP(\alpha, G_0)$ is almost surely discrete, even if G_0 is continuous.

The variable simulated from the Dirichlet process is actually from a unknown distribution, a feature which is desirable in the current context because of the unknown factor controlling the structural change in the relationship between inflation and monetary growth.

Sethuraman (1994) demonstrated that G can be represented as:

$$G = \sum_{k=1}^{\infty} \pi_k \theta_k \tag{5.1}$$

where $\{\theta_k\}_{k=1}^{\infty}$ represents a set of distinct regime-related parameters following G_0. The $\{\pi_k\}_{k=1}^{\infty}$ come from a stick-breaking process (denoted by $SBP(\alpha)$ hereafter), which can be written as:

$$\pi_k = V_k \prod_{l=1}^{k-1} (1 - V_l); \quad V_l \overset{iid}{\sim} \beta(1, \alpha) \tag{5.2}$$

where V_k represents part of a unit-length stick assigned to the kth value after the first $k-1$ values have been drawn. The smaller α, the smaller the remainder of the stick after the first $k-1$ values.

The Dirichlet process is frequently used as a prior on the parameters in the Dirichlet process mixture model (DPM model), which takes form as:

$$\pi \sim SBP(\alpha), \tag{5.3}$$

$$\theta_i \sim G_0, \quad i = 1, 2, ...\infty, \tag{5.4}$$

$$x_j \sim f(x_j|\theta_{\pi_i}), \quad j = 1, 2, ...N. \tag{5.5}$$

The observation x_j is followed by the conditional probability density function $f(x_j|\theta_{\pi_i})$, where parameter θ_{π_i} is generated from a mixture distribution G which is drawn from a Dirichlet process $G \sim DP(\alpha, G_0)$. However, the DPM model in our case could easily produce an unrealistic stochastic regime as it lacks regime persistence.

5.2.2 The structure of Infinite Hidden Markov Model

For the infinite Hidden Markov Model, a single Dirichlet process is not enough. We need a collection of Dirichlet processes where each group of the observation-linked Dirichlet process is conditionally-independent given a common base measure π_0 which follows a SBP $(\pi_0) \sim DP(\beta)$. The collection of Dirichlet processes is called a hierarchical Dirichlet process (HDP) which was introduced by Beal et al. (2002).

To describe the HDP, suppose the whole sample is divided into J groups. The group-specific distributions $\{\pi_j\}_1^J$ independently follow a Dirichlet process $\pi_j \overset{ind}{\sim} SBP(\alpha, \pi_0)$, where j represents the jth group of observations. Thus π_j measures the deviation from π_0 with α governing the amount of variability. Based upon the group-specific distribution π_j, each observation is assigned to a group of observations $s_{ij} \sim \pi_j$. The parameters $\{\theta_j\}_{j=1}^J$ for the jth group of observation is generated from a group-specific mixture distribution $\{H_j\}_{j=1}^J$. In summary, the HDP can be represented by:

$$\pi_0 \sim SBP(\beta), \tag{5.6}$$

$$\pi_j \overset{ind}{\sim} DP(\alpha, \pi_0), \tag{5.7}$$

$$s_{ij} \sim \pi_j, \tag{5.8}$$

$$\{\theta_j\}_{j=1}^J \overset{ind}{\sim} \{H_j\}_{j=1}^J, \tag{5.9}$$

$$x_{ij} \overset{ind}{\sim} f(x_{ij}|\theta_j) \tag{5.10}$$

where x_{ij} denote ith observation from group j which has a density function conditional on group j parameters.

Teh et al.(2006) introduced the iHMM based on the structure of hierarchical Dirichlet process. To move from the HDP to iHMM, we suppose there is an unobserved regime sequence $s = (s_1, ..., s_T)$. For each regime, it presents a linear relationship between inflation and money. Then, regime sequence denotes the development of relationship between inflation and money over time. In the iHMM, s_t can take on a number of distinct regimes: $1, ..., J$. Unlike HDP, where the group for each distribution is determined by the parameter α, the transition between regimes in iHMM is Markovian and parametrized by the transition matrix π with $\pi_{ij} = Pr(s_t = i|s_{t-1} = j)$, where the distribution π_j is determined by the previous regime s_{t-1}. Therefore, each row of the transition matrix, π, specifies a different mixture distribution over the same parameter set θ_{s_t}. Thus, we have a density function for x_t given the previous regime s_{t-1} as:

$$Pr(x_t|s_{t-1}) = \sum_{j=1}^{J} Pr(s_t|s_{t-1} = j)Pr(x_t|\theta_{s_t}).$$

The iHMM model is then shown as:

$$\pi_0 \sim SBP(\beta), \tag{5.11}$$

$$\pi_j \overset{ind}{\sim} DP(\alpha, \pi_0), \tag{5.12}$$

$$s_t|s_{t-1} = j \sim \pi_j, \tag{5.13}$$

$$\{\theta_j\}_{j=1}^{J} \overset{ind}{\sim} \{H_j\}_{j=1}^{J}, \tag{5.14}$$

$$x_t|s_t, X_{1,t-1} \sim f(x_t|\theta_{s_t}) \tag{5.15}$$

where $X_{1,t-1}$ represent the available information up to time $t-1$. For each row of the transition matrix, the distribution is drawn from the same Dirichelt process. In practice, the maximum number of regimes, J, is set to be much larger than the expected number of regimes. Conditional on the regime at time $t-1$, if the value of s_t occurred in the previous regime, we observe a regime switch. Otherwise, s_t represents a new regime and the transition is defined as a structure break. Thus, the change of regime in the sequence after simulation of whole regime sequence will be classed into regime switch and structural break. This characteristic of iHMM model is desirable and meets the requirement of our model in distinguishing different types of structural change.

However, as with the DPM model, the iHMM model does not distinguish between probability of regime unchanged over time (self-transitions) and probability of transitions to other regimes. To tackle this problem, Fox et al.(2011) introduced the so-called sticky iHMM model where the probability of self-transition is increased by adding a positive parameter k into the Dirichlet process for π_j. Then equation 5.12 can be rewritten as:

$$\pi_j|\pi_0 \overset{ind}{\sim} DP(\alpha + k, \frac{\alpha\pi_0 + k\delta_j}{\alpha + k}) \tag{5.16}$$

where δ_j is the Kronecker's delta which takes value 1 at π_{jj}. Adding k to the jth component of $\alpha\pi_0$ leads to an increased probability of self-transition. Song (2011) subsequently simplified this method by directly adding a positive value ρ to the jth

elements of the base probability for DP:

$$\pi_j|\pi_0 \sim DP(\alpha, (1-\rho)\pi_0 + \rho\delta_j). \tag{5.17}$$

As a result, we have the sticky iHMM by replacing equation (5.14) in iHMM with equation (5.17).

The Sticky iHMM consists of two hierarchical structures. The first hierarchical structure governs the transition probabilities between regimes, comprises (5.11) and (5.17). π_0 in (5.11) is drawn from stick breaking process and represents a vector containing probabilities for all regimes. We can then use vector $\pi_j = (\pi_{1j}, \pi_{2j}, ...)'$ in (5.17) to represent the probability vector drawn from a Dirichlet process, $DP(\alpha, (1-\rho)\pi_0 + \rho\delta_j)$, with the concentration parameter α and $(1-\rho)\pi_0 + \rho\delta_j$. Each element π_{ji} is the probability of regime at time t,s_t, taking the integer value j given that $s_{t-1} = i$ where i also takes integer value. If ρ is larger, it adds weight to δ_j. Then, π is expected to have a larger probability to sustain regime j. By increasing the probability of self-transition, the unrealistic high dynamics in the regime change can be ruled out. Also, the sticky iHMM will become the iHMM by setting $\rho = 0$.

The second hierarchical structure, which governs the parameters of conditional data density, includes (5.14)and (5.15). The conditional density parameters $\{\theta_j\}_{j=1}^J = (\phi_j', \sigma_j)'$, where ϕ_j and σ_j are the vector of coefficients and standard deviation of error for the regime j.

5.2.3 Inference of Sticky iHMM

The two hierarchical structures of sticky iHMM are too complex to be analyzed. Therefore, as shown in Fox et al.(2011), Song (2011) and Jochmann (2013), estimation is undertaken using a non-parametric Bayesian method which involves two steps: generating the regime sequence based on the prior conditional density parameters and estimating conditional density parameters based on the posterior regime sequence.

Simulation of the regime sequence in the sticky iHMM uses a Bayesian approach based on the method of Gibbs sampling which is applicable when the joint distribution of regimes in iHMM is unknown or difficult to simulate directly, but the conditional distribution of regime s_j based on remaining regimes is easy to obtain[1]. Given the regime vector $\{s_j\}_{j=1}^{J}$, the conditional distribution of s_j, $Pr(s_j|(X_n, s_{-j})$, is generated based on data X_n and remaining regimes s_{-j}. The procedure describing Gibbs sampling for $Pr(s_j|(X_n, s_{-j})$ is given by the following:

Step 1: generating the initial regime vector $G^0 = (s_1, ..., s_J)$ and set $i = 1$.

Step 2: simulate s_j^{i+1} from $Pr(s_j^{i+1}|s_1^{i+1}, ...s_{j-1}^{i+1}, s_{j+1}^{i}, ..., s_J^{i})$.

step 3: set $i = i + 1$, go back to step 2.

Repeating the above steps a large number of times, generates a Gibbs sequence $(G^0, ..., G^i)$, where i is sufficiently large so that the sampler converges. However, as mentioned in Chib(1996), standard Gibbs sampling is inefficient in the case where there are a large number of regime J while the repeating times of Gibbs sampling will be largely proliferated. To remedy this, Chib (1996) introduced a forward-filtering, backward-sampling

[1]The introduction of Gibbs sampling refers to Chapter 2.

scheme where all the regimes are treated as one block and the regime sequence is then sampled from a joint distribution $Pr(S_n|X_n,\theta)$, which can be written as:

$$Pr(S_n|X_n,\theta) = Pr(s_n|X_n,\theta) \times \cdots \times Pr(s_t|X_n, S^{t+1},\theta) \times \cdots \times Pr(s_1|X_n, S^2,\theta), \quad (5.18)$$

where $S_n = (s_1, ..., s_n)$,$S^t = (s_t, ..., s_n)$. Furthermore, $Pr(s_t|X_n, S^{t+1})$ is the product of the conditional probability of s_t given (X_t,θ) and the transition probability from s_t to s_{t+1},

$$Pr(s_t|X_n, S^{t+1}) \propto Pr(s_t|X_t,\theta) \times Pr(s_{t+1}|s_t,\theta). \quad (5.19)$$

The determination of $Pr(s_t|X_t,\theta)$ involves two steps. The first is the prediction step where the conditional probability of s_t is determined given X_{t-1} and θ. By the law of total probability,

$$Pr(s_t|X_{t-1},\theta) = \sum_{j=1}^{J} Pr(s_t|s_{t-1} = j,\theta) \times Pr(s_{t-1} = j|X_{t-1},\theta).$$

The update step subsequently determines $Pr(s_t|X_t, ,\theta)$ as

$$Pr(s_t|X_t,\theta) \propto Pr(s_t|X_{t-1},\theta) \times f(x_t|X_{t-1},\theta_{s_t})$$

where $f(x_t|X_{t-1},\theta_{s_t})$ represents the density function of x_t conditional on the dataset up to time t and the regime-related parameter θ_{s_t}.

To simulate the regime sequence, we first run the prediction and update steps recursively to compute $Pr(s_t|X_t,\theta)$, where s_n is simulated from $Pr(s_n|X_n,\theta)$. After s_n

is simulated, the remaining regimes beginning with s_{t-1} are simulated from equation (5.18).

The regime sequence based on Gibbs sampling follows a Markov chain, where nearby samples are correlated. Therefore, the prior base measure π_0 of the Dirichlet process in the iHMM, which is set to simulate the initial regime sequence, should also be updated in order to update the transition matrix based on the information from posterior regime sequence. Song (2011) updated π_0 by using a Polya Urn scheme where a indicator variable vector $\{I_t\}_{t=1}^{T}$ is deployed to update π_0[2]. From equation (5.12), each vector of transition matrix π_j conditional on posterior regime sequence $S_t = (s_1, ..., s_t)$ and prior π_0, is a Dirichlet distribution:

$$\pi_j|S_t, \pi_0 \sim Dir(c(1-\rho)\pi_{01} + n_{j1}, ..., c(1-\rho)\pi_{0,j} + c\rho + n_{jj}, ..., +c(1-\rho)\pi_{0J} + n_{jJ})$$

where n_{jk} is the number of transition from regime j to k in the prior regime sequence. Sampling the indicator variable, I, conditional on S_t follows Bernoulli distribution:

$$I_{t+1}|s_t = j, s_{t+1} = k, \pi_0 \sim Ber(\frac{c(1-\rho)\pi_{0k}}{n_{ji}^t + c\rho\delta_j(k) + c(1-\rho)\pi_{jk})}.$$

Cumulating I_t based on each regime will have $m_k = \sum_{s_t=i} I_{s_t}$. Adding the m_k to the corresponding element in α, we have the conditional posterior of π_0 given S_t and $\{m_k\}_{k=1}^{J}$:

$$\pi_0|S_t, \{m_k\}_{k=1}^{J} \sim Dir(\alpha_1 + m_1, ..., \alpha_J + m_J).$$

[2]Polya Urn scheme, name after George Polya, is a dichotomous sampling model to simulate hyper-parameters.

Then, conditional on π_0, each row vector of posterior transition matrix is a Dirichlet distribution by conjugacy:

$$\pi_j|S_t, \pi_0 \sim Dir(c(1-\rho)\pi_{01} + n_{j1}, ..., c(1-\rho)\pi_{0,j} + c\rho + n_{jj}, ..., +c(1-\rho)\pi_{0J} + n_{jJ}).$$

The posterior transition matrix is subsequently used to estimate the regime sequence next iteration.

Given the regime sequence, the conditional density parameters θ_{s_t}, which includes the regime-specific coefficient ϕ_j and variance σ_j, are simulated as following:

$$\phi_j \sim N(\mu, \Sigma),$$

$$\sigma_j^2 \sim Inv - Gamma(c_0, d_0)$$

where $\phi_j = (\phi_{0,j}, ..., \phi_{k,j})', k = 1, 2, ..., \infty$. For estimation of the coefficients, we follow the method in Jochmann (2013) and simulate μ and Σ from hyperprior distribution:

$$\mu \sim N(b_0, B_0),$$

$$\Sigma \sim inv - Wishart(S_0, m_0).$$

In our application, the prior is set as: $b_0 = 0_k, B_0 = diag(10, 1, 1, .., 1), S_0 = I_k, m_0 = 10, J = 10$. Here, the maximum number of regimes J is set to be 10. Also we set the prior in the step of sampling regimes as : $\pi_0 \sim Dir(1/J, ..., 1/J), c = 10, \rho = 0.9$. In practice, the maximum number of regimes in our application only reached five.

However, if the maximum number of regimes had been reached in practice, we would increase the maximum number of regimes to fulfill the requirements of the model.

Several other settings for the prior were also tried in the modeling exercise. For example, we changed the self-transition parameter ρ from 9 to 5 and further to 2. This change decreased the probability of self-transition in the transition matrix. We also chose $b_0 = diag(1, 0.1, 0.1, ..., 0.1)$ which implies a higher prior probability on self-transition. However, the results of our estimation stay stable over various prior setting.

5.3 Convergence of estimation

Due to lack of prior knowledge, the Gibbs sampling is often initialised at a random value which is often far from the true distribution. Then, the problem is not that early samples are invalid samples from Gibbs sampling, but rather that it is not likely to obtain samples from the true posterior distribution unless the Gibbs sampling runs long enough to reach convergence. In this section, we consider tests for the convergence of Gibbs sampling based on our data.

Convergence of the Gibbs sampler is influenced by two determinants, the length of the burn in period and the number of iterations that subsequently follow. First, we consider if our estimation discard sufficient burn-in period, which is the first part of Markov chain correlated to starting value. By discarding a sufficient burn-in period, the correlation between the remaining samples of the Markov chain give an as accurate estimate of the parameter as possible.

The Geweke test, which was introduced by Geweke (1991), compares the mean values in the early part of Monte Claro Markov Chain (MCMC) to those in the latter part. If the mean values of the parameter in the two sub-sequences of the Markov chain are close to each other, it is assumed that the two values come from the same distribution. Suppose two subsequences from Markov chain are represented by: $\theta_t^1 : 1 < t < n1$ and $\theta_t^2 : n_2 < t < n$, where $1 < n_1 < n_2 < n$. The mean values of two sub-sequences are defined as:

$$\bar{\theta_1} = \frac{1}{n_1} \sum_{t=1}^{n_1} \theta_t^1,$$

$$\bar{\theta_2} = \frac{1}{n - n_2} \sum_{n_2}^{n} \theta_t^2$$

where $\bar{\theta_1}$ and $\bar{\theta_2}$ are the mean values for two subsequences. The Geweke test statistic is then given by:

$$Z_n = \frac{\bar{\theta_1} - \bar{\theta_2}}{\sqrt{\frac{s_1}{n_1} + \frac{s_2}{n-n_2}}}$$

where Z_n is referred to z value based on normal distribution, s_1/n_1 and $s_2/n - n_2$ are the variances of θ_1 and θ_2. The Geweke test is a two tailed test by setting the hypothesis as:

$$H_0 : \theta_1 = \theta_2.$$

$$H_0 : \theta_1 \neq \theta_2.$$

Generally, the Geweke test is conducted by comparing the mean value of first 10% of chain and last 50% of chain. In our estimation, we discard a 5000 burn-in period. Table 5.1 and 5.2 list the p value of Geweke test for the coefficients of the model with optimal lag length in dataset. The results suggest that the null hypothesis of equal mean value for two sub-sequences can not be rejected in all cases based on 5% significant level. This means that 5000 burn in period is enough for the convergence of estimation.

Table 5.1: P value of Geweke test for M4

	m412					m403		
	regime2	regime3	regime8	regime9	regime10	regime6	regime7	regime10
α_0	0.078	0.835	0.846	0.682	0.901	0.058	0.293	.767
ϕ_0	0.09	0.229	0.782	0.570	0.226	0.132	0.275	0.836
ϕ_1	0.059	0.059	0.198	0.235	0.312	0.238	0.246	0.251
ϕ_2	0.483	0.548	0.886	0.42	0.191	0.147	0.266	0.521
ϕ_3	0.589	0.528	0.648	0.71	0.261	0.066	0.231	0.792
ϕ_4	0.53	0.07	0.841	0.724	0.27	0.107	0.109	0.107
ϕ_5	0.296	0.085	0.531	0.827	0.661	0.222	0.230	0.298
ϕ_6	0.433	0.967	0.832	0.715	0.431	0.094	0.186	0.138
ϕ_7	0.463	0.954	0.411	0.633	0.735	0.118	0.098	0.068
ϕ_8	0.777	0.426	0.423	0.686	0.261	0.181	0.099	0.215
ϕ_9	0.252	0.245	0.165	0.755	0.326	0.348	0.076	0.389
ϕ_{10}	0.948	0.162	0.237	0.339	0.464	0.073	0.265	0.395
ϕ_{11}	0.311	0.349	0.156	0.778	0.57	0.171	0.091	0.412

Table 5.2: P value of Geweke test for M0

	m012			
	regime1	regime2	regime3	regime9
α_0	0.781	0.067	0.057	0.088
ϕ_0	0.702	0.228	0.053	0.312
ϕ_1	0.411	0.129	0.112	0.282
ϕ_2	0.921	0.055	0.209	0.081
ϕ_3	0.721	0.077	0.096	0.247
ϕ_4	0.492	0.074	0.662	0.254
ϕ_5	0.137	0.151	0.081	0.239
ϕ_6	0.391	0.057	0.061	0.388
ϕ_7	0.695	0.191	0.201	0.335
ϕ_8	0.475	0.158	0591	0.214
ϕ_9	0.797	0.305	0.293	0.346

Except for sufficiency of the burn in period, we are also concerned whether the sequence length of the Gibbs sampling after the burn-in period is long enough to achieve convergence of estimation. The Rafery and Lewis test, introduced by Rafery and Lewis (1992,1996), is designed for evaluating the accuracy of the estimated percentiles for the parameters. We first calculate U_t, which is a function of the parameter θ, for each iteration, and then form $z_t = I(U_t \leq u)$, where $I(\cdot)$ is the indicator function. z_t is a binary 0-1 process that is derived from a Markov chain. u is the value we choose for precision r and probability p on our estimation. That is, we want our estimation $U_t \in [u - r, u + r]$ with probability p Hence, if we form the new process z_t^k, where z_t^k is k-thinned process from z_t, then z_t^k will be approximately a Markov chain for k sufficiently large[3]. For the determination of Gibbs sequence length n, we assume that:

$$P = \begin{pmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{pmatrix},$$

which is the transition matrix for z_t^k. The criterion to determine the number of iterations needed, n, is if the estimated probability is within $\pm r$ of the true cumulative probability q, with probability p, which in turn can be written as:

$$P[q - r \leq \bar{z}_n^k \leq q + r] = p \tag{5.16}$$

where $\bar{z}_n^k = \frac{1}{n}\sum_{t=1}^{n} z_t^k$. When n is large, $\bar{z}_n^k = \frac{1}{n}\sum_{k=1}^{n} Z_t^k$ is approximately normal distribution with mean q and variance $\frac{1}{n}\frac{\alpha\beta(2-\alpha-\beta)}{\alpha+\beta}$. Thus equation (5.16) will be satisfied if

$$n = \frac{\alpha\beta(2-\alpha-\beta)}{(\alpha-\beta)^3}\{\frac{\Phi(\frac{1}{2}(2+p))}{r}\}^2$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function.

To implement the test, Rafery and Lewis (1992) suggest running the sampler for an initial number of iterations, n_{min}, to determine if additional iterations are required.

[3] k thinned process is generated by taking value from sequence for every kth value

Then the n_{min} is given by:

$$n_{min} = \Phi^{-1}(\frac{1}{2}(1+p))^2 q(1-q)/r^2.$$

For example, $q = 0.025$,$r = 0.005$ and $p = 0.95$, then we have $n_{min} = 3748$. If n is less than n_{min}, then more iterations are required in order to achieve convergence of estimation. In our test, we also choose this standard to justify the sufficiency of estimation for coefficients.

Table 5.3 and 5.4 list results for Rafery and Lewis test of coefficient for the model with the optimal lag length in each dataset. All the results are larger than the minimum number 3748 of iterations. This result suggest that the number of iterations and the space between iterations for thinning the sequence of Gibbs sampling in our estimation is acceptable for achieving the convergence of estimation for parameters.

We further investigate the convergence of our estimation by increasing the number of iterations from 50000 to 100000. As shown from Figure 5.1 to 5.3, the model with optimal lag length in each dataset are used compared the regime sequence between 50000 iterations and 100000 iterations. The regime sequence in each case suggest that the regime sequences based on 50000 iterations are not significantly different from the cases based on 100000 iterations.

Table 5.3: Raftery test for M4

	m412					m403		
	regime2	regime3	regime8	regime9	regime10	regime6	regime7	regime10
α_0	3916	3916	4368	4449	3845	3845	3776	4212
ϕ_0	3987	3776	3776	4615	3776	3776	4449	4449
ϕ_1	3845	3776	4368	3916	3776	3916	4449	3845
ϕ_2	3916	3845	3776	4016	3776	3845	3845	3776
ϕ_3	4449	3845	3776	3776	4061	3776	4449	3776
ϕ_4	3845	3776	3787	4136	3776	3916	3987	3776
ϕ_5	3845	3776	3916	4061	3845	3776	3776	3916
ϕ_6	4061	3845	3776	3776	3987	4061	4449	3776
ϕ_7	3916	3776	4212	4061	3987	3916	4449	3776
ϕ_8	3845	3776	3833	3845	3916	3987	4449	3776
ϕ_9	3845	3845	4212	3916	3845	3987	4449	3916
ϕ_{10}	3845	3776	3883	3845	3776	3776	3776	3845
ϕ_{11}	3845	3776	3916	3776	3776	3845	3987	3987

Table 5.4: Raftery test for M0

	m012			
	regime1	regime2	regime3	regime9
α_0	4212	3776	3845	4212
ϕ_0	3776	3776	3776	3776
ϕ_1	3776	3845	3845	3776
ϕ_2	3845	4449	3776	3845
ϕ_3	3845	3845	3776	3845
ϕ_4	4449	3916	3776	4449
ϕ_5	3916	3916	3845	3916
ϕ_6	3776	3916	3845	3776
ϕ_7	3776	4449	3845	3776
ϕ_8	4212	4449	3776	4212
ϕ_9	4289	4449	3776	4389

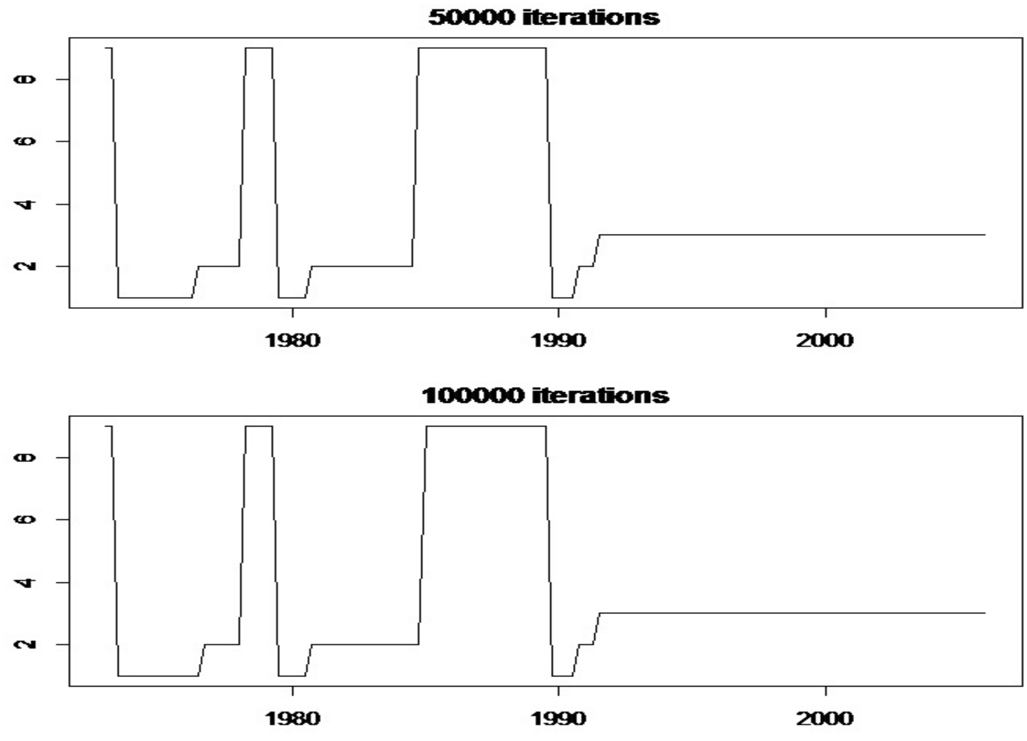

Figure 5.1: regime sequence based on M012 between different iterations

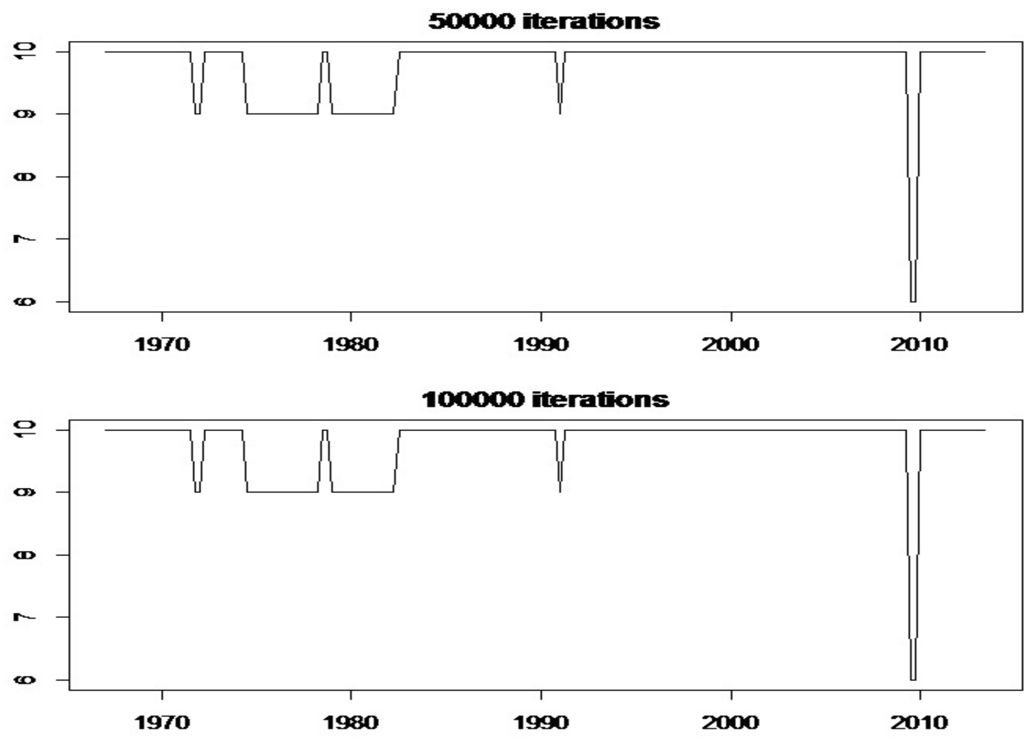

Figure 5.2: regime sequence based on M403 between different iterations

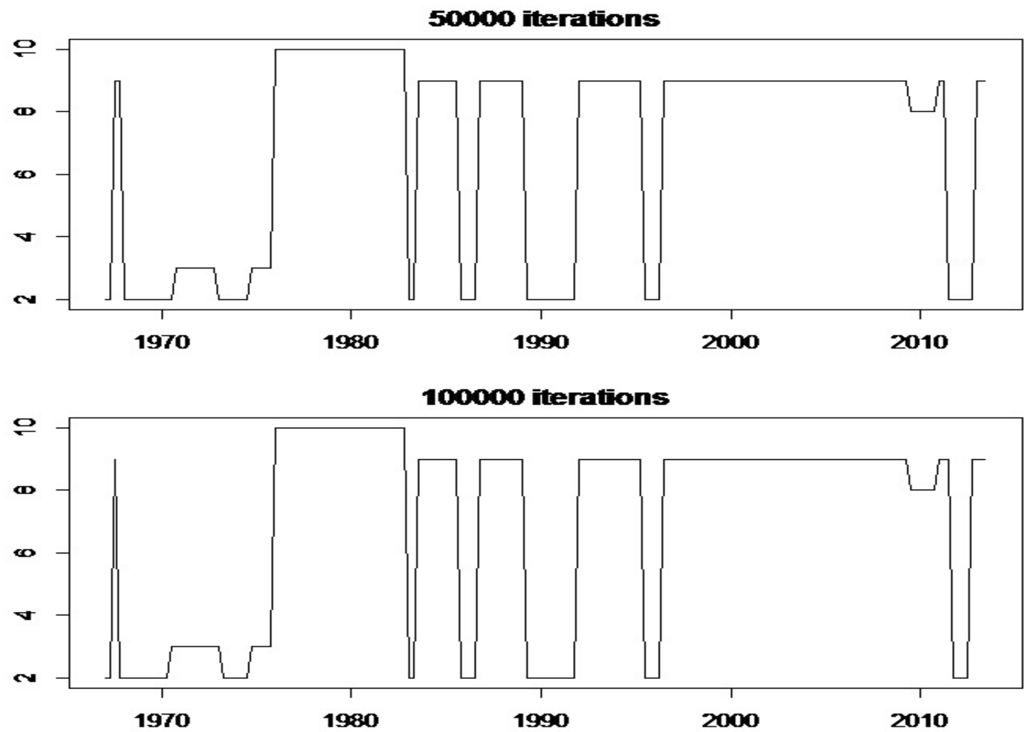

Figure 5.3: regime sequence based on M412 between different iterations

5.4 Estimation from iHMM model

The estimation results of iHMM from Gibbs sampling are based on taking every 10-th of 45000 draws from Monte Carlo Markov Chain (MCMC) output after an initial burn-in period of 5000 draws which are removed to reduce the effect of initial values. The reduced-form equation takes the same form as equation (4.3) discussed in Chapter 4. Since the relationship between monetary growth and inflation is nonlinear, the number of optimal lagged monetary growth(k) terms is unclear. In practice, we tried different number of lags in order to test the optimal number of lagged monetary growth in describing the movement of inflation.

Table 5.5: The value of MSE and MAE

	m403		m412		m012	
lags	MSE	MAE	MSE	MAE	MSE	MAE
k=1	14.402	3.233	8.322	5.954	10.404	6.959
k=2	2.342	1.045	4.101	1.206	2.863	1.145
k=3	4.113	1.644	4.152	1.172	2.105	0.979
k=4	4.915	1.575	2.753	0.961	7.758	4.519
k=5	2.148	0.951	4.492	1.161	5.733	1.317
k=6	0.903	0.643	3.423	1.426	1.591	0.761
k=7	0.623	0.581	3.772	1.581	0.654	0.597
k=8	0.658	0.595	4.578	1.314	0.608	0.571
k=9	0.689	0.592	3.494	1.143	0.370*	0.479*
k=10	0.713	0.591	2.56	0.761	2.604	1.328
k=11	0.490*	0.541*	0.983*	0.710*	5.986	1.427
k=12	3.170	1.051	10.864	1.273	7.286	1.459

From the results of iHMM as shown in Table 5.5, the estimation of iHMM outperformed Markov switching modelwhich was discussed in Chapter 4, in terms of having lower MSE and MAE for three different dataset. Furthermore, the result of iHMM, as shown from Figure 5.4[4],suggests that the number of regimes in the relationship between infla-

[4]The number on vertical axes as shown in Figure 5.4 is randomly selected by iHMM in the es-

tion and money is more than two regimes. This result is different from Markov switch model, as discussed in Chapter 4, which involves only two regimes. In this section, the difference between two models will be discussed in terms of the various monetary series.

5.4.1 The estimation based on different dataset

In the case of M012, we observe four regimes rather than the two regimes based on the Markov switch model over the period from 1960 to 2006. The regime sequence, as shown in Figure 5.4, also exhibits a more complex pattern than the one based on Markov switch model as shown in Figure 4.1. For both regime sequences from Markov switch model and iHMM, structural breaks and regime switches mainly happened in two periods.

As shown in Figure 5.4, the first period is from 1976 to 1985, which covers the operated period of monetary targeting policy. However, the regime sequences from the two models are quite different. First, we observe a structural change for both state sequences at the time when the UK introduced monetary targeting policy in July 1976. However, iHMM detects another structural change in 1978Q2 when inflation continued to rise reaching a level in excess of 20 per cent in 1980. The Conservative government introduced the Medium Term Financial Strategy (MTFS) in 1980 intending to strengthen control of inflation. Until 1982, as shown in Figure 4.1 and 5.4, regime sequences from both models went through a period of fluctuation. After 1982, there are no structural

timation.The random number is only used to indicate change of regimes rather than any order of regimes.

changes in the regime sequence based on the Markov switch model. However, another regime switch is detected by iHMM in 1985 when the UK government abandoned monetary targeting policy. Mishkin (2001) discussed the lessons from the failure of monetary targeting in the UK in terms of an unstable relationship between monetary growth and inflation over the time period. This, Mishkin argued, created difficulty for the monetary authority in conducting their monetary policy as the information about developments in the monetary aggregate for inflation was unreliable in the short run.

The second period is from 1985 to 1992. Compared to the result of Markov switch model, which has only one regime switch in 1989, iHMM contains four regimes after 1985. In the period from 1985 to 1989, the UK informally linked sterling to the German Deutsche Mark. During this period, as discussed in Chapter 4, UK monetary policy closely followed Bundesbank's monetary policy. Following the United Kingdom's departure from the Exchange Rate Mechanism in September 1992, iHMM, unlike the Markov switch model, also detects a structural break when a new policy of inflation targeting was announced in October 1992. Since then, the relationship has switched into a stable period where no further structural break or regime switch is detected.

In the case of M403, the regime sequence, as shown in Figure 5.4, is similar to the result of the Markov switch model. Most of the time, the relationship between inflation and money switches between two regimes. However, there are two major differences between the results of the two models.

First, in the period from 1970 to 1980, the relationship regained stable based on the Markov switching model. However, we observe a period of fluctuation in the relationship similar to the case of M012 based on iHMM, a regime switch occurring at the first oil crisis in 1973, the announcement of monetary targeting policy in 1976 and the second oil crisis in 1979.

Second, unlike the result from Markov switch model (see Chapter 4 for detail), iHMM detects a further structural break in 2008, switching the relationship into a new regime. However, the relationship between inflation and money only remainsin the new regime for two quarters before switching back to the pre-crisis regime.

In the case of M412, the regime sequence, as shown in Figure 5.4, exhibits the most complicated pattern of the cases based on both the Markov switch model and iHMM. For the Markov switch model, the regime sequences are similar between cases based on M412 and M403, as shown in Figure 4.1. However, when we apply iHMM to the relationship, as shown in Figure 5.4, the regime sequence are different in terms of the number of regimes and position of structural breaks.

There is a structural break in 1971, similar to the case based on Markov switch model, potentially associated with the major disturbance in the international monetary system with the US unilaterally terminating convertibility of the US dollar to gold. However, in order to redesign the exchange rate regime, the Smithsonian Agreement was signed to peg the dollar at $38 per ounce. This, however, collapsed in February 1973 – ending the Breton Wood fixed exchange rate system where a regime switch occurred in this year. In the same year, we see OPEC quadruple the price of oil in October thereby administering a supply-side inflationary shock to the UK.

Another major structural break was observed in 1976 when monetary targeting was introduced in the UK. However, compared to the case based on M403 shown in Figure 5.4, there are no structural breaks from 1976 to 1982. During this period, the second oil crisis and adoption of MTFS by the UK government in 1980 did not change the relationship between inflation and M412.

Another regime switch occurred in 1982 when MTFS was heavily revised by raising the £M3 target. Afterwards, the relationship switched into a regime which lasted for only two quarters. In this regime, inflation fluctuated accompanying the shocks in monetary policy and the economy. As shown in Figure 5.4, at the time of key disturbances, such as that associated with the shadowing of the D-Mark by Sterling and the economic crisis in 2008, iHMM detects a transitional switching in the relationship which is sustained for only a short period (generally less than four quarters) before the relationship reverts to a regime sustained for a longer time with inflation remaining relatively low compared to other regimes.

5.4.2 Comparing estimates from iHMM

Through a comparison of estimates based on alternative datasets, we can find some characteristics in the relationship between inflation and money. First, even though positions of structural break are shared between different datasets as discussed in previous sections, the definition of money is a important factor contributing to differences in the relationship between inflation and money.

For example, as shown in Figure 5.4, the relationship between inflation and M0 was in the fluctuation after introduction of monetary targeting. However, the relationship between inflation and M4 are relative stable before the MTFS was heavily revised in 1982.

Second, the choice of growth rate is also critical in the determination of the relationship between inflation and money. As shown in Figure 5.4, the relationship between inflation and M403 is more stable than the alternatives in terms of having fewer regimes and structural breaks.

Third, the shift in the monetary policy regime, such as announcement of monetary targeting or the introduction of inflation targeting, is the main reason for the change of relationship between inflation and money. Disturbances in the economy, such as the oil crisis and financial crisis, only switch the relationship into a transitional regime (generally two quarters) for a short time.

5.4.3 Difficulties faced by iHMM

However, iHMM shares some difficulties with Markov switch model in tackling the relationship between inflation and money. First, coefficients, as shown from Table 5.6 to 5.8, change with the structural change. However, some coefficients are negative which is suspicious as the effect of monetary growth is unlike to have a negative effect on inflation. This results suggest that the model may be inefficient in simulating the relationship between money and inflation. This unreasonable result also lead to another difficulty faced by the iHMM.

Second, the length of the lags of money to inflation is fixed for both models. From the quantity theory of money, the long run relationship between money and inflation is unity. However, the long run is not inflexible in calender time. As discussed in Chapter2, the long run relationship between money and inflation suggests economic conditions after prices fully adjusted to the effect from monetary growth. Therefore, the period of adjustment should be flexible in terms of lagging monetary growth. However, both iHMM and Markov switch model are not able to achieve this by changing the lag-length over time. However, by allowing the length of lags to be flexible increases the difficulty in the estimation of model. This difficulty will be left to future research.

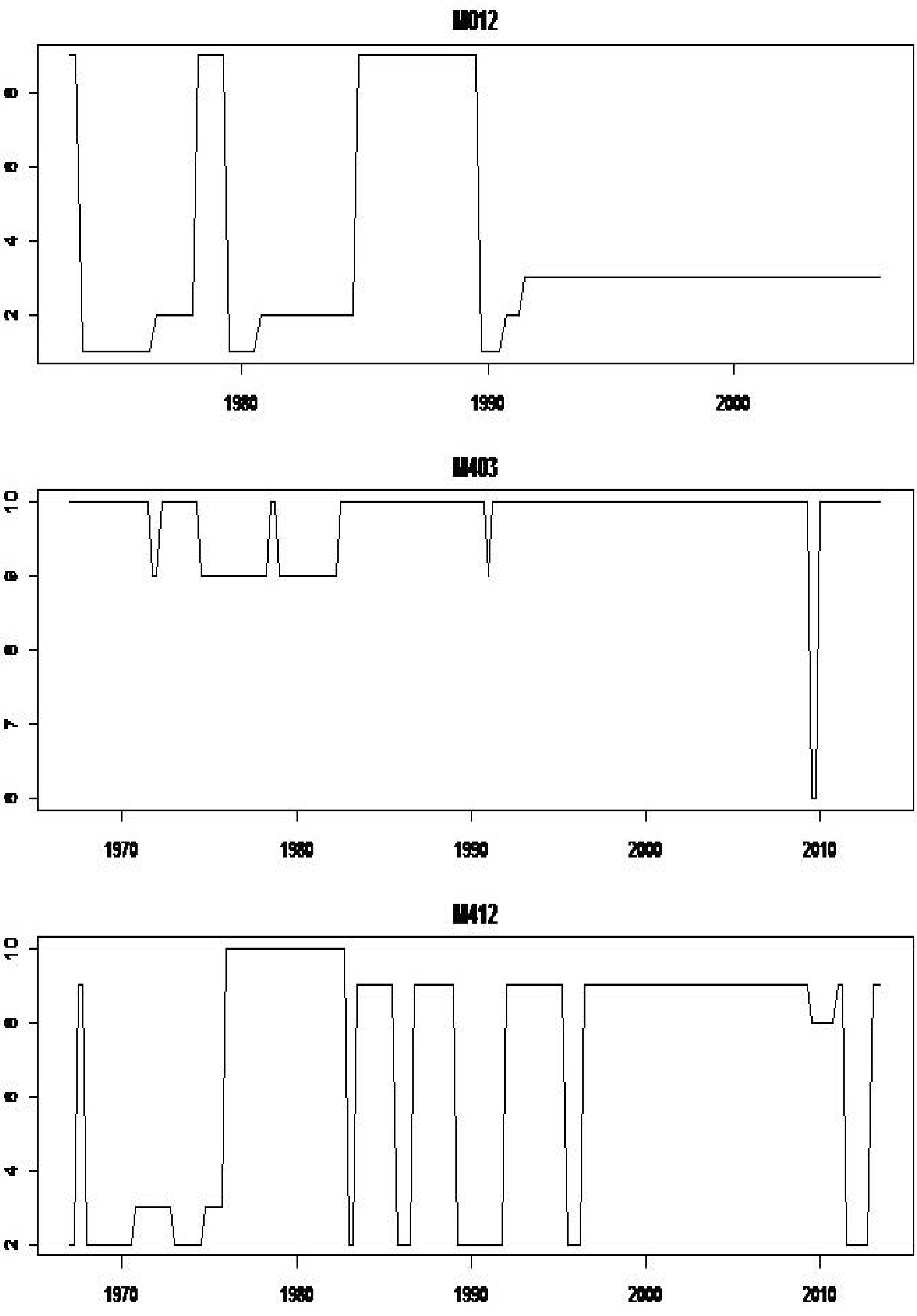

Figure 5.4: regime sequence for optimal model in each dataset

Table 5.6: Estimation of iHMM with 11 lags of M403

parameters	regime 6	regime 9	regime 10
α_0	0.0372	0.6551	0.2613
ϕ_0	0.3591	-0.0511	0.0183
ϕ_1	0.3892	-0.0541	0.0699
ϕ_2	0.6902	0.1407	0.0859
ϕ_3	-0.8592	-0.1544	-0.0574
ϕ_4	0.4052	-0.2441	0.0455
ϕ_5	-0.2912	-0.1489	0.0214
ϕ_6	-0.8001	0.4763	0.0472
ϕ_7	0.6231	0.3574	-0.062
ϕ_8	-0.5004	0.1841	0.0756
ϕ_9	-0.9201	0.1821	-0.0062
ϕ_{10}	-0.2892	0.2151	0.0636
ϕ_{11}	0.0781	-0.0635	0.0171
Sum of Coef.	-1.1152	0.8396	0.319

Table 5.7: Estimation of iHMM with 9 lags of M012

parameters	regime 1	regime2	regime3	regime9
α_0	3.2576	2.1355	3.8371	3.0801
ϕ_0	0.2890	0.1221	-0.0606	-0.3454
ϕ_1	-0.181	-0.3164	-0.0902	-0.1023
ϕ_2	0.1428	-0.1750	-0.0493	0.2458
ϕ_3	0.9347	0.4042	0.1605	0.4671
ϕ_4	0.8958	0.2222	0.0354	0.1336
ϕ_5	-0.6841	0.1949	0.1302	0.0719
ϕ_6	-0.3417	-0.1294	0.0605	0.1983
ϕ_7	-0.134	0.1134	0.0393	0.1214
ϕ_8	0.1392	-0.0573	-0.0588	-0.3320
ϕ_9	0.2985	0.5693	-0.0343	-0.1015
Sum of Coef.	1.3539	0.948	0.1327	0.3569

Table 5.8: Estimation of iHMM with 11 lags of M412

parameters	regime 2	regime 3	regime 8	regime 9	regime10
α_0	1.0959	0.6894	0.9712	1.3756	0.3523
ϕ_0	-1771	-0.3082	-0.3472	0.0113	-0.4908
ϕ_1	-0.2367	0.3341	-0.1872	-0.0257	-0.3525
ϕ_2	-0.0607	-0.1977	-0.4388	0.0579	-0.0275
ϕ_3	0.0988	0.3327	-0.0219	-0.1007	0.0718
ϕ_4	-0.0097	-0.1934	-0.0039	0.0783	-0.0366
ϕ_5	0.1510	0.2147	0.2463	0.0294	0.0116
ϕ_6	-0.0215	-0.0391	0.0753	0.0684	0.5374
ϕ_7	0.0049	0.0951	0.2305	-0.0185	0.5102
ϕ_8	-0.1003	-0.1523	0.1268	0.1397	0.3524
ϕ_9	0.0114	0.0585	0.0061	0.0271	0.344
ϕ_{10}	0.0122	0.3579	0.1537	-0.0153	0.03214
ϕ_{11}	0.1454	0.3272	0.1145	-0.0579	0.03229
Sum of Coef.	0.4126	0.8294	-0.0407	0.194	0.9845

5.4.4 Application of iHMM in detecting structural change

As discussed in section 5.3, a structural break/regime switch that indicates a change in the relationship between monetary growth and inflation. Therefore, for analysing the effect of monetary policy on the relationship, it will be useful if the change of relationship between monetary growth and inflation can be detected as early as it just happened. In this section, we truncate the sample size to let the structural breaks/regime switch happened at the end of sample in order to test the ability of iHMM in detecting the structural change. Since the M0 dataset discontinued at 2006, we choose the relationship between inflation and M4 to conduct the test. The positions of structural breaks, after financial crisis in 2008, are selected for both M4 datasets with lag length at $k = 11$.

First, the sample of M403 is truncated at the position of structural change of 2008Q4. As shown in Figure 5.5(a), iHMM can successfully detect the structural change even at the end of sample. Second, the sample of M403 is truncated at the position of structural change of 2009Q2, two quarters after previous structure break. However, as shown in Figure 5.5(b), the iHMM model failed to detect the structural change. For this reason, we extend the sample to 2009Q3, one quarter after structural change at 2009Q2. As shown in Figure 5.5(c), the regime sequence includes a structural change at 2009Q2.

In contrast, the regime sequence for M412 involves more structural changes after economic crisis. There are four structural changes at 2008Q4, 2010Q2, 2010Q4 and 2012Q2. As shown in Figure 5.6 and 5.7, the structural change at position of 2008Q4, 2010Q2 and 2012Q2 are successfully detected. However, the regime change at 2010Q4, two quarters after the structural change at 2012Q2, can not be detected at the end of sample. In this case, sample is truncated at 2011Q1, one quarter after the structural change at 2010Q4. The regime sequence, as shown in Figure 5.7(a), involves a structural break at 2011Q1 other than 2010Q4.

In general, the estimation from iHMM can effectively detect the structural change at the end of sample. However, if the structural change is very close to the previous structural change, for example, two structural changes happened within three quarters in our case, the position of structural change may not be detected at the end of sample.

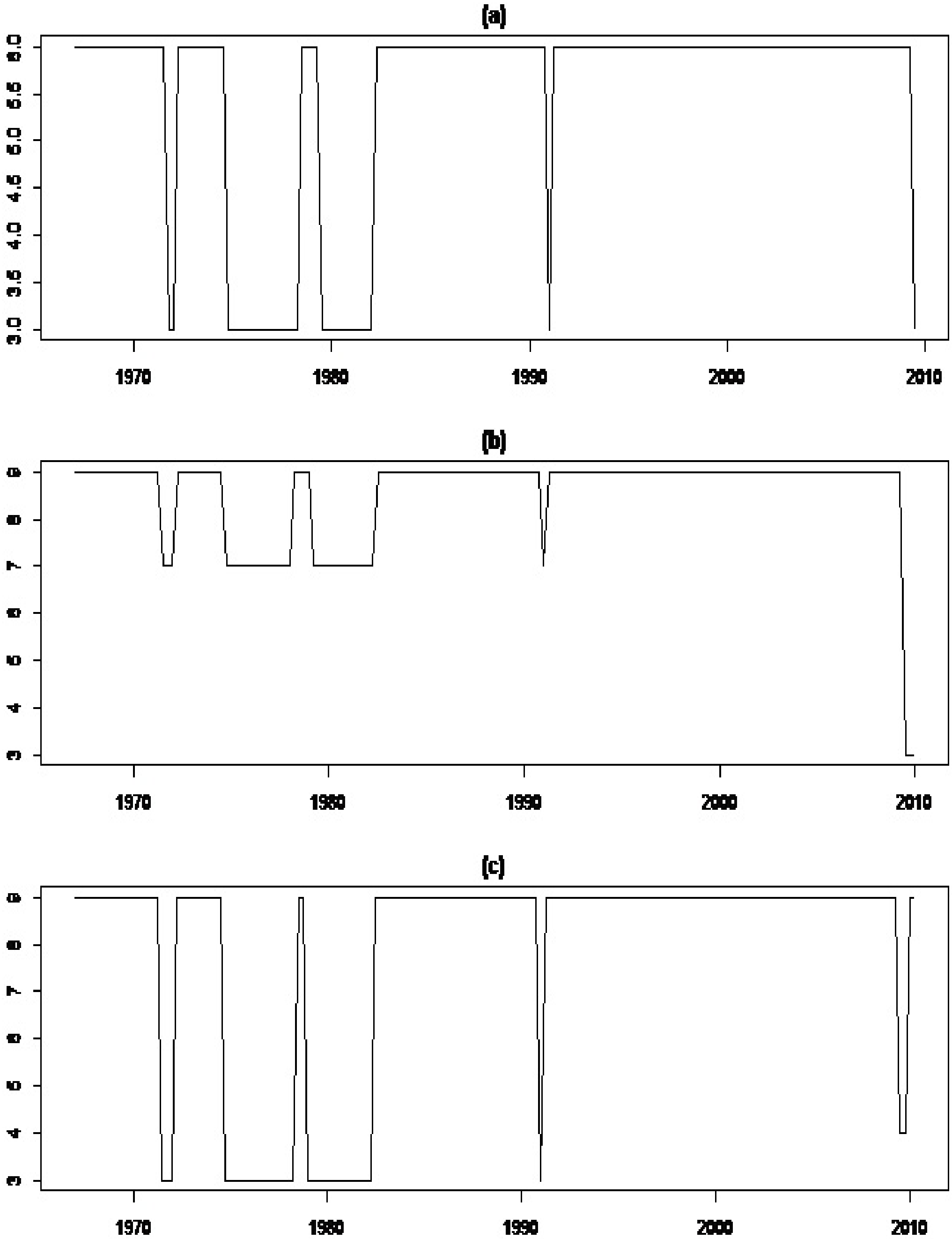

Figure 5.5: (a) regime sequence based on M403 truncated at 2008Q4; (b) regime sequence based on M403 truncated at 2009Q2; (c) regime sequence based on M403 truncated at 2009Q3

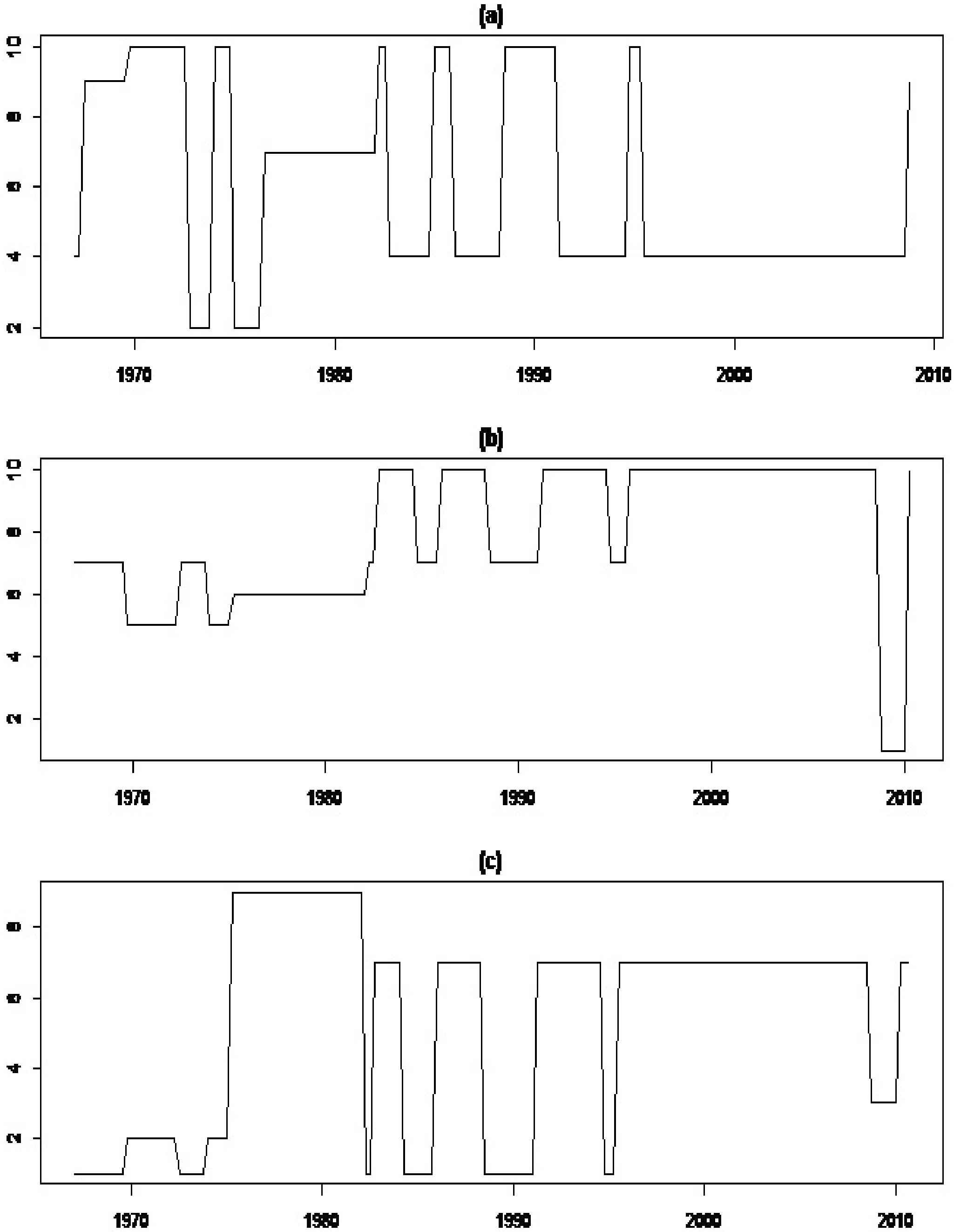

Figure 5.6: (a) regime sequence based on M412 truncated at 2008Q4; (b) regime sequence based on M412 truncated at 2010Q2; (c) regime sequence based on M412 truncated at 2010Q4

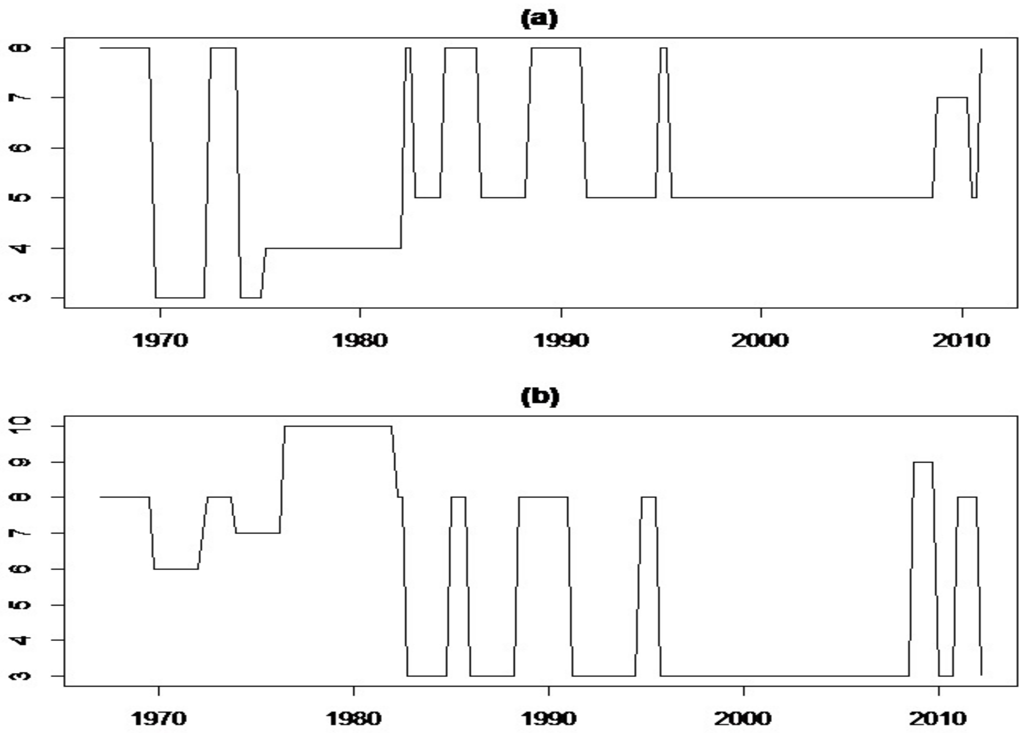

Figure 5.7: (a) regime sequence based on M412 truncated at 2012Q4; (b) regime sequence based on M412 truncated at 2011Q1

5.5 Summary

In this chapter, the relationship between monetary growth and inflation in the UK is divided into multiple regimes by applying the infinite Hidden Markov Model. In general, the leading period of monetary growth over inflation is looking backward up to 11 quarters for M4 and 9 quarters for M0. However, relationships between inflation and different definitions of money are varied. The change of regime in the relationship between inflation and quarterly 3 months growth rate of money is dominated by the change of inflation. Their regime sequences involves fewer regimes and structural changes when compared to the relationship between inflation and quarterly 12 months growth rate of money. Also, changing regime in the relationship between monetary growth and inflation is accompanied by the change of sum of coefficients on monetary growth. This suggests the change of aggregate effect of preceding monetary growth on inflation.

After introduction of inflation targeting in UK, the relationship between monetary growth and inflation become stable until the economic crisis in 2008. Otherwise, the recent economic crisis disrupted the relationship between monetary growth and inflation by introducing a structural break. However, the relationship between monetary growth and inflation only sustained in a new regime, after economic crisis, for a short time before switching back into the regime before the crisis.

In addition, the iHMM can effectively detect the structural change at the end of sample. This feature is very useful in monitoring a change in the relationship between monetary growth and inflation as it happened.

APPENDIX

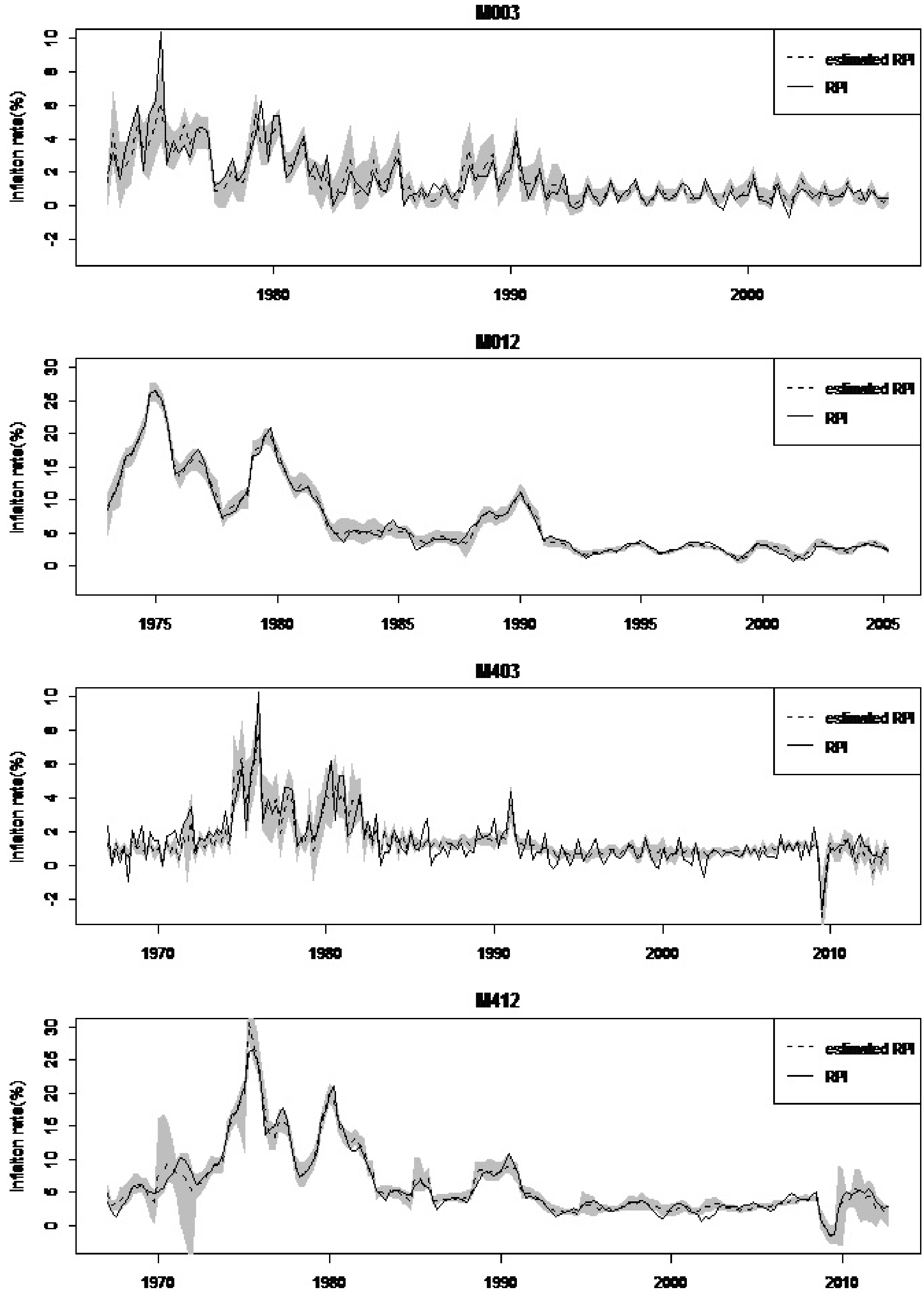

Figure 5.8: Estimation result for optimal model in each dataset

Chapter 6

Conclusion

In this thesis, we have investigated nonlinearity in the relationship between monetary growth and inflation. From the quantity theory of money, inflation should move one to one with monetary growth in the long run. However, in the short run, the relationship between inflation and monetary growth is also affected by other economic factors.

Currently, the mainstream model, the New Keynesian model, excludes a role for money in the determination of inflation. Woodford (2003) stated that there is no space for the monetary aggregate in the New Keynesian model, because the additional money balances beyond the optimal level, which is settled by the interest rate, provide no further liquidity services. However, there is a basic assumption in the New Keynesian model where all the non-monetary assets are perfect substitutes. In this case, the transaction friction provided by money supply can be safely ignored. On the contrary, introducing financial frictions into models of asset prices, and recognizing the role of money in reducing those frictions, provides a potentially significant role for money in the transmission mechanism (King,2002). The debate between Monetarist and Keynesian (New Keynesian after) about transmission mechanism has been focused on the range of assets provided. However, the continued debate make the position of money in structure model for inflation still uncertain.

In the absence of a structural model for the relationship between monetary growth and inflation, a reduced form equation provides a direct and convenient way for describing the relationship. However, the unstable relationship suggest that the relationship between monetary growth and inflation in the short run may be nonlinear. Existing empirical studies also suggest that the nonlinear model is more desirable than the linear model in describing the relationship between monetary growth and inflation.

In order to investigate the potential nonlinear relationship, we first consider a test of structural breaks in the linear model between inflation and monetary growth. The SQ and DQ test, which test for structural breaks not only in the mean but also in the quantiles of the distribution, are applied. The results of DQ test suggest the existence of structural breaks in the linear model between inflation and preceding monetary growth. However, the results of the DQ test only suggest a single structural break with a maximum test statistic across quantiles. Then, we used the SQ test to investigate if the structural break only exist in a specific quantile. The result of the SQ test detects a structural break in almost every quantile. Also, the positions of the structural break for different quantiles are varied. This suggests the existence of multiple structural breaks in the linear relationship between inflation and monetary growth. The result of the DQ test for sub-samples, which truncated the sample at the positions of structural break, supports the existence of two or more structural breaks in the relationship between monetary growth and inflation.

Qu (2008) introduced critical values for the SQ and DQ test based on the sample size, $n = 500$. Even though, Qu (2008,2010) suggested that critical values simulated based on $n = 500$ is reasonable for the sample size smaller than $n = 500$, Monte Carlo experiments shows that the power of test significantly decreased with sample size.

In many empirical studies, the sample size is less than 500. In our study, the sample size is less than 200 observations for both monetary growth and inflation. Therefore, we calculate critical values corresponding to various sample sizes at $n = 300$, $n = 200$ and $n = 100$. The critical values decreased with the sample size. Through the Monte Carlo experiment, we found that the power of test based on critical values corresponding to different sample size are significantly improved when compared to the power of test based on $n = 500$.

The results of DQ and SQ test suggests a potential nonlinear relationship between monetary growth and inflation. This result is supported by existing empirical studies which suggest the variability in the relationship between money and inflation. However, there are some constraints faced by existing empirical studies due to technical difficulties.

Before we discussing the iHMM in analysing the relationship between inflation and money, we conduct Makrov switch model to compare the results with iHMM. The results of Markov switch model support the findings from SQ and DQ tests. Also, Markov switch model found more structural breaks over time. However, Markov switch model is limited to have only two regimes. Therefore, it is still unknown if two regimes is enough to study the relationship between inflation and money. From the results of iHMM, we found that the limit on the number of regimes do affect the efficiency of estimation of Markov switch model as the estimation of iHMM involved more regimes over time and had lower MAE and MSE compared to Markov switch model.

Generally, there are three restrictions on the nonlinear model in existing studies. First, the presumable number of regimes for structural change are limited. Many nonlinear models only involve two regimes. Second, the reason for the structural change is specified in advance. However, without a clear transmission mechanism, the cause of structural change in the relationship between monetary growth and inflation remains unknown. Third, structural change can not distinguish between a new regime and the reoccurrence of a previous regime.

In order to tackle these difficulties in modeling a nonlinear relationship between monetary growth and inflation, we applied the iHMM to study the underlying relationship. From the estimation of iHMM, we found that the leading period of monetary growth over inflation is up to three years. Also, the definition and growth rate of data are critical in analysing the relationship between inflation and money as the relationship between inflation and money varied with different data. Otherwise, the change in the relationship between inflation and money is in accordance with the change of monetary policy regime. On the contrary, the disturbances in economy only cause a short term fluctuation in the relationship.

For the application, we found that the iHMM can efficiently detect a structural change in the relationship between money and inflation even at the end of sample. This feature is desirable for detecting the potential structural change in the relationship early.

In the future, there are several direction of studies that we can work on. First, the cause of structural change in the relationship between monetary growth and inflation is still uncertain. Therefore, it is difficult predict the possible structural change in the future. A possible solution is to analyse the segmentation of inflation and study and effect of money on different components of inflation.

Second, the length of lags in iHMM should be given before the estimation. This could cause the misspecification of model, such as negative coefficients in the estimatino of iHMM as discussed in Chapter 5. However, by setting the length to be flexible, the estimation of model will become even more difficult. Therefore, it will be interesting to study the estimation method which not only provide enough flexibility but also keep the efficiency in the estimation.

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