"ENERGY, ENTROPY AND ECONOMIC STRUCTURE"

BY

JOHN L. R. PROOPS

A THESIS SUBMITTED FOR A Ph.D (ECONOMICS)

DEGREE AT THE UNIVERSITY OF KEELE,

OCTOBER 1979.

CONTENTS

ABSTRACT

This thesis takes as its central theme the problem of why economies use energy. Some thermodynamic aspects of economic functioning are examined, particular attention being paid to the relationship between the "organisation" exhibited by economies and their rates of energy dissipation. An attempt is made to place this study of economies in the context of the evolution and interrelation of ideas in economics and thermodynamics. It is noted that some authors have suggested that as economies become more organised their rates of energy dissipation increase. As the notion of "organisation" is closely allied to that of "entropy", the entropy/information concept is examined in some detail, particularly in its applications to the social and biological sciences. The properties of general dissipative systems are explored, and the possibility of such systems becoming progressively more organised over time is examined with reference to the Second Law of Thermodynamics and the "Evolutionary Arrow of Time". It is argued that if a physical viewpoint is taken, economies can be considered as self-organising dissipative systems. To allow suitable measures of organisation and dissipation to be defined for economies an examination is made of the application of Input-Output analysis to the formulation of Energy Intensities and the Energy Coefficient. Several measures of organisation and dissipation are devised, and using them an empirical analysis is carried out. The conclusion reached is that two effects are present. Comparison between economies confirms that energy dissipation by economies is greatest for those that are most organised. For any given economy over time, though, it seems that as that economy becomes more organised it also becomes more "efficient" in its use of energy.

CHAPTER ONE

Introductory Remarks

Recently a great deal of interest, and publication, has centred upon the problem of how economies use energy. Current difficulties of oil supply certainly justify this interest, and undoubtedly the new field of Energy Analysis is producing stimulating suggestions and prognostications, in the context of the central role energy plays in modern economies.

In a more abstract sense, though, it is not at all clear why economies should use energy at all. An indication of a possible line of explanation of this phenomenon may be had by noting that precisely the same problem presents itself in terms of the functioning of organisms. The anatomy and physiology of an organism may be known with some certainty, and it may be recognised that for its maintenance and reproduction an inflow of matter and available energy is necessary. However, such a description does not provide a thermodynamic explanation of the functioning of the organism. What this study seeks, therefore, is a thermodynamic description of economies, their self-maintenance, and their observed structural evolution.

In Chapter Two the intellectual debt of economic theory to thermodynamics is outlined, and it is noted that although much of the similarity between aspects of economic theory and thermodynamic analysis is there through the conscious or unconscious use of analogy, it might be the case that thermodynamics has more to offer economic theory than just analogical tools. In particular, some authors have suggested that fruitful statements might be made relating the degree of "organisation" exhibited by an economy to its use, or rate of dissipation, of energy. This suggested relationship is taken up as the central theme in the succeeding chapters.

"Organisation", as used in this sense, is an ill-defined concept. A possibly related concept is that of entropy, a physical variable measuring a system's "mixed-upness". In Chapter Three the entropy concept is explored; not only is the thermodynamical sense examined there, but also the use of the entropy concept in Information Theory.

In Chapter Four some applications of the entropy concept in the Social and Biological sciences are reviewed, in particular the use of the Information Theory sense of entropy as a means of estimating the changes in the thermodynamic entropy of developing organisms.

Having outlined the entropy concept, the next step is to relate the notion of entropy to that of organisation, and also to suggest ways in which systems can become more organised, and maintain their organisation. This is attempted in Chapter Five, where the work of Prigogine and the Brussels School of Thermodynamicists is used to outline a theory of self-organising dissipative systems. Such systems maintain themselves in an organised, low entropy, state by taking in low entropy energy and dissipating high entropy energy. Further, such systems are far from thermodynamic equilibrium, and may proceed to other configurations that are also far from equilibrium. Several modes of such self-organising behaviour are outlined, and the relationship of such behaviour to the Second Law of Thermodynamics, of increasing entropy, and the "Evolutionary Arrow of Time", of increasing organisation, are examined.

In Chapter Six the paradigmatic nature of the economic discipline is discussed. It is suggested that although the four major paradigms of economic thought do not require the concepts of energy and entropy, the physical activities of economies can be viewed as those of self-organising dissipative systems, and as such the problem of <u>why</u> economies use energy can be answered in terms of the necessary properties of such systems.

Having proposed a conceptual framework for viewing the physical activities of economies, the last two chapters attempt to establish suitable measures of organisation and dissipation for economies, and empirically test the proposition, noted in Chapter Two, that as economies become more organised they tend to dissipate energy at a higher rate. In Chapter Seven the use of Input-Output analysis is discussed with respect to the formulation of Energy Intensities, and the decomposition of the Energy Coefficient, detailing both the aggregated and disaggregated relationships between financial transactions and energy dissipation in economies. A representation of economic activity purely in terms of energy dissipation is also suggested.

In Chapter Eight the Input-Output framework is used to devise several measures of organisation and dissipation of economies. These measures are applied to some available economic data in an attempt to analyse the relationship between organisation and the energy dissipation rate of economies.

Throughout this work economies are viewed in terms of physical, rather than social, relations. As such a novel view of the "economic world" is proposed, and before the ideas outlined above are presented in detail, it would be useful to suggest where such a view might fit into the scheme of modern intellectual and academic discourse.

Two intellectual trends seem to have been prevalent over the past century. The first is the mode of "reduction", whereby complex "notions" are analysed into constituent parts interacting through definable relations. The application of quantum theory to chemistry, and of biochemical techniques to the analysis of biological functioning are examples of this trend.

The second trend is towards a "Gestalt" or holistic approach to system functioning and behaviour, as exemplified by the blossoming of "Systems Science".

However, it would be a mistake to see these trends as being opposed, or even independent. The reductionist mode of analysis is vacuous if the relations between elements which are thus revealed do not then contribute to a synthesis, from which models of the nature of the entire system can be extracted.

Similarly, a premature Gestalt must be reassessed in the light of new details of a system's internal structure.

The relation between the reductionist and the holistic approaches should therefore be viewed as dialectical, or mutually supporting.

In this work the functioning of particular sorts of systems, economies, are scrutinised from the view-point of physical functioning. As such, a charge of reductionism might be levelled, the functioning of economies being seen as subsequent upon purely physical interaction. There may be some truth in this, but perhaps no more than in the accusation that economics "reduces" the study of economies to "just" the interactions between individuals, classes and social institutions.

On the other hand, the attempt to examine economies through their physical functioning can also be seen as contributing to a new Gestalt. In particular, the case is argued in this thesis for seeing economies as exhibiting, in their physical aspects, properties in common with organisms, flames, and convection cells. That is, it is suggested that economies can be viewed as self-organising dissipative systems, such that through the "reduction" of economies to their physical aspects, a more complete whole can be encompassed in the mental models of such physical behaviour.

CHAPTER TWO

Thermodynamics and Economic Analysis

The analysis of society, and in particular the analysis of economic systems, requires a theoretical framework. One element of this theoretical framework will be certain conceptual tools and structures, the manipulation of which gives rise to mental models, which may or may not be testable against reality. These conceptual tools and structures are abstracted from experience, and one would suppose that their nature will be largely determined by the social and intellectual milieu of the abstracter, ie. the books he reads. Further, one would expect that conceptual equipment derived from one area of intellectual endeavour can often be transposed to another area. Indeed, the intellectual history of mankind indicates that such transposition is the rule rather than the exception.

In the particular case of economic theory, one set of conceptual equipment available to the nineteenth century intellectuals, from whose insights much modern economic theory stems, was Newtonian mechanics. Indeed, in 1834 Hamilton completed the work of Lagrange to give a general maximising formulation to Newtonian mechanics. Using the calculus of variations, this allows concise definition and classification of equilibrium states for mechanical systems. Now common experience indicates that the quantities of goods offered for sale and the prices they are sold for are both inter-related, and roughly constant from year to year. In these circumstances the recasting of price and quantity relations into an equilibrium problem immediately suggests itself. This was fully exploited by Walras[1] in 1871, and the similarity between Walras's method of specifying market equilibrium and Lagrange's approach to mechanical equilibrium has been well

1. L. Walras, Elements d'économie politique pure, 1871, (trans.) Elements of Pure Economics, George Allen and Unwin, London, 1954

expounded by Pikler[2] and Amoroso[3].

A more general analogy between field theory and utility theory was attempted by Edgeworth[4], whose indifference maps were consciously modelled on the fields of forces, devised by Maxwell, which are so useful in analysing the equilibrium states of bodies interacting at a distance.

A further extension of the utility-mechanics analogy was made by Pareto[5], where he went from the analogy between mechanical equilibrium and general economic equilibrium, to that between mechanics and individual human behaviour, as discussed by Pikler[2].

Thus the framework of theoretical mechanics had a powerful influence on the definition of at least some schools of economic thought. This influence need not be thought of as the forcing of analogies between mechanics and human behaviour, but rather as the recognition that certain aspects of production and exchange are amenable to mathematical representation in terms of previously explored functional relations. This is not to say that some authors have not detected forcing of analogies. To quote Samuelson:

> "There is nothing more pathetic than to have an economist or retired engineer try to force analogies between concepts of physics and the concepts of economics."[6].

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2. A.G. Pikler, "Utility Theories in Field Physics and Mathematical Economics" British Journal for the Philosophy of Science 5, 1954, p.47 and p.303.
3. L. Amoroso, "Theorie mathematique de l'équilibre économique: Equations générale de la dynamique", Econometrica 18, 1950, p.64.
4. F.Y. Edgeworth, Mathematical Psychics (1871) reprinted in Reprints of Scarce Tracts in Economics and Political Science, London School of Economics, London, 1932.
5. V. Pareto, Manuel d'économie politique, 1909, (trans.) Manual of Political Economy, MacMillan, London, 1972
6. P. Samuelson, "Maximum Principles in Analytical Economics", American Economic Review, 62, 1972, p.249

But we should note that in his seminal work Foundations of Economic Analysis[7] Samuelson himself expresses certain economic relations as being a form of Le Chatelier's principle. This principle, loosely put, says that if constraints are imposed on a system, that system will react so as to counteract, or negate, those constraints. The commonly given example is of a weight hanging from a block of ice by a fine wire. The intense pressure on the ice causes it to melt (though still below $0^{\circ}C$), eventually allowing the wire to pass through the block, which is left intact. One possible representation of Le Chatelier's principle is in terms of the difference between isothermal (constant temperature) and adiabatic (thermally isolated) changes i.e.:

$$(\partial v/\partial p)_t \leq (\partial v/\partial p)_s \qquad (1)$$

constant temperature (isothermal) constant entropy (adiabatic)

Here v is volume, p is pressure, t is temperature and s is entropy. Samuelson compares this relationship with that of the effect of quantity and price changes on a two input production process. If the goods have prices p_1 and p_2, and the process employs quantities q_1 and q_2 respectively, then if good 2 is constrained there are two possible cases to consider.

Case 1 Short Run Hold q_2 constant. If we raise p_1 then Samuelson has shown (under very general conditions) that q_1 will be reduced i.e.:

$$(\partial q_1/\partial p_1)_{q_2} \leq 0$$

Case 2 Long Run Hold p_2 constant. If we raise p_1 again, in this case q_1 will reduce to less than in case 1. i.e.:

$$(\partial q_1/\partial p_1)_{p_2} \leq (\partial q_1/\partial p_1)_{q_2} \leq 0 \qquad (2)$$

7. P. Samuelson, Foundations of Economic Analysis, Harvard University Press, Cambridge, Mass., 1948.

The analogy between (1) and (2) is obvious if we associate the quantity of good 1 with volume, the price of good 1 with pressure, the quantity of good 2 with entropy (of which much more later) and the price of good 2 with temperature.

Here we have a case of the identification of the analogy coming after the economic principle has been formulated in its own right, and Samuelson obviously believes that such <u>a posteriori</u> identification is permissible. Here he would seem to be applying higher standards of rigour than, at least, Edgeworth and Pareto.

The analogy used by Samuelson is with a relationship between thermodynamic variables. That is, variables identifying the macroscopic properties of matter which is subject to work and heat (e.g. pressure, volume, temperature). Perhaps the earliest tabulation of isomorphisms between thermodynamic and economic theory is due to Fisher[8], who related marginal utility to force, utility to energy and disutility to work. More recent work by Davis[9] and by Lisman[10] in the theory of budgets has related entropy to money utility, which leads to the correspondence of income with heat supplied, savings with internal energy, expenditure with external work done by the system, etc. Pikler[11], without limiting himself to the theory of budgets, has also identified entropy with utility and income with heat.

Contemporary with Fisher's work mentioned, Winiarski[12] was attempting

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8. I. Fisher, "Mathematical Investigations in the Theory of Values and Prices", <u>Trans. of the Connecticut Academy of Arts and Sciences</u> <u>9</u>, 1892, p.85.

9. H.T. Davis, <u>The Theory of Econometrics</u>, Principia, Bloomington, Mass., 1941.

10. J.H.C. Lisman, "Econometrics and Thermodynamics: A Remark on Davis' Theory of Budgets", <u>Econometrica</u> <u>12</u>,1949, p.59.

11. A. Pikler, "Optimum Allocation in Econometrics and Physics", <u>Weltwirtschaftliches Archiv</u> 66, 1951, p.97

12. L. Winiarski, "L'energie sociale et ses mensurations", <u>Revue Philosophique</u> <u>49</u>, 1898-9, p.113 and p.237.

to apply results from statistical mechanics to social phenomena. He compared human beings to the molecular particles of physical theory, the interaction of these human particles being due to human attractions, such as that between members of opposite sexes. He commented on the obvious analogy between the flow of heat and social movement, as well as introducing some rather unpleasant speculation on the applicability of the Second Law of Thermodynamics (see next chapter) to the "reversible" and "irreversible" interactions between races. In this strange mixture we find analogy of which not only Samuelson would disapprove.

The treatment of social phenomena as deriveable from a statistical mechanics of interacting human particles is found even among modern authors. Daniel[13,14] has applied this approach to the appearance of cooperative phenomena in human groups. She uses the analogy of the interaction between particles in solids, and liquids, which can be expressed in terms of the relationship between the observed degree of order and the corresponding level of disorganising activity (heat).

In a slightly less arbitrary way, Kerner[15,16] has shown that the Volterra equations describing the populations of interacting species in a predator-prey model, can be cast into statistical mechanical form, so that species "temperatures" and "heat flows" can be identified.

That isomorphisms exist between physical theory and economic theory is clear. But the problem as to the use of these isomorphisms, and even

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13. V. Daniel, "Physical Principles in Human Co-operation", Sociological Review 44, 1952, p.107.
14. V. Daniel, "The Uses and Abuses of Analogy", Operations Research Quarterly 6, 1955, p.32.
15. E.H. Kerner, "A Statistical Mechanics of Interacting Biological Species", Bulletin of Mathematical Biophysics 19, 1957, p.121.
16. E.H. Kerner, "Gibbs Ensemble and Biological Ensemble", Annals of the New York Academy of Science 96, 1962, p.975.

more fundamental, their origin, remains.

The most compelling application must be that outlined by Franksen[17,18], following the early paper by Pedersen[19]. Franksen has suggested that both economic theory and electrical network theory can be formulated as branches of mathematical programming, so that circuit analogues of economic systems can be established and analysed. The analogies he suggests are of voltages with prices, currents with commodity flows, electrical admittance with elasticity, mutual inductance with cross elasticity and the Law of Conservation of Energy (the First Law of Thermodynamics) with Walras's Law (supply equals demand at equilibrium). So Franksen is suggesting that insights into economic behaviour can be gained through the study of electrical circuits. However, as these electrical circuits will almost inevitably be studied by expressing their pertinent features mathematically, there seems little to be gained by inserting electrical network theory between the abstraction of models from economic systems and the mathematical analysis of these models.

Here, as with Pikler[11], we seem to have unification of theory for the sake of unification, where tidiness is established in the universe by making one theory do for two distinct situations. This tidiness is, in its place, a very good thing as it will obviously save the leg-work of establishing already known mathematical principles. But tidiness is also constricting if one works on the principle that models for physical systems

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17. O.I. Franksen, "Mathematical Programming in Economics by Physical Analogies", Simulation, June, July and August 1969, p.297, p.25 and p.63.

18. O.I. Franksen, "Basic Concepts in Engineering and Economics" in Physical Structure in Systems Theory (ed. J.J. van Dixhoorn and F.J. Evans), Academic Press, London, 1974.

19. P.O. Pedersen, "Et produktionsdynamisk problem", Nordisk Tidsskrift for Teknisk Økonomi No.1, 1935, p.28.

can be used in the social sciences, and that independent modes of thought are unnecessary. It is precisely this constricting of conceptual models which seems to give rise to the physical-social isomorphisms, the intellectual background of the scholars preparing them for the perceiving of certain structural relations between social variables. In this sense, the only merit in Samuelson's use of Le Chatelier's Principle as against Winiarski's wild statistical speculation is that Samuelson was not consciously seeking isomorphisms, while Winiarski was.

A rather different approach to the association of thermodynamics with economic systems has been well expressed by Jevons[20]:

> "Not an article of furniture or ornament, not a thread of our clothes, not a carriage we drive in, nor a pair of shoes we walk in, but is partly made of coal..."[21].

If for "coal" we substitute "energy", we have a statement which points out the physical nature of economies, particularly the universal requirement for free energy to perform the functions of production and transformation of goods which constitutes economic activity. Indeed, Jevons was so far advanced in his understanding of the role of energy in economies that a few quotations will delineate most energy problems of interest today. On the role of energy in the cosmos:

> "It has been rendered apparent that the universe, from a material point of view, is one great manifestation of a constant aggregate of energy."[22].

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20. W.S. Jevons, The Coal Question, McMillan, London, 1906 (First Edn. 1865).
21. Ibid. p.444.
22. Ibid. p.161.

On society's need for energy:

> "Day by day it becomes more apparent that the coal we happily possess in excellent quality and abundance is the mainspring of modern material civilisation."[23].

On the increased per capita demand for energy with economic development:

> "... the quantity consumed by each individual is a composite quantity, increased either by multiplying the scale of former applications to coal, or finding wholly new applications."[24].

On the relation between technological change and energy use in an economy:

> "These views lead us at once to look upon all machines and processes of manufacture as but the more or less efficient modes of transmitting and using energy."[25].

On the interdependence of productive processes in a technologically advanced economy:

> "None of our inventions can successfully stand alone - all are bound together in mutual dependence."[26].

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23. Ibid. p.1
24. Ibid. p.196.
25. Ibid. p.161.
26. Ibid. p.412.

On natural resource depletion and the importance of renewable energy sources:

> "A farm, however far pushed, will under proper cultivation continue to yield forever a constant crop. But in a mine there is no reproduction: the produce, once pushed to the utmost will soon begin to fail and sink towards zero."[27]

and:

> "... while other countries mostly subsist upon the annual and ceaseless income of the harvest, we are drawing more and more upon a capital which yields no annual interest, but once turned to light and heat and motive power, is gone forever into space."[28].

Jevons restricted himself to the role of energy in nineteenth century England. Modern writers have tended to use a broader canvas, but do not seem to reveal much that Jevons had not already made clear. Cottrell[29] sees his task as:

> "... trying to discover the relations between the energy converters and fuel men use and the kinds of society they build."[30].

To this end Cottrell assumes that energy use is the major (even only) determinant of social behaviour:

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27. Ibid. p.201.
28. Ibid. p.412.
29. F. Cottrell, Energy and Society, McGraw-Hill, London, 1955.
30. Ibid. p.3.

> "The thesis is that the amounts and types of energy employed condition man's way of life materially and set somewhat predictable limits on what he can do and on how society will be organised."[31].

In other words, Cottrell is asserting an energy theory of history, with energy use the motivating influence behind technological development, rather than seeing increased energy use as a resultant of technological change. The truth is, of course, that both influences take effect, and may be mutually reinforcing. For example, the drilling of oil wells to obtain a replacement for whale oil in lighting, required a certain level of technological development of drilling equipment, etc. Once available, oil found other uses, such as in the internal combustion engine, which have substantially influenced the structure of modern society, and stimulated improvements in the technology of oil extraction.

Having formulated the principle that history follows energy, Cottrell goes on to suggest that social development inevitably waits upon the (fortuitous?) availability of an excess of energy over current social needs.

> "Civilisation waited upon the appearance of such energy surpluses."[32]

Or put another way:

> "... it is generally true that as energy available to man increases, the variety of his activities increases."[33].

31. Ibid. p.vii.
32. Ibid. p.32.
33. Ibid. p.31.

In the light of recent experiences with easily accessible oil and gas supplies there may be some merit in these sentiments. But despite this caveat we would probably prefer to re-express the relationship between economic activity and energy use in the form:

> As the variety of man's activities increases
> it is generally true that the energy required
> by man increases.

As well as his rather exaggerated insistence on the causative nature of the energy supply to societies, Cottrell also expresses several sentiments where the term "energy" could perfectly well be replaced with the phrase "natural resources". When this replacement has been effected there remains only the platitudinous, or at best the obvious e.g.:

> "The preservation of a system of values requires a continuous supply of energy equal to the demands imposed by that system of values..."[34].

> "An elaborate organisation which was able to direct the flow of energy when one set of converters was used may prove quite incapable of channelling the flow from others. So new structures must emerge or the flow will diminish."[35].

Similar expressions are found in the work of White,[36,37] who also wishes to see cultural change as one-dimensional.

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34. Ibid. p.4.
35. Ibid. p.115.
36. L.A. White, The Science of Culture, Grove Press, New York, 1949.
37. L.A. White, "The Energy Theory of Cultural Development", in The Ghurye Felicitation Volume, (ed. K.M. Kapadia) Popular Book Depot, Bombay, 1955.

> "All cultural systems, therefore, like all physical biological systems, can be reduced to a common denominator: energy. Energy is a universal dimension of culture."[38].

The gist of his argument is the perfectly reasonable one, mentioned in the discussion of Jevons, that the production and transport of goods requires the dissipation of energy. But White then goes on:

> "The things accomplished by a cultural system, the degree of development of a culture, is proportional to the amount of energy harnessed and put to work, other factors being constant."[39].

Here White has changed his ground, and is now defining the energy use (per capita per year) of a social system as a measure of its cultural development. He moderates this definition by including an efficiency factor, which when multiplied by the energy use gives a measure of the society's degree of culture. However, it is quite obviously impossible to define this efficiency factor in a way that would satisfy all social systems simultaneously. What, for example, is the efficiency associated with a candle burning in front of a Madonna, or the fuel used by warplanes on exercises? Indeed, it is not counter-examples but rather examples of adequately defined social energy efficiencies that are hard to conceive of.

Illich[40] has also expounded on the role played by energy use in society, and he suggests that there is a certain level of per capita energy use above which equity between members of that society will be diminished.

> "Further energy affluence means decreased distribution of control over that energy."[41].

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38. Ibid. p.1.
39. Ibid. p.2.
40. I.D. Illich, Energy and Equity, Calder and Boyars, London, 1974.
41. Ibid. p.17.

One may have sympathy with the notion that increased consumption of raw materials, including energy, will lead to centralised control of those raw materials. However, that energy is special in this respect, that increased energy dependence necessarily leads to a reduction in equity, and that Illich has correctly identified the per capita energy level above which equity is impossible, are all cases so loosely analysed that one must regard Illich's arguments as inconclusive.

A more measured approach to the role of energy in society is taken by Cipolla[42], though his starting position is equally emphatic:

> "Mais les disposibilités d'energie représentent la base nécessaire à l'organisation de la matière et à tout developpement de l'histoire des hommes."[43].

Cippola is concerned to understand the neolithic and industrial revolutions from the standpoint of energy use before and after their occurrence, with the subsequent alterations to the patterns of social development. Here, unlike Cottrell and White, he sees the increase in energy use per capita as consequent on the improvements in agricultural and industrial technology, rather than as causing these changes.

The most ambitious attempt to express social phenomena in terms of society's energy use is undoubtedly that by Odum[44,45]. As an ecologist, his fundamental model is that of the ecosystem, in particular the structure of energy flows in such ecosystems. In such a system energy enters by plant photosynthesis. These plants are eaten by herbivores, which are in turn eaten by carnivores, where eating transfers energy from the eaten to the

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42. C. Cipolla, "Sources d'énergie et histoire de l'humanité" Annales E.S.C. 16, 1961, p.521.
43. Ibid. p.521.
44. H.T. Odum, "Energy, Ecology and Economics", Ambio, 1973 (2), p.220.
45. H.T. Odum, Environment, Power and Society, Wiley, London, 1971.

eater. The metabolism of plants and animals dissipates energy, thus maintaining an energy balance for a system in equilibrium. So the interaction between the various sections of an ecosystem can quite reasonably be viewed in terms of energy transfers within that ecosystem.

Odum extends this method, together with its rather complicated symbolic representation, to economic phenomena, and also to political and religious systems, by applying the suggestion that money and energy "flow in opposite directions".

> "We receive food from the grocery store by passing money in the opposite direction to the grocer. We receive money when we put energy into work that makes an energetic contribution to the function of at least one other unit."[46].

From this assumption a vast theory of the energy representations for social interaction results. But it is an edifice ill-founded. Consider the following three cases.

(1) When a lion eats an antelope it gains energy.

$$\text{ANTELOPE} \xrightarrow{\text{energy}} \text{LION}$$

(2) When a man buys food he exchanges money for energy.

$$\text{SHOPKEEPER} \underset{\text{money}}{\overset{\text{energy}}{\rightleftarrows}} \text{MAN}$$

(3) When a man buys a motor car he exchanges money for a lump of metal which is a potential energy <u>sink</u>.

$$\text{CAR SALESMAN} \underset{\text{money}}{\overset{?}{\rightleftarrows}} \text{MAN}$$

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46. Ibid. p.174.

If Odum's model is correct then the man in case (3) is getting a bad deal. But we know he is not, as he set out to buy a car and not a chocolate bar or a can of paraffin, and we do not therefore expect energy to change hands with the money. If we wish, we can associate the amount of energy that has been dissipated in its manufacture with a good, so that it is as if energy changes hands. But then what about the energy associated with the production of the antelope, or the food from the grocer's? Obviously this is no solution.

A car is a good and food is a good. Food also, and coincidentally, contains useful energy. Indeed, the satisfaction we obtain from this energy is why we buy food, but it is not why we buy motor cars. So we see that wherever Odum associates energy flows with thetransfer of goods he must either be using an "as if" ism or be mistaken. As he nowhere suggests that these energy flows are purely imaginary we can only conclude that he is mistaken. Our faith in this conclusion is strengthened when we notice that in the section on religion and energy he has a diagram with one energy flow labelled "Dogmas" [47].

From asserting that history is determined by society's use of energy it is a relatively small step to asserting that the value of goods is determined by their energy "content". By "content" is usually meant either energy that can be released from the good, as with oil, coal and food, or the energy used up in the manufacture of the good.

One of the earliest exponents of this theory was Winiarski[12], who also suggested that the measure of the "transformation" (dissipation?) of energy in an economy is the expenditure of gold. His reasons for asserting that the gold, or currency, value of a good is an accurate reflection of its energy content are obscure, but at least this approach has the merit of being correct by definition.

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47. Ibid. p.241.

A little later Ostwald[48] proposed an energy theory of value as part of his general energy theory of the cosmos, the "New Energetics" so severely criticised by Boltzmann[49].

> "... value in general rests upon the transformation of energy."[50].

Even more explicit was Soddy[51,52,53] in his identification of energy with all things in this world worth having:

> "But energy and wealth are synonymous."[54].
>
> "Wealth ... is essentially the product of useful or available energy."[55].
>
> "If he (Marx) had left out from his definition of wealth the word "human" and had said that wealth had originated in labour in the sense the physicist uses the word for work or energy, he would have anticipated modern views."[56].

The final quotation is interesting, suggesting as it does that the labour theory of value is a special case of the energy theory of value. Now as has been discussed by Desai[57], the relation between labour inputs and prices

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48. W. Ostwald, "The Modern Theory of Energetics", The Monist 17, 1907, p.481.
49. L. Boltzmann, Populare Schriften, Essay 3, 1886, reprinted in Theoretical Physics and Philosophical Problems (ed. D. Reidel), Dordrecht, Holland, 1974.
50. Ostwald, op.cit., p.513.
51. F. Soddy, Matter and Energy, Thornton Butterworth, London, 1912.
52. F. Soddy, Wealth, Virtual Wealth and Debt, Allen and Unwin, London, 1933.
53. F. Soddy, The Role of Money, Routledge, London, 1934.
54. Matter and Energy, op.cit., p.34.
55. Wealth, Virtual Wealth and Debt, op.cit., p.102.
56. The Role of Money, op.cit., p.14.
57. M. Desai, Marxian Economics, Gray-Mills, London, 1974.

of outputs has long been recognised, and the derivation of one from the other is normally known as the Transformation Problem. Samuelson[58], among others, has shown that under certain conditions labour inputs can be reconciled with the equilibrium market prices of neo-classical theory. But rather than reinforcing Soddy's claim for energy, it seems to leave it completely untenable.

First, if "labour" is thought of as "human physical work", there seems to be no value left for other energy sources to account for.

Second, Samuelson's approach quite fairly sees labour as time foregone in exchange for goods. Whether this is time foregone hewing coal or rifling papers is immaterial. It is the social rather than the physical aspect of work which is used in the social phenomenon of establishing price levels.

Strongly influenced by, even derivative from, Soddy's publications was the Technocracy[59-62] movement in the USA between the wars. This movement was founded by Howard Scott, who claimed an extensive background in engineering but seems more probably to have been a cement gang foreman and a floor polish salesman[63], and was aimed at establishing the supremacy of technical man over economic, and particularly stock exchange, man. The

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58. P. Samuelson, "Understanding the Marxian Notion of Exploitation: A Summary of the so-called Transformation Problem between Marxian Values and Competitive Prices", Journal of Economic Literature 9, 1971, p.399.

59. H. Scott, Introduction to Technocracy, John Lane, London, 1933.

60. A. Raymond, What is Technocracy?, McGraw-Hill, New York, 1933.

61. F. Arkwright, The ABC of Technocracy, Hamish Hamilton, London , 1933.

62. Technocracy Study Course, Technocracy Inc., New York, 1944.

63. H. Elsner, The Technocrats, Syracuse University Press, Syracuse, N.Y., 1967.

Wall Street Crash of 1929 seems to have provided the stimulus needed to establish Technocracy as the movement of that moment. The confused aims and background of Technocracy are well expressed in one of the movement's introductory texts:

> "Technocracy further states that, as all organic and inorganic mechanisms involved in the operation of the social macrocosm are energy-consuming devices, therefore the basic metrical relationships are: the factor of energy conversion or efficiency; and the rate of conversion of available energy of the mechanism as a functional whole in a given area per unit time."[64].

Despite the enormous popular acclaim that Technocracy received in the early thirties it was a short-lived movement. Criticism by Irving Fisher[65], among others, quickly established the weak (even non-existent) theoretical basis of this approach, and the technical utopia offered by Howard Scott soon lost the headlines to the infantile utopia of Shirely Temple.

Very recently, mainly since the rapid oil price rises of 1973, the role of energy in productive processes has again become an active field of study under the title of Energy Analysis.

The aim of energy analysis is to map the direct and indirect energy use of an economy in the production of goods and services, and the very early work of Podolinski[66] into the energy "return" on cultivation by human and animal effort, can be seen as a forerunner of this approach. For example, in the production of a motor car energy is used in the stamping out of body

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64. Scott, op.cit. p.39.

65. J.G. Frederick (ed.), For and Against Technocracy, Business Bourse, New York, 1933.

66. S. Podolinski, "La socialisme et l'unité des forces physiques", La Revue Socialiste 20th June 1880, No.8, p.353. Also published as "Alenschlicke Arbeit und Einheit der Kraft", Die Neue Zeit 1, 1883, p.413.

parts from sheet metal, the welding together of the components, the grinding and boring of the cylinder block and cylinder head, etc. The indirect inputs are the energy used in the smelting, purifying and rolling of the steel, manufacturing of the rubber, etc. The way the energy should be measured and accounted for has been thoroughly discussed in an international workshop document[67].

But even if the method of energy analysis is clear, the motive behind this type of study may not be, certainly not to some "classical" economists. Slesser[68] has well expressed what seems, by consensus, to be the raison d'être of energy analysis. He uses the parable of an aircraft that has crashed on a desert island, and outlines the physical resources available to the crash survivors.

> "But of all the residual supplies, the unburnt fuel in the jetliner's tanks would be the most precious. In clever hands it could do many things. The fuel would be the only intensive energy available to them, capable of melting metal, running motors, sending radio signals, or making fertilizers. They might, if they had the know how, decide to devote their last remaining supplies of fuel to making solar cells, and thus ensure a perpetual supply of somewhat more intense energy. But whether they did this or not, the stored energy on this island would be limited."[69].

The parable is clear. Modern society depends almost entirely upon fossil fuels for the production of the goods it consumes. Fossil fuels

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67. Energy Analysis, IFIAS Workshop Report No.6, Stockholm, 1974.

68. M. Slesser, "Energy Analysis and Technology Assessment", Technology Assessment 2, 1974, p.201.

69. Ibid. p.201.

are in limited supply, and if society has not adjusted itself to using non-depletable energy sources by the time the tanks run dry, then its members will soon be reduced to gnawing coconuts. Thus the interest in the use of energy in the workings of economies is stimulated not only by its ubiquity in the production process, but principally by the limited nature of fossil fuels.

The applications suggested for energy analysis are numerous and diverse, reflecting the wide range of backgrounds and interests of workers in this field. One application is in the study of the effect of price changes of various fuels on the prices of market commodities. The effect on a commodity will obviously be dependent on the direct and indirect energy use in the economy necessary for the manufacture of that good. Such "energy intensities" are discussed in much greater detail in a later chapter.

A second application is into the estimation of the energy requirements of an economy in the light of the anticipated demand by consumers for the produced goods. This is, of course, closely allied with the assessment of the rates of depletion of non-renewable fossil fuels under given technologies and consumer demand schedules.

One particular branch of energy analysis is concerned with discovering the overall energy output to the economy by fuel producing processes, once the imputed energy inputs of tools, transport, etc. have been deducted. This type of study is known as Net Energy Analysis, and has been applied to fossil fuel and nuclear installations, and also agriculture. More ambitious suggestions by Gilliland[70] for the use of energy analysis are related to measuring the environmental impact of economic activities, using energy as a common unit which would allow the internalising within the economic process of the previously external environmental factors.

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70. M.W. Gilliland, "Energy Analysis and Public Policy", Science 189, 1975, p.1051.

> "For example, the energy value of the environment is the amount of the sun's energy used by the eco-system in providing services and products, just as the value of a manufactured commodity is the amount of fossil fuel used by the machines in making the product."[71].

Now Gilliland's approach would seem to invoke an energy theory of value, though she denies this[72]. Nevertheless it is this aspect, real or imagined, of energy analysis that has drawn the strongest fire from its critics[73]. Having disposed of this easy target they move on to the (often self-generated) suggestion which naturally follows, that energy analysis is attempting to supplant economics. This despite the repeated protestations by workers in this field that <u>they</u> do not assume an energy theory of value, and their work should be seen as a complement to, rather than a substitute for economics[74].

Perhaps the most extreme critical position has been taken up by Webb and Pearce[75], who continuously compare the analytical framework of energy analysis unfavourably with that of neo-classical theory in a perfect market. Now energy analysis, as we have seen, is expressly formulated as an empirical tool which can, under simplifying assumptions necessary for calculation, give insights into the importance of energy in production. Neo-classical theory, on the other hand, is a conceptual tool to aid in the understanding of the establishment of economic equilibrium, with the attainment of

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71. Ibid. p.1056.
72. M.W. Gilliland, <u>Science</u> <u>192</u>, 1976, p.12.
73. D.A. Huettner, "Net Energy Analysis: An Economic Assessment", <u>Science</u> <u>192</u>, 1976, p.101.
74. <u>Workshop on Energy Analysis and Economics</u>, IFIAS Report No.9, Stockholm, 1975.
75. M. Webb and D. Pearce, "The Economics of Energy Analysis", <u>Energy Policy</u>, Dec. 1975, p.318.

maximum satisfaction by consumers and maximum efficiency by producers. So one might say that comparing neo-classical theory with energy analysis is like comparing the theory of equations with a theodolite.

This concentration on the concept of the neo-classical market has led Webb and Pearce into a very strange criticism of the usefulness of energy analysis in assessing the effect of an "energy tax".

> "... a tax on energy consumption can be implemented without carrying out elaborate exercises to identify energy use. If, say, some tax proportionate to energy consumption was introduced, energy-intensive activities would automatically bear the heaviest tax burden, simply because energy costs comprise part of the costs of economic activity and because these costs are shifted forward from the most basic economic sectors such as resource extraction to the final product."[76]

As Common[77] has pointed out, Webb and Pearce are here confusing the analysis of how an energy tax would rebound on prices through the economy, with the estimation of the impact of such taxation. The former is a theoretical, even analytic, result of the workings of a market system. The latter is an empirical, synthetic, statement about probable outcomes.

Other similar criticisms of energy analysis by Webb and Pearce are well dealt with by Common and by Chapman[78]. Common's final admonition of some economists' reaction to energy analysis illuminates the problem whose neglect by most economists stimulated the current interest in energy analysis:

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76. Ibid. p.320.

77. M. Common, "The Economics of Energy Analysis Reconsidered", Energy Policy, June 1976, p.158.

78. P. Chapman, "The Economics of Energy Analysis Revisited" Energy Policy, June, 1977, p.161.

> "What is at issue is the nature of the stylised facts which the vast majority of economists take as adequate descriptions of the state of nature. Economics has recently rediscovered the finite nature of the environment within which economic activity occurs, and this is all to the good; but it is not the case that many economists have got very far with working out all the implications for economic analysis of that discovery. This being so, a little humility towards the efforts of others is needed."[79].

We started this chapter by examining isomorphisms between theories of economic behaviour and thermodynamic variables and relationships. Having in the central portion examined some approaches to energy in history, energy and value, and energy in production we return to the inter-relation of thermodynamics and economic theory, which to some extent synthesises these preceding concerns. Our interest will now centre on some aspects of the thermodynamic formulation of physical theories of economic activity and economic development, though as will become obvious, in some authors' eyes such physical theory may be inextricably bound up with one or more of the previously discussed relationships between economics and thermodynamics.

Perhaps the first author to concentrate explicitly on the physical aspects of economic behaviour was Davidson[80], in 1919. Davidson was concerned with the law of diminishing returns, which he saw as due to the physical interaction between various proportions of productive agents, such as fertilizer and land, machines and men, etc.

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79. Common, op.cit., p.165.

80. J. Davidson, "One of the Physical Foundations of Economics", Quarterly Journal of Economics 33, 1919, p.717.

"The law of diminishing returns is based, as I shall try to show, upon chemistry and physics, and, like certain chemical and physical laws, is capable of being reduced to a phase of the law of probabilities."[81].

Davidson attempted to justify this assertion by considering the random combination of two different sized sets of elements, noting that if one set increases in size continually, the likely number of "mixed pairs" also increases, but at a decreasing rate. Hence, he supposes by analogy, continually adding more of one productive element to a production process will yield diminishing returns per unit added.

This consideration of the combination of diverse elements in economic activity has recently been generalised by a consideration of economic activity in terms of the entropy concept. Entropy, which is discussed in detail in the next chapter, is a thermodynamic variable which is found to increase as an isolated system tends towards thermodynamic equilibrium. Such equilibrium is, of course, characterised by the lack of any tendency for thermodynamic change in the system. An alternative formulation of the entropy law is that the system becomes maximally "disordered", all energy and material concentrations having been dispersed.

As noted above, many writers have recognised the necessity of energy "use", more properly termed energy dissipation, to the functioning of economic systems. Now energy dissipation consists of the dispersal of the energy contained in the fuel as waste heat, and so constitutes an entropy generating process. This has led Georgescu-Roegen[82,83], following

81. Ibid. p.718.

82. N. Georgescu-Roegen, The Entropy Law and the Economic Process, Harvard University Press, Cambridge, Mass., 1971.

83. N. Georgescu-Roegen, "Energy and Economic Myths", Southern Economic Journal 41, 1975, p.347, also published in Ecologist 5, 1975, p.164 and p.242.

Schroedinger[84], to write:

> "Casual observation suffices to prove that <u>our whole economic life feeds on low entropy</u>."[85].

He follows this statement with what seems to be an obvious corrolary:

> "We may take it as a brute fact that low entropy is a <u>necessary</u> condition for a thing to be useful."[86].

His chain of reasoning seems to be:

(1) Economies feed on low entropy.

(2) Things needed by economies are "useful".

Therefore

(3) Useful things have low entropy

But obviously (3) does not necessarily follow from (1) and (2). We must also take into account the hidden premises:

(a) All useful things have similar levels of entropy.

(b) All useful things have similar uses.

That Georgescu-Roegen's conclusion does not correspond with the world we observe is instanced by the usefulness of air, fluoridated as opposed to pure water, and omelettes.

However, the point which Georgescu-Roegen seems to be making is that the production of goods requires machines. These are not likely to be made of naturally occurring materials, but rather of processed and purified, and therefore low entropy, substances. So he is suggesting that economies can be thought of as consisting of low entropy devices, the functioning of which generates entropy. This idea is different, and importantly different,

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84. E. Schroedinger, <u>What is Life?</u> Cambridge University Press, London, 1944.
85. <u>The Entropy Law and the Economic Process</u>, op.cit., p.277.
86. Ibid. p.278.

from that put forward by Boulding[87], and echoed by Overbury[88] and English[89,90]. This is that the production of goods corresponds to an entropy decreasing process, as far as the goods are concerned, and the consumption of goods corresponds to an entropy increasing process. It is certainly the case that production goods are likely to have low entropy, as discussed, and their consumption, through use, will cause their entropy to increase. However, as mentioned, the usefulness of goods in general is not necessarily related to their entropy, and in at least the case of iron ore, the consumption of the good can lead to an entropy decrease in that good.

Perhaps more fundamental than the considerations of the physical entropy of the producing goods, and the entropy created by their use in the production process, is the problem of the organizational relations within an economy, as reflected in the energy dissipation by that economy. This problem has been recognised, but not very closely analysed, by Boulding[87], English[89,90] and Adams[91].

> "It so happens that there is a higher correlation between the throughput of production and consumption in a society, as measured for instance by its GNP, and the complexity and elaborateness of the state which it maintains from moment to moment."[92]. (Boulding)

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87. K.E. Boulding, Economics as a Science, McGraw-Hill, London, 1970.
88. R.E. Overbury, "Features of a Closed-System Economy", Nature 242, 1973, p.561.
89. J.M. English, "Economic Concepts to Disturb the Engineer", Engineering Economist ASEE 20, 1974, p.141.
90. J.M. English, "Economic Theory - New Perspectives", in Physical Structure in Systems Theory, op.cit.
91. R.N. Adams, Energy and Structure, Texas University Press, Austin, Texas, 1975.
92. Boulding, op.cit., p.45.

> "At the same time as (man's) productivity is increasing his real needs may also be growing. This may be a result of the changing complexity of society that dictates an increasing consumption level as a concomitant of organisational growth."[93] (English)

> "The complexity, and therefore much of the form, of social and political organization is directly determined by the amount of energy that is being converted in the system."[94] (Adams)

So all three authors see a positive correlation between the degree of "organization" in a society and the energy it dissipates, though while Boulding and English see the organization as determining the energy use, Adams sees energy use as determining organization, in line with the energy theories of history put forward by Cottrell and White. However, none give more than outline arguments as to why this effect takes place, nor indeed do they give empirical evidence to show that it takes place at all. It is this problem that we shall adopt for the rest of this study. That is, how can the physical functioning of economies be expressed so as to give insights into the relationship between the organizational structure of economies and their energy use? As the entropy concept will obviously be of central importance in such considerations the next chapter will be devoted to an analysis of entropy, and the associated concept of "information".

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93. English, "Economic Theory", op.cit., p.284.

94. Adams, op.cit., p.304.

CHAPTER THREE

Entropy and Information

In this chapter we shall be concerned with charting the foundations of the Entropy concept in Classical Thermodynamics, Statistical Mechanics and Information Theory. This discussion will follow the chronology of the concept's development, beginning with thermodynamics.

Thermodynamics began as an empirical science, its development largely stemming from Carnot's early nineteenth century studies of the possible efficiency of steam engines. Being based on empiricism, only observable, macro-variables are invoked in the analysis and explanation of thermodynamic phenomena, no microscopic theory of matter being needed for the formulation and solution of problems. Thus thermodynamics attempts to relate observables to observables. These observables are known as thermodynamic variables. Almost any measurable feature of the world may be termed such a variable, though those most often used (as they seem to be the most useful) are pressure, volume and temperature.

Thermodynamics, like many branches of physics, can be regarded as an axiomatic structure, the results of thermodynamic analysis being the theorems derivable from a few fundamental axioms. The three principal axioms of thermodynamics are known as the Zeroth, First and Second Laws (though at least four laws and up to seven can be specified; See Brostow[1]). Let us discuss these three laws in turn.

The Zeroth Law is so called because it was formulated after the First and Second Laws, when it was realised that these two depended upon a more primitive concept. Calling this principle the Zeroth Law

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1. W. Brostow, "Between the Laws of Thermodynamics and Coding of Information", Science 178, 1972, p.123.

saved renumbering. The Zeroth Law states:

> When two systems are at the same temperature as a third, they are at the same temperature as each other.

The Zeroth Law introduces the concept of "temperature" as applied to a "system". In thermodynamics a system refers to a certain quantity of matter bounded by a closed surface. This surface may be actual or imaginary, and serves only to allow us to distinguish between our "system", with which we are particularly concerned, and the system's "environment", which is of less interest to us. If matter can neither enter nor leave the system it is called "closed". If a system is closed and also energy cannot enter or leave it, it is called "isolated". The temperature is the "degree of hotness" with which we are all familiar, and this law serves to state precisely what is so obvious that it received only belated recognition as an axiom.

The First Law of Thermodynamics states:

> If a certain amount of heat, Q, is added to a system, and a certain amount of work, W, is also added, then the internal energy, U, of the system is increased by Q + W.

So here we are discussing a conserved variable called energy. In particular, the energy <u>internal</u> to the system, rather than that due to the position or velocity of the entire system.

If the internal energy of the system is initially U, and the final internal energy is U_2, then the First Law can be written as:

$$U_2 - U_1 = Q + W \qquad (1)$$

Thus work and heat are specified as interconvertible forms of energy. We can restate (1) in a more useful infinitesimal form:

$$dU = \delta Q + \delta W \qquad (2)$$

The right-hand side of (2) uses "δ" rather than "d" to indicate that while dU is an exact differential δQ and δW are not.

That dU is an exact differential is quickly proved. We recall that an exact differential has the form:

$$dz(x,y) = (\partial z/\partial x)_y dx + (\partial z/\partial y)_x dy$$

$$= X\,dx + Y\,dy$$

Now: $\int_1^2 (X\,dx + Y\,dy) = \int_1^2 dz = z_2 - z_1$

So the integral of (X dx + Y dy) is independent of the path in x-y space over which the integration is taken. So we can consider <u>any</u> cyclic integral over (X dx + Y dy) as composed of two line integrals over different paths, but with the same end points. i.e.:

$$\oint (X\,dx + Y\,dy) = {}_{I}\!\int_1^2 (X\,dx + Y\,dy) + {}_{II}\!\int_2^1 (X\,dx + Y\,dy)$$

$$= (z_2 - z_1) + (z_1 - z_2)$$

$$= 0$$

i.e.: $\oint dz = 0$

Thus any exact differential has a zero cyclic integral, and vice versa. Now as, by definition, the Internal Energy U is a conserved quantity, then:

$$\oint dU = 0$$

i.e. dU is an exact differential. But Q and W are inter-convertible, hence it is possible to conceive of integrals over descriptive variables for δQ and δW (e.g. temperature, pressure, volume) which have the same end-points but different paths which, due to interconversion of heat and work, take different values. Thus $\oint \delta Q \neq 0$ and $\oint \delta W \neq 0$.

The two laws stated so far have given us a static description of the pertinent features of our thermodynamic system, but like any static (or at least quasi-static) description they reveal little about "process". So far we have imposed some restrictions on possible processes, but have said nothing about what defines the process that actually occurs. This gap is filled by the Second Law of Thermodynamics. However, a description and explanation of the Second Law requires some previous familiarity with the distinction between reversible and irreversible system changes.

A reversible change, as its name suggests, can with equal ease cause a system to progress from State I to State II, and from State II to State I. This implies that whatever alterations are imposed upon the system or its environment during the phenomenon can be completely undone. A basic requirement that any system change reversibly is that the change takes place infinitely slowly, with infinitesimal temperature, chemical concentration, etc. gradients. Such changes are, by their nature, purely imaginary, and most real systems exhibit changes which impose irreversible alterations on the system and/or its environment.

We shall present three formulations of the Second Law. The first is due to Clausius:

> Heat can never, of itself, flow from a lower to a higher temperature.

The second formulation is due to Thomson (and is often known as the Kelvin-Planck Law):

> It is impossible to extract heat from a reservoir and convert it wholly into work, without causing other changes in the universe.

The third formulation is a mathematical one (also due to Clausius):

> In every reversible cycle the integral $\oint(\delta Q/T) = 0$, for an irreversible cycle $\oint(\delta Q/T) < 0$. Cycles in

which $\oint(\delta Q/T) > 0$ are impossible. (T≡temperature).

The first formulation is a simple expression of our common experience that the heat flows from the hotplate to the cold saucepan, and not vice versa.

The second formulation says that although work can be wholly changed into heat, heat can never be wholly changed into work. This formulation therefore denies the possibility of constructing a perpetual motion machine which feeds off waste heat.

The third formulation seems impenetratable at first sight, but it is easy to see that it corresponds to both the Clausius and the Thomson formulations. First, suppose that we had a device able to extract a quantity of heat Q from a reservoir at temperature T_2, and deliver it to another reservoir at a higher temperature T_1. Obviously this device would work in defiance of the Clausius formulation. If this device performs a cycle then:

$$\oint(\delta Q/T) = Q/T_2 - Q/T_1 > 0 \text{ (as } T_1 > T_2\text{)}.$$

This operation is denied by the third formulation, so the Clausius and mathematical formulations are seen to correspond.

Second, suppose we had a device which could execute a cycle in which it extracted heat from a reservoir at temperature T and converted it wholly into work, in defiance of the Thomson formulation. In this cycle no heat is given up by the device, so:

$$\oint(\delta Q/T) = (Q/T) > 0$$

So the Thomson formulation also corresponds with the mathematical one.

We have identified two easily observable phenomena in the real world which allow us to define a natural direction for processes, and which are expressible in terms of the cyclic integral of a differential. We note that for a reversible change $\oint(\delta Q/T) = 0$, so for a reversible change $\delta Q/T$ is an exact differential, even though δQ is not. That is $\delta Q/T$ is a conserved quantity for reversible changes. The substitution $\delta Q/T \equiv dS$ is normally used, where we call S the "Entropy" of the system. So the entropy of a system is unaltered by a cycle of a reversible change of the system.

But for an irreversible change $\oint dS < 0$. What does this tell us about the effect on the system induced by a real, irreversible change? Consider a cycle consisting of a reversible section and an irreversible section. Suppose the irreversible change takes place with no transfer of heat to or from the environment (i.e. $\delta Q = 0$), while the reversible section does involve heat transfer. In that case:

$$\oint_{\text{irrev}} dS = \int_{1\ \text{rev}}^{2} (\delta Q/T) + \int_{2\ \text{irrev}}^{1} (\delta Q/T) < 0$$

Now: $$\int_{2\ \text{irrev}}^{1} (\delta Q/T) = 0 \text{ as } \delta Q = 0$$

i.e.: $$\oint_{\text{irrev}} dS = \int_{1\ \text{rev}}^{2} dS < 0 \qquad (3)$$

Now dS is an exact differential for a reversible change, so we can write:

$$\int_{1\ \text{rev}}^{2} dS = S_1 - S_2 \qquad (4)$$

So combining (3) and (4): $S_1 - S_2 < 0$

i.e.: $S_2 > S_1$

The entropy of the final state is greater than the entropy of the initial state. Now it was an irreversible change that transformed the system from state 1 to state 2, though actually <u>what</u> these states are is arbitrary. So we see that any real, and therefore irreversible, spontaneous change in a system will cause the system to proceed to a state of higher entropy. This implies that if a system is not undergoing spontaneous change it must have achieved the highest entropy possible under the currently imposed constraints. So a system in equilibrium will exhibit macroscopic properties corresponding to the state of maximum entropy.

To sum up, the Zeroth and First Laws gave us the concepts of temperature and energy. The Second Law has now given us a direction, by telling us that there is a non-conserved quantity, entropy, which always increases for spontaneous changes in isolated systems. This is a very powerful and useful result, especially as no particular types of systems have been invoked and the proof can therefore apply to <u>any</u> isolated system. But the problem now confronts us: What is entropy? The definition $dS = {}^{\delta}Q/T$ may be formally immaculate, and operationally applicable, but it seems to be telling us little that is intuitively meaningful about the world. But what are the features that give direction to naturally occurring changes? For example, if we mix 1 litre of hot water with 1 litre of cold water we get two litres of warm water. How is this final state different from the initial state? We have lost the <u>difference</u> between the hot and cold. The system is more "mixed-up". It may occur to us to ask why the water is now mixed-up, rather than co-existing as a section of hot water and a section of cold. A simple reply is "That's just the way things are", and classical thermodynamics can go no further than this. A more detailed reply would require a theory of the micro-structure of matter, which is not contained in model independent classical thermodynamics.

To approach the problem of what entropy <u>is</u>, and why systems tend to become mixed up, we must establish a micro-theory, and thereby enter the realm of Statistical Mechanics.

The micro-theory of matter found to be the most suitable is the Atomic theory. This suggests that matter exists as discrete particles (atoms), which are themselves structured. These atoms can combine into molecules which are, by definition, the smallest freely existing particles of matter. This statement of the theory is, of course, a wild over-simplification of the real nature of the world, but it will serve our purposes well enough. Our micro-theory must now be applied to systems consisting of a very great number ($\approx 10^{23}$) of particles to give us a microscopic description and explanation of macroscopic phenomena.

But first we must have a model of the behaviour of the systems of particles we are considering. In the simple case of a gas we consider the particles to be in free motion at great speed, bouncing off the walls of the container and off each other. In a closed system the total number of particles remains constant, while in an isolated system the total energy of all the particles is also constant. How best can we describe the particles in the system? One way would be by a series of "snapshots" of the system, the position coordinates of each particle being tabulated at successive instants. But position alone will be insufficient description to give us a correspondence between the microscopic and macroscopic descriptions of the system. For example, a "perfect" gas, with no internal potential energy (i.e. no particle-particle interactions) has all its internal energy in the form of the kinetic energy of the particles, so heating up the gas will increase the average velocity of the particles. But a position coordinate description takes no account of particle speeds and will therefore not distinguish between a hot and a cold gas. A complete microscopic description of the state of the system will require a tabulation of spatial <u>and</u> velocity coordinates for each particle.

But even this complete tabulation is unsatisfactory, as it tells us both too much and too little about the state of the system to be relevant to macroscopic observations. It tells us too much in that the slightest deviation of any coordinate of any particle will define a new state for the system in terms of microstructure. This leaves us with an infinity of infinities of states to bring into correspondence with the macro-observables which define the "macrostate". This is mathematically difficult to handle, to say the least. The tabulation tells us too little in that we still have no level of discrimination between the microscopically described states of the system, so we cannot say whether two such descriptions are or are not significantly different from each other. Both these problems can be resolved by introducing the concepts of "cells" in "phase space".

So far we have described the system by spatial and velocity coordinates for each particle; i.e. (x,y,z,v_x,v_y,v_z). So the previous tabulation for the N particles in the system can be graphically represented by N points in a 6-dimensional "phase space". If we divide up the phase space into a large number (n) of cells, then we can assign a "microstate" to the system by a complete description of which point in phase space is in which cell. This description allows us to associate an energy per phase point with each cell, as the energy of the particle will be either kinetic and expressible in terms of the (v_x,v_y,v_z) descriptor, or potential and expressible in terms of the (x,y,z) descriptor. So a microstate is an allocation to each particle of a point in a phase cell, which has associated with it a definite energy for that phase point. To a 6-dimensional observer the behaviour of the phase points in the phase space will be not unlike the behaviour of gas molecules in a 3-dimensional container.

Using the phase points in phase space description of our system, how do we now specify an observable macrostate? We do this by simply saying that macrostates are identical if and only if each corresponding cell in phase space contains the same <u>number</u> of phase points. This is a reasonable assertion as, by definition, a macrostate is independent of the behaviour, and hence phase position, of individual particles. Now if the cells are extremely small, so that there is rarely more than one phase point per cell, then little has been achieved by this device. On the other hand, if we choose our phase cells large enough so that there are a very large number of phase cells and each phase cell contains a very large number of phase points, then we can begin to establish a relation between microstates, which are established by the identification of each phase point in each phase cell, and the macrostates, which are established only by the number of phase points in each phase cell.

For instance, let us consider a simple system composed of two phase cells (A and B) for four phase points. If N_A and N_B are the number of points in the cells, the possible macrostates are:

Macrostate	1	2	3	4	5
N_A	4	3	2	1	0
N_B	0	1	2	3	4

Now there may be more than one microstate corresponding to each macrostate. For example, labelling the four phase points a,b,c,d we see there are four microstates corresponding to macrostate 2 ($N_A = 3$, $N_B = 1$). i.e.:

Cells		Points			
	A	abc	abd	acd	bcd
	B	d	c	b	a

We note that once a macrostate is specified we can, in general, easily calculate the number of microstates which correspond to it. A specified macrostate can be thought of as an array of slots for the phase points, any slot accepting any phase point. So if there are N phase points, and therefore N slots, there are N! arrangements of phase points possible. But within a phase cell the order in which the phase points are specified is immaterial to the identification of the microstate (i.e. abd, bad, dba, etc. all correspond to the same cell occupancy as far as the microstate is concerned). So the total number of phase point arrangements must be reduced to account for the rearrangements that are possible within phase cells. If there are n cells this causes a reduction by a factor of $\prod_{i=1}^{n} N_i!$, where N_i is the number of phase points in cell i. So we see that the number of microstates corresponding to a given macrostate is given by:

$$W = \frac{N!}{\prod_{i=1}^{n} N_i!} \qquad (5) \qquad (\text{N.B. } \sum_{i=1}^{n} N_i = N)$$

Here W is known as the "Thermodynamic Probability", though it is not a probability nor even specifically thermodynamic. What W is, though, is an indication of the likelihood of the occurrence of a particular macrostate. To see why this is so we need to make the assumption that all microstates are equally likely to occur i.e. a microstate with all the gas molecules spread evenly throughout a container is as likely to occur as a microstate with all the gas molecules gathered up into a small clump in one corner of the container. These are equally likely outcomes for the same reason that drawing four successive Aces from a shuffled pack is just as likely as drawing the Two of Clubs, Six of Diamonds, Jack of Hearts and the Seven of Spades. In both cases the probability is 48!/52!. Here the "non-randomness" of the first draw against the second is purely in the eye of the beholder.

If we accept that microstates are equally probable to occur, then the probability of a given macrostate occurring is precisely proportional to the number of microstates which correspond to that macrostate. Also, the state to which a system will seem to tend will be the macrostate with the highest probability of occurrence. But our classical thermodynamics tells us that the state to which a system will naturally tend is the state with the highest entropy. We immediately see that there is a close relation between the thermodynamic probability, W, and the entropy, S. In fact, by considering the entropy change and thermodynamic probability change for two gases when they are allowed to mix, it can be shown (see Fast[2]) that:

$$S = k \log W + C \qquad (6)$$

Here k is Boltzmann's constant and C is an arbitrary constant. If, for convenience, we set C = 0, then we obtain the satisfyingly simple relation

$$S = k \log W$$

We recall from (5) that: $W = \frac{N!}{\prod_i N_i!}$

So: $\log W = \log N! - \sum_i \log N_i!$

To examine log W more closely we need to replace the analytically difficult factorials with more tractable functions. We can do this by noting Stirling's approximation (see Courant and John[3]):

$$m! \rightarrow (2\Pi)^{\frac{1}{2}} m^{m+1} e^{-m} \quad \text{as} \quad m \rightarrow \infty$$

So: $\log m! \simeq (m+1) \log m - m + \frac{1}{2} \log 2\Pi$

For m > 70 this can be reduced to a simpler approximation, which still gives an error of < 1%:

$$\log m! \simeq m \log m - m$$

For $m \simeq 10^{23}$, as in the cases of interest, this is a splendid approximation. So we can write:

$$\log W = N \log N - N - (\sum_i N_i \log N_i - \sum_i N_i)$$

$$= N \log N - \sum_i N_i \log N_i \qquad (\text{as } \sum_i N_i = N)$$

Now we can define a set of probabilities $\{p_i\}$ by $\{N_i/N\}$, where p_i

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2. J.D. Fast, Entropy, Philips Technical Library, Eindhoven, 1962.
3. R. Courant and F. John, Introduction to Calculus and Analysis, Wiley, London, 1965.

represents the likelihood that a given phase point will be in the i th phase cell. Using these probabilities we see that:

$$\log W = N \log N - \sum_i (p_i N) \log (p_i N)$$

$$= N \log N - N(\sum_i p_i \log p_i + \sum_i p_i \log N)$$

$$= N \log N - N\sum_i p_i \log p_i - N \log N$$

i.e. $$\log W = -N\sum_i p_i \log p_i \qquad (7)$$

So we see that the entropy of a given macrostate is dependent on a function related to cell occupancy probabilities. Intuition would indicate that the more evenly spread-out among the phase cells are the phase points the greater is likely to be the number of microstates corresponding to that macrostate. On the other hand, if all the phase points are in one phase cell then only one microstate corresponds to that macrostate, so the entropy would be a minimum. That log W does have these intuitively obvious properties is easy to see. If we define: $H = -\sum_i p_i \log p_i$

Then as $0 \leq p_i \leq 1 \; \forall \; i$, $p_i \log p_i \leq 0$ so $H \geq 0$. i.e. min $H = 0$
When $p_j = 1$, $p_i = 0 \; \forall i \neq j$, $H = -1 \log 1 = 0 = \min H$
So, in accord with intuition, log W is a minimum when all the phase points are in one phase cell.

To find the maximum possible value of log W we require that $dH = 0$.

Now: $$dH = \sum_i (\partial H/\partial p_i)\, dp_i$$

$$= -\sum_i p_i(\partial(\log p_i)/\partial p_i)\, dp_i - \sum_i (\log p_i)\, dp_i$$

$$= -\sum_i (p_i/p_i)\, dp_i - \sum_i (\log p_i)\, dp_i$$

$$= -\sum_i dp_i - \sum_i (\log p_i)\, dp_i$$

We can also impose the boundary condition that $\sum_i p_i = 1$. i.e. $\sum_i dp_i = 0$.

So we have: $-\sum_i (\log p_i)\, dp_i = 0$

and: $\sum_i dp_i = 0$

We can solve for p_i using the method of Lagrangean multipliers (see Apostol[4]).

Multiplying the second equation by $\log\alpha$ and adding the equations we get:

$$\sum_i (\log\alpha - \log p_i)\, dp_i = 0$$

For this relation to be satisfied the coefficient of each dp_i must equal zero. i.e.: $\log\alpha - \log p_i = 0$

i.e.: $p_i = \alpha$

So H is maximised when $p_i = \alpha \;\forall\; i$.

Now:
$$\sum_i p_i = n\alpha$$
$$= 1$$

i.e.: $\alpha = 1/n$

So the maximum entropy is obtained when the occupancy of each cell is equally probable, as we had anticipated. The function $\sum_i p_i \log p_i$ is in fact an excellent measure of the "spread-outness" of a set of probabilities, and its more general use will be discussed later.

In most cases of interest another boundary condition can also be applied, to restrict further the distribution of the probabilities. This condition is that the internal energy, U, is a constant. We can associate an internal energy of ε_i per particle with each phase cell, where this energy is dependent upon both the spatial and velocity coordinates, as discussed above. We then have:

$$U = \sum_i \varepsilon_i N_i$$
$$= N \sum_i \varepsilon_i p_i$$

4. T.M. Apostol, <u>Mathematical Analysis</u>, Addison-Wesley, London, 1965.

Now we know that for an isolated system U is constant.

i.e.: $dU = 0$

Now: $dU = N\sum_i p_i \, d\varepsilon_i + N\sum_i \varepsilon_i \, dp_i$

But ε_i is a constant for each phase cell, so $d\varepsilon_i = 0 \; \forall i$.

i.e.: $\sum_i \varepsilon_i \, dp_i = 0$

We can again use: $\sum_i dp_i = 0$

and: $-\sum_i (\log p_i) \, dp_i = 0$

So the use of an isolated system imposes a further condition on the maximum possible value of log W. We can solve this system by using Lagrangean multipliers $-\beta$ and $\log\alpha$ on the first and second equations respectively. Adding the three equations we get:

$$\sum_i (-\beta \, \varepsilon_i + \log\alpha - \log p_i) \, dp_i = 0$$

i.e.: $-\beta \, \varepsilon_i + \log\alpha = \log p_i$

or: $p_i = \alpha e^{-\beta \, \varepsilon_i}$

We see that β must be non-negative as $p_i \leq 1$, but ε_i can be arbitrarily large for large enough (v_x, v_y, v_z). So if $\beta < 0$ we could find an ε_i large enough so that $\alpha e^{-\beta \, \varepsilon_i} > 1$. So we conclude that $\beta \geq 0$.

This indicates that the extra effect of isolating the system is to limit the probability of occupation of a phase cell to be negative exponentially related to the energy per particle associated with that cell. So in this case the entropy of the system is given by:

$$\begin{aligned} S &= k \log W \\ &= -kN\sum_i p_i \log p_i \\ &= -kN\sum_i e^{-\beta \, \varepsilon_i} (\log\alpha - \beta \, \varepsilon_i) \end{aligned}$$

To evaluate this sum a particular model for the ε_i will have to be established, and it is the establishment of these ε_i and the corresponding

$\sum_i e^{-\beta\varepsilon_i}$ that leads to an association between general statistical theory and models for real physical systems. In particular, it can be shown (see Sears[5]) that we can identify the temperature (T) of the system in equilibrium with a function of β i.e. $T = 1/k\beta$. However, the application of statistical theory need not detain us here, as we have found what we were seeking, a microscopic description of the macroscopic variable, entropy. We have seen that the physically observable state of a closed isolated system may be defined by the assignment of values to certain macro-variables (e.g. pressure, temperature, volume), the values actually observed being restricted by the condition that the system has the maximum possible entropy. We have also seen that macrostates, corresponding to these assigned values of the macro-variables, can be defined in terms of many equally likely microstates. The macrostate actually observed will be that corresponding to the greatest number of microstates, subject to the constraints imposed. Thus the state of maximum entropy can be understood to correspond to the most likely of a multitude of possible macrostates.

This probabilistic aspect of entropy may seem disturbing, as it allows that, very occasionally, a macrostate may be observed which does not correspond to a state with maximum entropy. However, it can be shown (see Fast[2]) that the likelihood of such an occurrence is infinitesimally small. For example, the probability of finding a room with 99% of its volume full of air and 1% a vacuum is roughly $10^{-44\times10^{20}}$. This number is almost unimaginably close to zero.

One aspect of the thermodynamic formulation of entropy which seems unsatisfactory is the defining of a state of zero entropy.

We recall that we defined: $dS = \delta Q/T$

so: $S = f(Q,T) + C$

5. F.W. Sears, Thermodynamics, Addison-Wesley, London, 1966.

Here C is an arbitrary constant. That is, only changes in entropy are defined by classical thermodynamics. The zero point of entropy is left as arbitrary.

The statistical formulation may seem to be an improvement in this respect, as it tells us $S = k \log W$, where W is a definite and well defined number. But we recall that this was only obtained by arbitrarily setting a constant to zero in eqn. (6). Indeed, consideration of the nature of W soon shows that log W is dependent not only upon the number of particles (N) in the system, but also upon the number of phase cells (n) into which the systems phase space has been arbitrarily partitioned. We recall eqn. (7):

$$\log W = -N\sum_i p_i \log p_i$$

Here log W is a maximum when $p_i = 1/n \ \forall i$.

i.e.: $$\max \log W = -N\, n((1/n) \log (1/n))$$
$$= N \log n$$

So the maximum feasible value of the entropy is dependent upon the partitioning of the phase space. Similarly, a coarse grained partitioning may allow $p_j = 1$, $p_i = 0 \ \ \forall i \neq j$, giving a minimum entropy, but a finer grained partition may make this $p_j = ½$, $p_k = ½$, $p_i = 0$ $\forall i \neq j,k$, which is not a minimum (see Grunbaum[6]). So the zero entropy is also arbitrary under the statistical analysis, and a finer partitioning will always cause the entropy of the system to increase. We can express this in more detail by considering the phase cell occupation probabilities as an array $\{p_i\}$, with corresponding entropy log W. If we then allow a finer partitioning within each cell, we can obtain an array of probabilities for these new finer cells within each of the coarser cells. i.e. we define $\{p_{ij}\}$, with $\sum_j p_{ij} = p_i$.

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6. A. Grunbaum, "Is the coarse-grained entropy of classical statistical mechanics an anthropomorphism?", in Entropy and Information, (Ed.) L. Kubat and J. Zeman, Elsevier, Oxford, 1975.

So we can define a new entropy for the system by:

$$\log W' = -N\sum_{ij} p_{ij} \log p_{ij}$$

Now we can write:

$$\sum_j p_{ij} \log p_{ij} = p_i \sum_j (p_{ij}/p_i) \log (p_{ij} p_i/p_i)$$

$$= p_i \sum_j (p_{ij}/p_i) (\log(p_{ij}/p_i) + \log p_i)$$

So: $$\log W' = -N\sum_i p_i \sum_j (p_{ij}/p_i) \log (p_{ij}/p_i) - N\sum_i p_i \sum_j (p_{ij}/p_i) \log p_i$$

$$= -N\sum_i p_i \sum_j (p_{ij}/p_i) \log (p_{ij}/p_i) - N\sum_i p_i \log p_i$$

$$= \log W - N\sum_i p_i \sum_j (p_{ij}/p_i) \log (p_{ij}/p_i)$$

The term on the right is the weighted sum of the internal entropies of each cell. So we see that any level of cell specification can be related to any other level via the notion of internal cell entropy. Therefore, by making the cells sufficiently small a system may have an arbitrarily large entropy. But by reducing the cell size we are effectively requiring that the position and velocity of a particle can be specified with that required degree of accuracy. Now, quantum mechanics has shown that the position ($\underline{x}$) and momentum ($\underline{p}$) of a particle can only both be specified within limits imposed by $\Delta\underline{x}.\Delta\underline{p} \simeq h^3$, where h is a very small but finite number (Planck's constant). Hence there is a minimum phase cell size which can be used in partitioning, so a maximum entropy can be defined for a system (for a fuller discussion see Harrison[7]).

We see that the entropy concept can be thought of as something of a triumph for physics. An extremely useful quantity can be defined for arbitrary systems, and statistical and quantum mechanics can be applied to this quantity to make its properties intuitively accessible and "well behaved". The story of entropy is not yet complete though, for

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7. M.J. Harrison, "Entropy concepts in physics", in Kubat and Zeman, op. cit.

insights into the nature of entropy, as well as a generalisation of its concept, are available from Information Theory.

Information theory was founded by Shannon[8] on foundations somewhat prepared by Wiener[9] and von Neumann[10], and deals with "finding out what we don't already know". The theory posits that from certain "messages" we find out more than from other messages, and that there is a way of expressing the difference in the information we gain. Consider the following examples:

Case A A farmer has two fields, in one of which there is a cow that needs milking. The cow is as likely to be in one field as the other. When he receives a message that the cow is in field 2 he has received only a small amount of information, as there was a 50% chance he would have guessed it was there anyway.

Case B A farmer has 64 fields, in one of which is a cow etc. When he receives a message that the cow is in field 27 he has received quite a lot of information, much more than in case A, as there was only a 1.56% chance that he would have guessed correctly this time.

How can we measure and compare the amounts of information the farmer has received from the two messages? One way is to compare the number of questions he would have had to ask someone in the know to find out where his cow was. In the first case it is obvious that one question will suffice e.g.:

Question	Reply	Outcome
Is the cow in field 1?	No	Cow is in field 2

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8. C.E. Shannon and W. Weaver, The Mathematical Theory of Communication, University of Illinois Press, Urbana, 1949.

9. N. Wiener, Cybernetics, Wiley, New York, 1948.

10. J. von Neumann, Mathematical Foundations of Quantum Mechanics, Princeton University Press, Princeton, 1955.

The questions to be asked in the second case are less obvious. One might be tempted to try the sequence:

(1) Is the cow in field 1?

(2) Is the cow in field 2?

(3) Is the cow in field 3?

etc.

Here we would expect the farmer to have to ask, on average, 32 questions. However there is a much more efficient method than this. Instead of eliminating one field at a time it is obviously better to eliminate as many fields as possible at a time. A moment's reflection shows that the most that can be eliminated at one time is half the fields under consideration. So the questions and replies might be:

	Question	Reply	Outcome
(1)	Is the cow in fields 1 - 32?	No	
(2)	Is the cow in fields 33 - 48?	No	
(3)	Is the cow in fields 49 - 56?	Yes	
(4)	Is the cow in fields 49 - 52?	Yes	
(5)	Is the cow in fields 49 - 50?	No	
(6)	Is the cow in field 51?	No	Cow is in field 52

We see that this method allows the cow's whereabouts always to be discovered with six questions, which is also the minimum number that will be necessary.

So here is an easy way of comparing the amount of information in messages. We just compare the minimum expected number of questions we would have had to ask to be sure of eliciting the information in the message. We see that for two equiprobable outcomes we need one question ($1 = \log_2 2$), while for 64 equally probable outcomes we need 6 questions ($6 = \log_2 64$).

Alternatively, we can say that the a priori probability that in the first case the cow would be in field 1 was 1/2, so the information gained is $- \log_2 1/2 (=1)$. In the second case the a priori probability for the cow to be in field 52 was 1/64, and the information gained is $- \log_2 1/64$ (=6). Thus we might generalise to say that if the a priori probability of a certain outcome is p_i and the message tells us that indeed that is the outcome, the information gained from that message is $- \log_2 p_i$. At this stage we should also note that a measure of "information received" is also a measure of "previous uncertainty", as if we weren't "uncertain" then the message we received could not have contained any "information" for us.

Having seen what information theory is about we are now in a position to take a more rigorous approach to the definition of an information, or uncertainty, function. What are the properties of an uncertainty function, H, that we would regard as necessary? (See Shannon[11]).

Property 1 The uncertainty about the outcomes of an event should be a function of the probability assigned to each outcome. i.e. For n outcomes, with probabilities p_i $(1 \leq i \leq n)$, then:

$$H = H(p_1, p_2, p_3, \ldots\ldots, p_n)$$

i.e. Our uncertainty must involve the whole set of possible outcomes.

Property 2 If all outcomes are equally likely, H should be a monotonically increasing function of n.

i.e. For equiprobable outcomes our uncertainty must increase as the number of possible outcomes increases.

Property 3 If two independent events, A and B, have a priori probabilities of occurrence p_A and p_B, and are defined to contribute a single event C, with a priori probability $p_A p_B$, then our uncertainty about the occurrence

11. C.E. Shannon, "A Mathematical Theory of Communication", Bell Systems Technical Journal 27, 1948, p.379 and p.623.

of C must equal the sum of our uncertainties of the occurrence of A and B.

i.e.: $H(p_A p_B) = H(p_A) + H(p_B)$

Property 4 The numerical value of H should not depend on how a problem is set up. i.e. The uncertainty should be independent of the way we specify the set $\{p_i\}$.

For example, if we specify n equally likely outcomes with

$$H = H(1/n, 1/n, \ldots\ldots, 1/n) = A(n)$$

then if we put $n = s^m$ and $n = t^p$, then $A(s^m) = A(t^p)$

If we combine Property 3 with Property 4 we see that if $H = A(n)$, then:

$$A(X^m) = mA(X). \qquad (8)$$

If we differentiate equation (8) with respect to m we get:

$$\frac{dA(X^m)}{dm} = A(X) \qquad (9)$$

Now the Left Hand Side of (9) can be written as:

$$\frac{dA(X^m)}{dX^m} \frac{dX^m}{dm} = \frac{dA(X^m)}{dX^m} \log X \; X^m$$

So we get:

$$\frac{dA(X^m)}{dX^m} \log X \; X^m = A(X) \qquad (10)$$

If we differentiate equation (8) with respect to X, though, we get:

$$\frac{dA(X^m)}{dX} = \frac{mdA(X)}{dX} \qquad (11)$$

Now the L.H.S. of (11) can be written as:

$$\frac{dA(X^m)}{dX^m} \frac{dX^m}{dX} = \frac{dA(X^m)}{dX^m} m \; X^{m-1}$$

So we get:

$$\frac{dA(X^m)}{dX^m} \; X^{m-1} = \frac{dA(X)}{dX} \qquad (12)$$

So differentiation has given us two equations (10) and (12) from which we can eliminate the term $\frac{dA(X^m)}{dX^m}$. i.e.:

$$\frac{A(X)}{X \log X} = \frac{dA(X)}{dX}$$

i.e.: $$\frac{dA(X)}{A(X)} = \frac{dX}{X \log X} \tag{13}$$

So on integrating (13) we get:

$$\log A(X) = \int \frac{dX}{X \log X}$$

$$= \log(\log X) + \text{const}$$

i.e.: $$A(X) = K \log X$$

So for n equally likely outcomes:

$$A = K \log n$$

Now in this case the probability of each outcome is $p_i = 1/n$, so we could write: $A = -K \log p_i$

This expression is of the same form as the one we derived earlier by our intuitive "question asking" approach, except that there we are using $\log_e$ (here written log) rather than $\log_2$. But $\log_e$ and $\log_2$ differ only by the multiplicative constant $\log_2 e$, which can be absorbed in the arbitrary constant K if we wish, when the intuitive and formal approach give the same result.

Now if there are n possible outcomes, but the <u>a priori</u> probability of each is not equal, what is our expected uncertainty for each outcome? That is, if we are told what the outcome is, what is the average amount of information we would gain? This is obviously just the weighted sums of the individual uncertainties. i.e.:

$$H_{avg} = -K \sum_i p_i \log p_i$$

If we re-examine the required properties of our uncertainty function we see that all are satisfied by this function. We also note that this is precisely the function we explored in conjunction with the entropy (S) of an N particle system, where:

$$S = -k N \sum_i p_i \log p_i$$

We conclude that the average uncertainty associated with the position of a phase point in phase space is the same as the average entropy associated with the corresponding particle, apart from an arbitrary multiplicative constant.

Now it may seem puzzling that these two expressions are identical, in view of the fact that the entropy equation was obtained by using Stirling's approximation, which is only valid for a large number of particles, while the information (or uncertainty) formula was obtained by taking an average, and does not seem to presuppose any particular number of possible outcomes. However, we should note that the information properties assume that the outcomes are independent. Thus any pair of probabilities p_i and p_j, associated with outcomes i and j, can be considered as independent of each other. However, $\sum_i p_i = 1$, so this assumption is only a good approximation when there are a large number of possible outcomes. So we see that both formulations assume a large number of elements in the probability set, and the seeming inconsistency vanishes.

Let us now ask what this uncertainty (or information) function means, and how it can be used. H supplies us with an objective measure of our uncertainty about the actual outcome of a series of variously probable available outcomes. Now if we wish to make any statement about the probabilities of these outcomes, we can ensure that we don't make any unwarranted assumptions about these probabilities by insisting that whatever we assert about the set $\{p_i\}$ corresponds with the maximising of our uncertainty about the actual outcome. i.e. We must insist that we maximise $H = -K \sum_i p_i \log p_i$, subject to any boundary conditions we may impose. Laplace's "Principle of Insufficient Reason" asserts that, unless we know to the contrary, we assume all probabilities to be equal (i.e. all outcomes to be equally likely). We are here making a less restrictive assertion, in that we only presume that we are maximally uncertain about the outcome. Let us see what we can deduce about the $\{p_i\}$ using this principle. We wish to maximise:

$$H = -K \sum_i p_i \log p_i$$

This maximisation will be subject to the condition:

$$\sum_i p_i = 1$$

This is, of course, the problem we saw earlier, which we solved with Lagrangean multipliers to give:

$$p_i = 1/n \ \forall i$$

So in this simplest possible case, the assumption of maximum uncertainty is equivalent to Laplace's principle. Let us now suppose that we can impose another constraint, which can be expressed in terms of some average, and therefore easily observable, property of the system of probabilities. i.e.

$$\langle f(x) \rangle = \sum_i p_i \, f(x_i)$$

Solving here with Lagrangean multipliers we get:

$$(\log p_i - \log\alpha + \lambda \, f(x_i) \,) = 0$$

i.e.

$$p_i = \alpha \, e^{-\lambda f(x_i)}$$

If a further property of the system is available we can also impose the condition:

$$\langle g(x) \rangle = \sum_i p_i \, g(x_i)$$

which gives:

$$p_i = \alpha \, e^{-\lambda f(x_i) - \mu g(x_i)}$$

So we see that the more that we assume we know about the gross, overall properties of the system of probabilities, the more sharply defined is the negative exponential distribution function describing these probabilities. This is obviously because our knowledge of the overall properties of the system is, to some extent, reducing our uncertainty about the system, which allows us to assert that some outcomes are more probable than others.

The above Lagrangean calculation is, of course, identical in form with that performed earlier to relate the phase space cell occupation probabilities to the particle energy associated with each cell. Indeed, the similarity between the uncertainty function, H, and the entropy function, S, has led Jaynes[12], and later Tribus[13,14], to propose that the physical entropy of a system is in fact a measure of our uncertainty about the state of that system. This seems to suggest that the entropy of a system is dependent not only upon the system, but also upon the observer's knowledge about the system. (For an interesting discussion of the role of the observer in thermodynamics see Bhandari[15]). This statement seems less unreasonable when we note that this will only refer to the absolute entropy of a system, and not to the physically important (and measurable) entropy changes. Such an approach is precisely in accord with the problem we explored of coarse and fine partitioning, and their relationship via internal cell entropy. The information theory approach is only suggesting that the entropy we observe will correspond to the situation we can most reasonably expect to encounter. i.e. The state of maximum uncertainty within the constraints of our knowledge about the macroscopic properties of the system. We have already noted the reciprocal relationship between information and uncertainty. Combining this with the above relationship

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12. E.T. Jaynes, "Information Theory and Statistical Mechanics", Physical Review 106, 1957, p.620 and 108, 1957, p.171.
13. M. Tribus, "Information Theory as the basis for Thermostatics and Thermodynamics", Transactions of the American Society of Mechanical Engineers Section E, Journal of Applied Mechanics 83, 1961, p.1.
14. M. Tribus, P.T. Shannon and R.B. Evans, "Why Thermodynamics is a logical consequence of Information Theory", American Institute of Chemical Engineers Journal 12, 1966, p.244.
15. R. Bhandari, "Entropy, Information and Maxwell's Demon after Quantum Mechanics", Pramana 6, 1976, p.244.

between the entropy of a system and our uncertainty about it has led Brillouin[16,17] to identify information with "negentropy", though the relationship between information and entropy had been noted by Lewis[18] as early as 1930.

Shannon's original work was prompted by the telecommunications problems of "noise" and "bandwidth" relationships, and this stimulus is reflected in the title of Shannon's original paper[11], "A Mathematical Theory of Communication". One type of communication which we can fruitfully consider is the observation of natural phenomena, normally termed "measurement", where information (negentropy) is generated. This notion of observation as a negentropy generating process can be applied to the longstanding thermodynamic problem of "Maxwell's Demon" (for a full discussion see Ehrenberg[19]).

This particular demon was born in 1871 in Maxwell's Theory of Heat[20], and constituted a problem whose resolution was initiated in 1927 by Szilard[21] and only completed after the formulation of information theory.

Maxwell's Demon is a molecule sorter, his domain being an isolated volume of gas. The gas container is divided into two sections by a partition. In the partition there is only one hole, and this is normally covered by a trapdoor, which the demon operates. If the trapdoor is

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16. L. Brillouin, Science and Information Theory, Academic Press, London, 1956.
17. L. Brillouin, Scientific Uncertainty and Information, Academic Press, New York, 1962.
18. G.N. Lewis, "The Symmetry of Time in Physics", Science 122, 1930, p.573.
19. W. Ehrenberg, "Maxwell's Demon", Scientific American 217, 1967,,p.163.
20. J.C. Maxwell, Theory of Heat, Textbooks of Science, London, 1891.
21. L. Szilard, "Uber die Entropieverminderung in einem thermodynamischen system bei Eingriffen intelligenter wessen", Zeitschrift fur Physik 53, 1929, p.840.

left open the gas soon reaches thermodynamic equilibrium (maximum entropy), the properties of the gas on either side of the partition becoming identical. The trapdoor can now be shut, leaving the total mass of gas still in the state of maximum entropy. At this stage the demon can begin his work. There are two ways he can function.

The first, and simplest, mode is to allow molecules to travel from the left hand section into the right hand section of the container, but not vice versa. Very quickly the distribution of the molecules between the two sections will become uneven, being reflected in a pressure difference across the partition. This pressure difference can be used to do useful work, which implies a decrease in the entropy of the system. As the system is isolated this result is counter to the Second Law.

The second mode the demon can use is to discriminate not only between molecules moving to the left and right, but also between fast and slow molecules. The demon now allows fast molecules to leave the left section of the container, while slow molecules are allowed to leave the right section. Very soon the average velocity of molecules is higher in the right section than in the left, so a temperature difference is established. Again, the entropy of the system has been decreased.

Now the fact that Maxwell's demon type devices have never been observed does not deny the possibility of their existence; if they were discovered we would just have to rewrite the Second Law, as discussed by Popper[22]. But we can be fairly sure that the Second Law will not need rewriting on Maxwell's account, as it can be shown that Maxwell's demon is inconsistent with the combined application of information theory and quantum mechanics. As the former has been shown to be a uniquely satisfactory method of guarding against bias, and the latter is very firmly

22. K.R. Popper, "Irreversibility; or Entropy since 1905", British Journal for Philosophy of Science 8, 1957, p.840.

based upon a great deal of experimental evidence, the discovery of a Maxwell's demon begins to seem unlikely, though some authors (e.g. Ubbelohde[23]) still seek him in living systems. This aspect will be dealt with more fully in a later chapter.

Szilard's approach was to note that to be able to discriminate between molecules the demon must first observe them. To do this he will have to illuminate them with a "torch", which will be producing photons and thereby creating entropy. Using quantum principles Szilard showed that the minimum expected entropy production by the torch must exceed the maximum entropy decrease possible through the demon's operation of the trapdoor. Thus in this isolated system of "gas-torch" the operation of the trapdoor can only increase the entropy of the system.

The operation of the demon (and his torch) can also be expressed in terms of information theory. Discrimination is a form of measurement, which means that the demon causes the negentropy of the system to increase. But a measurement is a physical, irreversible process, which therefore generates entropy. Brillouin[16] has shown that the entropy generated by the measurement must always outweight the negentropy represented by the decreased uncertainty about the state of the system.

As well as this close relationship with statistical mechanics, information theory has also found application in many other fields. One early use of the theory was by Shannon[24] himself, when he applied it to the "long-range" ordering of letters and words in the English language. MacDonald[25] has suggested that information theory might be used in the theory of "filing", while a more recent application is by Evans

23. A.R. Ubbelhode, Time and Thermodynamics, Oxford University Press, London, 1947.

24. C.E. Shannon, "Prediction and Entropy in Printed English", Bell System Technical Journal 30, 1951, p.50

25. D.K.C. MacDonald, "Information Theory and its application to Taxonomy", Journal of Applied Physics 23, 1952, p.529.

and Langholz[26] to systems analysis. Perhaps its most ambitious application is to scientific method, by Rothstein[27]. Some of the many applications of information theory to the Social and Biological Sciences are discussed in the next chapter.

So far this chapter has dealt with the continuity of theory between thermodynamics, statistical mechanics and information theory, which might lead one to think that the acceptance of their identity is complete. However, this is far from being true.

The identification by Brillouin of information with negentropy has been questioned by Wilson[28]. He suggests that information is better identified with entropy, though this objection seems to rest upon Brillouin's rather loose formulation of the "uncertainty" - "information" relationship. Other authors, such as Georgescu-Roegen[29], have gone further, and questioned whether Brillouin's identification has any content at all. In view of some of the more extravagant claims made by Brillouin and others, these objections have some merit. However, if we are willing to restrict the entropy-information identification to the description of "uncertainty" about the position of phase points in phase space, then the derivation above seems to be valid.

More fundamental is the doubt about the fundamental relationship between thermodynamics and statistical mechanics, which is well summarised by Georgescu-Roegen[29]. Thermodynamics asserts that for an isolated system the entropy S is a maximum, so for a system near equilibrium $dS/dt \geq 0$.

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26. F.J. Evans and G. Langholz, "Uncertainty, Measurement and Thermodynamics of Information", International Journal of Systems Science 6, 1975, p.281.
27. J. Rothstein, "Information and Organisation as the Language of the Operational Viewpoint", Philosophy of Science 29, 1962, p.406.
28. J.A. Wilson, "Entropy, not Negentropy", Nature 219, 1968, p.535.
29. N. Georgescu-Roegen, The Entropy Law and the Economic Process, Harvard University Press, Cambridge, Mass., 1971.

Boltzmann[30] attempted to show that statistical mechanics leads to the same result, where he defined $H = \sum_i p_i \log p_i$ with $dH/dt \leq 0$. This result is consequently known as the Boltzmann H-Theorem.

Two immediate objections can be raised to the H-theorem. The first is due to Loschmidt[31], who pointed out that there is nothing to stop us, in our imagination, reversing the motion of all the particles constituting the system, which would cause the system to reverse its behavioural evolution, giving $dH/dt \geq 0$.

The second objection is due to Zermelo[32], who used a theorem by Poincaré to show that an isolated system of particles will eventually come arbitrarily close to any specified state, so a low entropy state may be attained, this being proceeded by $dH/dt \geq 0$.

Both of these results can be thought of as reflecting the statistical character of statistical mechanics. As to whether the statistical mechanical description, with its entropy fluctuations, is to be preferred to the thermodynamic description, with entropy inexorably increasing, must surely be decided by experiment and observation. However, as real systems are never perfectly isolated, and also as theory predicts that large entropy fluctuations are extremely rare (though small ones are common and easily observable as Brownian motion) there seems to be little lost if we accept the provisional identification of the statistical and thermodynamic formulations of the entropy concept.

The property that the entropy of an isolated system increases to a maximum led Eddington[33] to talk of entropy as the "arrow of time". This same phenomenon also led Clausius to predict the "heat-death" of the

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30. L. Boltzmann, "Weitere Studien uber Warmegleichgewicht unter Gasmolekulen (H-Theorem)", Sitzungberichte der K. Wiener Akademie 73, 1876. p.139.

31. J.Loschmidt"Uber den Zustand des Warmegleich gewichtes eines Systems von Korpern mit Rucksicht auf die Schwerkraft", Sitzungberichte der K. Wiener Akademie 73, 1876, p.139.

32. E. Zermelo, "Uber einen Satz der Dynamik und die mechanische Warmetheorie" Annalen der Physik und der Chemie 57, 1896, p.485.

33. A.S. Eddington, The Nature of the Physical World, Cambridge University Press, Cambridge, 1928.

universe, all energy eventually being degraded to heat at a uniform temperature. Both these points have been examined by Popper[34,35], who has noted that production of entropy need not, and in view of its possible fluctuations, should not be used as an indicator of time's passage. He suggests that it is the irreversibility of phenomena which defines a direction for time, and it is possible to imagine irreversible processes which do not generate entropy. The example Popper gives is the outward spread of ripples on a pond from a central disturbance. If the (entropy generating) damping terms are ignored we have a process which can be reversed only by the centrally coordinated action of an infinite loop of wave generators. Thus the suggestion that the direction of time is defined by the increase in entropy of the universe would seem unfounded, for an entropy free, but still irreversible universe can be imagined.

As for the heat-death of the universe, Popper has noted:

> "..the entropy of almost all known regions of our universe either remains constant or decreases, although energy is dissipated (by escaping from the system in question)"[35].

The crux of the argument for the heat-death of the universe is that the universe can be regarded as an isolated system. Whether the universe contains sources or sinks for energy or matter is at present unknown, though Gal-Or[36] has suggested that a sufficiently fast expanding universe could act as a heat (and therefore entropy) sink. What is known, though, is that the universe is very far from thermodynamic equilibrium, and the evidence for an overall entropy increase in the universe is not convincing; in fact, as Popper has indicated, the evidence is to the contrary.

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34. K.R. Popper, "The Arrow of Time", Nature 177, 1956, p.538.
35. K.R. Popper, "Time's Arrow and Entropy", Nature 207, 1965, p.233.
36. B. Gal-Or, "Entropy, Fallacy and the Origin of Irreversibility", Annals of the New York Academy of Science 196, 1972, p.305.

CHAPTER FOUR

The Entropy Concept in the Social and Biological Sciences

In the previous chapter we saw that the thermodynamic concept of Entropy and the statistical concept of Information could be expressed in identical mathematical form. This correspondence allows us to follow common useage and use the term "entropy" to denote either, the implied meaning being clear from the context.

In this chapter we shall examine the application of the entropy concept, in both senses, in the social and biological sciences. These two areas have been selected for study firstly because, outside of the physical sciences and communication engineering, the entropy concept seems to have been most widely used in economic and regional studies, and in biology (though considerable use has also been made of it in psychology; see Attneave[1]). More important, this study is about certain physical features of economies, which it will be argued are best examined by the use of the entropy concept, and which resemble, in a non-trivial way, properties exhibited by many biological systems.

As we noted, the entropy of a set of probabilities seems to be a very good measure of the "spread-outness" of these probabilities. But also, as discussed, the entropy concept allows the formulation of a uniquely efficient means of guarding against statistical bias, and further, can provide us with a physical description of the likely state of a system. These three aspects allow us to, rather coarsely, differentiate between three types of useage of this concept in the social and biological sciences.

First, entropy can be used simply as a measure of diversity, inequality or concentration. In particular, its aggregative properties, relating

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1. F. Attneave, Applications of Information Theory to Psychology, Holt, Rinehart and Winston, London, 1959.

within sets and between sets entropies to the total entropy, allow it to be used most effectively in problems of classification, either by aggregation or differentiation.

Second, entropy maximisation subject to constraints allows the solution of certain problems of system specification, where knowledge of the state of the system is incomplete or incoherent.

Third, the frequent, though perhaps contentious, identification of information entropy with thermodynamic entropy has encouraged the thermodynamic formulation of certain aspects of the theory of production and self-replication.

To reiterate, these types of usage are neither complete nor distinct, but their statement allows us a frame of reference for this chapter.

Let us recall that we define the entropy measure, H, on a set of probabilities $\{p_i\}$, by:

$$H = - \sum_{i=1}^{n} p_i \log p_i$$

We saw that: $H_{max} = \log n$ when $p_i = 1/n \; \forall i$

and: $H_{min} = 0$ when $p_i = 1, \; p_j = 0 \; \forall j \neq i$

Obviously the more "even" are the p_i, the greater is H. But we should note that the maximum potential "evenness" increases with the number of cases, n, under consideration. As will be discussed in a later chapter, this is in itself no bad thing. But if we wish to compare the evenness of two different probability sets, we obviously cannot compare the resulting H values if the numbers of components of the sets are different. To get around this problem, we can instead define the "Relative Entropy", R, by:

$$R = H/H_{max} = H/\log n$$

We see that $R \varepsilon [0,1]$. Using R the relative evenness of probability sets with differing numbers of elements can be meaningfully assessed.

Should H be constrained in any way, such that $H_{min} > 0$, then we can devise a related measure, V, where:

$$V = \frac{H - H_{min}}{H_{max} - H_{min}}$$

This V usually has the somewhat misleading name of "Evenness". We note that V reduces to R when $H_{min} = 0$.

Another technique that has found favour is to express the entropy of a set of probabilities in terms of the number of elements that would be required to give that entropy were all of the p_i the same for those elements; i.e., what is f where $H = \sum_{i=1}^{n} p_i \log p_i = \log f$? We see that $f = \exp(\sum_{i=1}^{n} p_i \log p_i) = e^H$. This f is known as the "Numbers Equivalent" of H.

We should also recall the aggregative property of H noted in the previous chapter. In the context of thermodynamic entropy we saw that the phase space could be subject to finer partitioning, allowing us to define a new set of probabilities $\{p_{ij}\}$ where $\sum_j p_{ij} = p_i$. This allowed us to define a new entropy, H', by:

$$H' = -\sum_i p_i \log p_i - \sum_i p_i \sum_j (p_{ij}/p_i) \log(p_{ij}/p_i)$$

$$= H + \sum_i p_i H_i$$

Here H is the entropy due to the relations between sets and $\sum_i p_i H_i$ is the probability weighted sum of the within sets entropies. That is, a dissaggregation of probabilities within subsets still allows the entropy of the entire set of probabilities to be represented by a simple additive expression.

This decomposition of entropy to between sets and within sets entropies allows us also to introduce the idea of conditional entropy, an idea which is naturally enough closely related to that of conditional probability. For example, let us construct a table of probabilities describing the weather at some moment in, say, August:
We describe the day in terms of its being "Sunny" or "Not Sunny", and "Rainy" or "Not Rainy".

.1	.3	.4	Sunny
.1	.5	.6	Not Sunny
.2	.8	1.0	
Rainy	Not Rainy		

We see that the probability of Sunny <u>and</u> Rainy is 0.1, Not Sunny <u>and</u> Not Rainy is 0.5, etc., while the overall probability of Sunny is 0.4, of Not Rainy is 0.8, etc. These latter marginal probabilities are of course of the form $\sum_j p_{ij}$, or $p_{i.}$ for short.

Now if $p_{ij} = p_{i.}p_{.j}$ then the state of Sun and No Rain is determined <u>independently</u> by these two factors. Thus $p_{ij} = p_{i.}p_{.j}$ tells us that the factors determining components i and j are independent, while conversely $p_{ij} \neq p_{i.}p_{.j}$ tells us of their dependence. This dependence of factors for the entire set of probabilities can be assessed by the use of conditional entropies. Here we need some definitions:

Total Entropy $$H(X,Y) = -\sum_i \sum_j p_{ij} \log p_{ij}$$

Marginal Entropies $$H(X) = -\sum_i p_{i.} \log p_{i.}$$

and $$H(Y) = -\sum_j p_{.j} \log p_{.j}$$

Conditional Entropies $$H_Y(X) = -\sum_i \sum_j p_{ij} \log(p_{ij}/p_{.j})$$

$$\text{and} \quad H_X(Y) = -\sum_i \sum_j p_{ij} \log(p_{ij}/p_{i.})$$

$$\text{Mutual Entropy} \quad H_{XY} = -\sum_i \sum_j p_{ij} \log(p_{ij}/p_{i.}p_{.j})$$

Using the technique of expansion described in the previous chapter it is relatively simple to show that:

$$\begin{aligned} H(X,Y) &= H(X) + H_X(Y) \\ &= H(Y) + H_Y(X) \\ &= H(X) + H(Y) + H_{XY} \end{aligned}$$

with the obvious corollaries:

$$H(X) = H_Y(X) - H_{XY}$$

$$H(Y) = H_X(Y) - H_{XY}$$

We see that these decompositions are an application and extension of the notion of within and between sets entropies, and the attractively simple algebraic relations among these entropies allows considerable scope for their use in both theoretical and empirical analyses.

The last entropy measure we shall introduce is that of "Entropy Gain", or as it is more frequently known, "Information Gain", T.

Quite simply, if our a priori supposition about a set of probabilities is that it is $\{q_i\}$, but examination indicates it to be $\{p_i\}$, how much information (entropy) have we gained by this examination? We might be tempted to write this as $-\sum_i p_i \log p_i + \sum_i q_i \log q_i$ i.e. what we have a posteriori minus what we had a priori. But it might be suggested that rather than use the q_i as weights, we should, a posteriori, use all the information at hand and use the p_i as weights in both summations. So the Entropy Gain is defined by:

$$\begin{aligned} T &= -\sum_i p_i \log p_i + \sum_i p_i \log q_i \\ &= -\sum_i p_i \log(p_i/q_i) \end{aligned}$$

If $p_i = q_i \; \forall \; i$ then we gain no information and $T = 0$.

We can now consider some of the applications of these entropy measures.

In economics Horowitz and Horowitz[2,3] have used overall entropy, H, to measure the change in the concentration of manufacturing in the brewing industry in the USA, while Horowitz[4,5] has used the Numbers Equivalent, f, to examine concentration in manufacturing in the USA, and also internationally. In these applications the $\{p_i\}$ were taken to be the market shares of the firms and, incidentally, in each study concentration was found to be increasing. Industrial concentration has also been examined with the simple entropy measure by Finkelstein and Friedberg[6], in this case with particular reference to the functioning of US anti-monopoly legislation.

Using within and between entropies, Horowitz[7] has also examined unemployment in various industries in the Common Market,while Hexter and Snow[8] have used the same technique to examine the inequality of output among industries in the USA. Theil[9] has used within and between entropies to investigate inequalities of income and employment in several countries, and also to examine the concentration of car production in the

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2. I. Horowitz and A. Horowitz, "Structural Changes in the Brewing Industry", Applied Economics 2, 1970, p. 1.
3. A. Horowitz and I. Horowitz, "Entropy, Markov Processes and Competition", Journal of Industrial Economics 16, 1967, p. 196.
4. I. Horowitz, "Numbers Equivalents in US Manufacturing Industries", Southern Economic Journal 37, 1971, p. 396.
5. I. Horowitz, "On Numbers Equivalents and the Concentration Ratio", Quarterly Review of Economics and Business 2, 1971, p. 55.
6. M.O. Finkelstein and R.M. Friedberg, "The Application of an Entropy Theory of Concentration to the Clayton Act", Yale Law Journal 76, 1966, p. 677.
7. I. Horowitz, "Employment Concentration in the Common Market: An Entropy Approach", Journal of the Royal Statistical Society (A), part 3, 1970, p. 463.
8. J.L. Hexter and J.W. Snow, "An Entropy Measure of Relative Aggregate Concentration", Southern Economic Journal 36, 1970, p. 239.
9. H. Theil, Economics and Information Theory, North Holland, Amsterdam, 1967.

USA.

The above studies have examined concentration mainly by economic activity, but perhaps an even more widespread application of the simple entropy measure has been to concentration in geographic terms. For example, the simple entropy measure has been used in the study of geographic concentration of manufacturing in the Tennessee Valley by Garrison and Paulson[10]. Within and between entropies have been used to assess spatial income inequalities in Brazil by Semple and Gauthier[11], the spatial concentration of corporate headquarters in the USA by Semple[12], and the structure of Soviet-Comecon trade by Semple and Demko[13]. The more straightforward spatial organisation of people has been examined by Chapman[14,15] also using within and between entropies.

Perhaps the most interesting use of the simple entropy concentration measure in spatial analysis is by Medvedkov[16,17], who attempted to test the hypothesis, due to Christaller and Losch, that settlements will establish themselves in a hierarchy of towns and villages, arranged in a simple geometric pattern, not unlike wallpaper. Medvedkov partitions the actual dispersion of settlements into a regular sub-pattern and an

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10. C.B. Garrison and A.S. Paulson, "An Entropy Measure of Geographic Concentration", Economic Geography 49, 1973, p. 319.

11. R.K. Semple and H.L. Gauthier, "Spatial-Temporal Trends in Income Inequalities in Brazil", Geographical Analysis 4, 1972, p. 169.

12. R.K. Semple, "Recent Trends in the Spatial Concentration of Corporate Headquarters", Economic Geography 49, 1973, p. 309.

13. R.K. Semple and G.J. Demko, "An Information Theoretic Analysis: An Application to Soviet-COMECON Trade Flows", Geographical Analysis 9, 1971, p. 51.

14. G.P. Chapman, "The Application of Information Theory to the Analysis of Population Distributions in Space", Economic Geography 46, 1970, p. 317.

15. G.P. Chapman, "The Spatial Organisation of the Population of the United States and England and Wales", Economic Geography 49, 1973, p. 325.

16. Yu.V. Medvedkov, "The Concept of Entropy in Settlement Pattern Analysis", Papers of the Regional Science Association 18, 1966, p. 165.

17. Yu.V. Medvedkov, "Entropy: An Assessment of Potentialities in Geography", Economic Geography 46, 1970, p. 306.

irregular one. To obtain his set of probabilities he divides the area of study into a net of regular quadrats and derives a (discrete) distribution of population concentrations for these quadrats. For a regular spacing of settlements, providing the repetition distance of the pattern is less than the quadrat dimensions, we will obviously get a sharply peaked distribution; i.e., $p_i = 1$, all others $= 0$. So the even distribution of settlements corresponds to zero entropy, thus defined.

The random pattern with which he contrasts this regular one is that of a Poisson distribution for the probabilities, so his method is to see whether this entropy for a region is significantly different from zero. Semple and Golledge[18] have applied Medvedkov's method to the distribution of settlements in the Canadian prairies between 1911 and 1961, and found that only 15% of the settlements were evenly spread in 1911, but this had risen to 70% by 1961.

In the biological sciences the entropy measure has been extensively used in ecological studies, the most well-known of its proponents being Margalef[19,20]. Its application in ecology has probably been stimulated by the venerable, but apparently unproven assertion that "simple" ecosystems will, if left to themselves eventually become "complex" ecosystems. Also associated with this maturation is the expectation that complex ecosystems are more "stable" than simple ones. Obviously, if the probability set is associated with, say, the relative biomass contributions of the various species, or their importance in the energy metabolism of the ecosystem, an entropy can be defined on that set will serve as a measure of the complexity of the ecosystem. Unfortunately

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18. R.K. Semple and R.G. Golledge, "An Analysis of Entropy Changes in Settlement Pattern over time", Economic Geography 46, 1970, p. 157.

19. R. Margalef, "Information Theory in Ecology", General Systems Yearbook 3, 1958, p. 36.

20. R. Margalef, "On certain Unifying Principles in Ecology", American Naturalist 97, 1963, p. 357.

MacArthur[21] and later Rutledge, Basore and Mulholland[22] identified such entropies not only with the "complexity" of the ecosystem but also with its stability. Stability and complexity may, under certain circumstances, go hand in hand, but work by May[23] has indicated that complexity is neither a necessary nor sufficient condition for ecosystem stability. Further, some recent work by Moore[24] indicates that the other basic tenet of ecological theory, that of increasing complexification, is not always and everywhere true.

Even the straightforward use of the entropy measure in ecosystem analysis has been subject to criticism by Pielou[25] and Hurlbert[26], criticism which applies equally to its applications in the social sciences.

Pielou notes that the formulation of H as $-\Sigma_i\ p_i \log p_i$ is in fact an approximation in the large of the small N_i measure $H' = \frac{1}{N}(\log N! - \Sigma_i \log N_i)$, and suggests that as tables of $x!$ are easily available the use of an approximation is unnecessary. Of course, this only applies to the cases where the N_i are known i.e. when population information, rather than sample information, is available. If only sample data are accessible, then the use of $H'' = -\Sigma_i\ (n_i/n) \log (n_i/n)$ is necessary (where the n_i are sample sizes), although Pielou points out that this is only a biased maximum likelihood estimator of H; i.e., H" is a biased estimator of an approximation to Pielou's "true" entropy, H'.

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21. R. MacArthur, "Fluctuations of Animal Populations and a Measure of Community Stability", Ecology 36, 1955, p. 533.

22. R.W. Rutledge, B.L. Basore and R.J. Mulholland, "Ecological Stability: An Information Theory Viewpoint", Journal of Theoretical Biology 57, 1976, p. 355.

23. R.M. May, Stability and Complexity in Model Ecosystems, Princeton University Press, Princeton, 1973.

24. P.D. Moore, "Dominance and Diversity", Nature 259, 1976, p. 13.

25. E.C. Pielou, "Shannon's Formula as a Measure of Specific Diversity: Its Use and Misuse", American Naturalist 100, 1966, p. 463.

26. S.H. Hurlbert, "The Non-Concept of Species Diversity: A Critique and Alternative Parameters", Ecology 52, 1971, p. 577.

Pielou is quite correct to point out that H is a large number limit of H', but as H and H' have identical aggregation properties and give identical orderings of probability sets, this does not seem a strong criticism. That H" is a biased estimate of H is more important and perhaps deserves wider recognition.

Hurlbert's criticism is aimed at the use of the entropy measure, as opposed to any other measure of concentration. Indeed, concentration measures are legion and many have been described and compared by Cowell[27]. Such measures rarely give the same concentration ordering of probability sets. If one has no means of distinguishing between concentration measures then obviously any will serve. Pielou[28] notes that in fact H and $\log \sum_i p_i^2$ (log of Herfindahl's index) are both special cases of $H_\alpha = (\log \sum_i p_i^\alpha)/(1 - \alpha)$ with $\alpha = 1$ and $\alpha = 2$ respectively. (N.B. The proof that $H_1 = H$ requires the use of L'Hôpital's Rule). Notwithstanding that H is a special case of a more general measure, it still has superior aggregative properties to all other H_α. Indeed as was shown in the previous chapter, H is unique in its simple additive properties under aggregation.

Although we have already mentioned some studies which use these aggregative properties to derive within and between sets entropies, the entropies were used only to allow description of concentrations. A more subtle approach is to use such compound entropies to analyse the internal organisation of the probability set, with the further possibility of using such internal structure for the formulation of taxonomic schema.

Pielou[28] uses the notion of conditional entropy to produce some ecologically useful measures. He considers the classification of ecological "objects" (flora and/or fauna) by species and by habitat,

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27. F.A. Cowell, Measuring Inequality, Philip Allan, Oxford, 1977.
28. E.C. Pielou, Mathematical Ecology, Wiley, New York, 1977.

giving a table of probabilities of occurrence for each species in each habitat, with the corresponding marginal probabilities. If X represents the species classification, and Y the habitat classification, then we recall that:

$$H(X,Y) = H(X) + H_X(Y) = H(Y) + H_Y(X)$$

If X and Y are independent classifications then $H(X) = H_Y(X)$ and $H(Y) = H_X(Y)$ i.e. marginal entropies equal conditional entropies, as species and habitat are entirely uncorrelated. If X and Y are totally dependent then $H_Y(X) = H_X(Y) = 0$ and $H(X,Y) = H(X) = H(Y)$. Each species corresponds to one and only one habitat, so $p_{ij} = 0 \; \forall \; i \neq j$. So we see that $0 \leq H_X(Y) \leq H(Y)$ and Pielou therefore defines $W = H_X(Y)/H(Y)$ $(\epsilon [0,1])$ which he suggests as a measure of average niche width. Similarly he defines $L = H_Y(X)/H(X)$ as a measure of niche overlap.

Interestingly, Gabor[29] derives precisely the same measure as Pielou's L, though in the context of social freedom of choice. Here the p_{ij} are the proportion of the population which, starting in the factor-group j (e.g. IQ, father's social class, etc.) make choice i (occupation). Thus in this context the measure can give some indication of freedom of socioeconomic choice, given the existence of prior constraints. Gabor uses this measure to suggest that once IQ is taken into account, father's occupation in Sweden has little influence on social mobility.

Chapman[30] has proposed $H_{XY}/H(X)H(Y)$ as a measure of "Relative Organisation" of the probability set, indicating as it does the importance of overall "ignorance" about the $\{p_{ij}\}$, even when the marginal probabilities are specified, as compared with the overall "ignorance" about the marginal probabilities.

29. D. Gabor, The Mature Society, Professional Library, London, 1972.
30. G.P. Chapman, Human and Environmental Systems, Academic Press, London, 1977.

Theil[9] uses similar reasoning to Chapman to suggest the adoption of mutual entropy, H_{XY}, as a measure of the "information content" of an Input-Output (I-O) table. An I-O table is a device for displaying the inter-industry transactions in an economy, and their use will be discussed extensively in a later chapter. If x_{ij} is the value of sales by industry type i to industry type j, then Theil defines the probability set by $p_{ij} = x_{ij}/ \sum_i \sum_j x_{ij}$. Thus H_{XY} is an indication of the independence of the actual transactions from the marginal totals, $x_{i.}$, $x_{.j}$; i.e.,independence from the overall outputs from, and inputs to, the various industrial sectors. This measure, therefore tells us how "unguessable" the table is, even when marginal totals are known.

The most interesting application of this method that Theil discusses is its use in a technique for updating I-O tables.

He supposes we have a previous year's I-O table, reduced to probabilities; i.e.,we have $\{p_{ij}\}$ and hence $p_{i.}$ and $p_{.j}$. He also supposes we have current information relating to marginal totals i.e. $q_{i.}$ and $q_{.j}$, and are seeking an estimate, $\hat{q}_{ij}$, for the unknown probabilities $\{q_{ij}\}$. He defines $q'_{ij} = p_{ij}\ (q_{i.}q_{.j}/p_{i.}p_{.j})$ as the initial approximation to q_{ij}. However $\sum_i \sum_j q'_{ij} \neq 1$ in general, so he defines a better approximation as $q''_{ij} = q'_{ij}/ \sum_i \sum_j q'_{ij}$. (i.e. $\{q'_{ij}\}$ is normalised).

He then suggests that the updating problem can be expressed as:

$$\min \sum_i \sum_j q''_{ij} \log (q''_{ij}/\hat{q}_{ij})$$

subject to $\hat{q}_{i.} = q_{i.}$

$$\hat{q}_{.j} = q_{.j}$$

The set $\{\hat{q}_{ij}\}$ which effects this minimisation is then the "minimum information" estimate. Theil goes on to show that this approach results in updated tables very similar to those produced by the more popular RAS updating technique.

Another application by Theil of the "information content" measure is to the problem of "information loss" when tables are aggregated. He shows that information loss must always be expected when two or more industrial sectors are aggregated into one sector. What he does not do, however, is use this information loss under aggregation to suggest a technique for "efficient" aggregation. That is, if an n x n I-O table is to be aggregated into an (n-m)x(n-m) table, one might suggest that the most "efficient" aggregation would be that involving the minimum information loss, as defined by Theil. The present author is unaware of the use of such an aggregation technique in I-O analysis. This lacuna is made more remarkable by the growing popularity of just such a minimum information technique in spatial and ecosystem analysis.

In both geography and ecology the rationale for the use of this technique is the provision of an objective taxonomic criterion, allowing the hierarchical classification of multi-attribute elements. The method involves the distinguishing of within sets and between sets entropies, of which the conditional and mutual entropies are special cases. If one uses an aggregative method, between sets entropy is converted to within sets entropy as aggregation proceeds, so between sets entropy is lost. Minimising this loss will ensure that the aggregation that takes place will be between elements that are most "similar". If a technique of disaggregation is used, between sets entropy is gained, so the correct taxonomic procedure here would be to disaggregate so as to maximise this gain.

Orloci[31,32] has used the aggregative approach to identify the spatial clustering of species, as well as examining habitat-species classifications for various ecosystems. Pielou[28] notes that Lance and Williams[33], and

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31. L. Orloci, "Information Analysis in Phytosociology: Partition, Classification and Prediction", Journal of Theoretical Biology 20, 1968, p.271.
32. L. Orloci, "Information Analysis of Structure in Biological Collections", Nature 223, 1969, p. 483.
33. G.N. Lance and W.T. Williams, "Note on a New Information-Statistic Classificatory Program", Computer Journal 11, 1968, p. 195.

Wallace and Boulton[34], have produced divisive or disaggregative taxonomic algorithms, though these do not seem to have gained widespread use.

In geographical studies Gurevich[35] and Nutenko[36] have discussed the minimising of entropy loss as a method for classifying spatial properties, and using a similar technique Marchand[37] has examined the information content of regional maps. Gatrell[38] has tried to show that the reason the "old" intuitive ways of devising regional classifications and the "new" ways, involving factor analysis etc., seem to produce almost identical schema is that most of the entropy (or information) of a map is conditional, thus the information to be "minimised", albeit implicitly, by any classificatory technique is extremely limited. As a result different taxonomic methods produce very similar patterns of spatial differentiation.

Batty[39,40] has approached the problem of spatial aggregation in a more fundamental way than other workers in this field, in that he is unhappy about accepting spatial quadrats as "given", suggesting that one should consider the aggregation problem as more akin to the crystalisation of hierarchical subregions from an homogeneous spatial solution, rather than as an aglutination of heterogeneous regions. That is, he tries to establish whether the entropy of a system can be represented by the integral of a

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34. C.S. Wallace and D.M. Boulton, "An Information Measure for Classification", Computer Journal 11, 1968, p. 185.
35. B.L. Gurevich, "Geographical Differentiation and its Measures in a Discrete System", Soviet Geography 10, 1969, p. 387.
36. L.Ya. Nutenko, "An Information Theory Approach to the Partitioning of an Area", Soviet Geography 11, 1970, p. 540.
37. B. Marchand, "On the Information Content of Regional Maps: The Concept of Geographical Redundancy", Economic Geography 51, 1975, p. 117.
38. A.C. Gatrell, "Complexity and Redundancy in Binary Maps", Geographical Analysis 9, 1977, p. 29.
39. M. Batty, "Spatial Entropy", Geographical Analysis 6, 1974, p. 1.
40. M. Batty, "Entropy in Spatial Aggregation", Geographical Analysis 8, 1976, p. 1.

continuous function, rather than as a summation over a discrete function. The obvious extension of the entropy formula over a probability density function would be:

$$H^* = -\int p(x) \log p(x)\, dx$$

with $\int p(x)\, dx = 1$

However, Batty shows that whereas one might anticipate:

$$H^* = \lim_{\Delta x_i \to 0} - \sum_i p(x)_i \Delta x_i \log(p(x)_i \Delta x_i)$$

in fact:

$$H^* = \lim_{\Delta x_i \to 0} - \sum_i p(x)_i \log(p(x)_i \Delta x_i)$$

$$= - \sum_i p_i \log p_i + \lim_{\Delta x_i \to 0} \sum_i p_i \log \Delta x_i$$

In the limit the last term on the right is undefined.

We see that, not surprisingly, the continuous representation of the entropy function requires explicit recognition of the limitation of quadrat size, Δx_i. Batty exploits this formulation to obtain spatial regionalisations of New York and Reading which are not subject to the constraint of preceding quadrat specification.

In the above taxonomic procedures we have used the notion of "information gain" with reference to the within sets entropy. Let us recall, though, that earlier in this chapter we defined an information gain as $-\sum_i p_i \log (p_i/q_i)$, a measure of the difference between the a priori and a posteriori probability sets. This type of information gain has been used by Berry and Schwind[41] to analyse the accuracy of a model of population migration. The greater the information gain due to the measurement of the actual flows, the lower is the accuracy of the predicting model.

The model used by Berry and Schwind is the "Gravity Model", where

41. B.J.L. Berry and P.J. Schwind, "Information and Entropy in Migrant Flows", Geographical Analysis 1, 1969, p. 5.

migrant flows between towns i and j are posited to be of the form:

$$m_{ij} = K\, s_i s_j / d_{ij}$$

Here s_i and s_j are the town sizes and d_{ij} is a "distance" function. K is a scale constant. This formulation is obviously based upon the Newtonian equation for gravitational interaction at a distance:

$$f = G\, m_1 m_2 / r^2$$

Berry and Schwind found this model quite unsatisfactory, not least because it predicts precisely symmetrical migrant flows between towns.

Gravity models of population movement have given rise to the application of the entropy concept in the second sense we initially discussed. That is, as a means of guarding against bias when only partial information as to the state of a system is available. First we need to establish the notation normally used in these gravity models.

T_{ij} - number of journeys made from location i to location j

$O_i = \sum_j T_{ij}$ - total number of departures from i

$D_j = \sum_i T_{ij}$ - total number of arrivals at j

The particular gravity model relevant here is:

$$T_{ij} = K\, O_i\, D_j / d_{ij} \qquad (K \equiv \text{constant})$$

Here O_i and D_j, the arrival and departure figures, are replacing s_i and s_j, the town sizes. The equation for the T_{ij} is subject to the conditions:

$$O_i = \sum_j T_{ij}$$

and

$$D_j = \sum_i T_{ij}$$

But substitution and rearrangement gives:

$$K = \left(\sum_j D_j / d_{ij} \right)^{-1}$$

and

$$K = \left(\sum_i O_i / d_{ij} \right)^{-1}$$

In general, both equations for K cannot hold simultaneously, so the model is inconsistently formulated. This inconsistency is normally overcome by splitting K into two sets of constants, $\{K_i\}$ and $\{K_j\}$, when the formulation becomes:

$$T_{ij} = K_i K_j O_i D_j / d_{ij}$$

where $$K_i = \left(\sum_j K_j D_j / d_{ij} \right)^{-1}$$

and $$K_j = \left(\sum_i K_i O_i / d_{ij} \right)^{-1}$$

If the O_i and D_j are known and a distance function d_{ij} is specified we then have three sets of linearly independent equations in three sets of unknowns (T_{ij}, K_i, K_j) which can be solved, usually iteratively.

Wilson[42] regarded the gravity model thus specified as rather ad hoc, both in its adoption of a simple Newtonian form, and in the introduction of the K_i and K_j as mere balancing variables. Instead of estimating the T_{ij} from the O_i and D_j by this simplistic method, Wilson sought to establish the T_{ij} which best satisfied the general "Principle of Insufficient Reason" i.e. had maximal entropy subject to the constraints of prior knowledge of the marginal totals. However, it quickly transpires that entropy maximisation subject to these constraints is insufficiently restricted to give an interesting distribution to the T_{ij}. Wilson therefore introduced an additional constraint, this being a limitation on the total transport cost. He supposed that the cost of trip T_{ij} was c_{ij}, and that the entire society had a limited travel budget, C. i.e. $C = \sum_i \sum_j T_{ij} c_{ij}$. As Wilson has shown, if we identify $p_{ij} \equiv T_{ij} / \sum_i \sum_j T_{ij}$, then the transport problem becomes:

$$\max \; - \sum_i \sum_j p_{ij} \log p_{ij}$$

subject to $$O_i = \sum_j T_{ij}$$

$$D_j = \sum_i T_{ij}$$

42. A.G. Wilson, Entropy in Urban and Regional Modelling, Pion, London, 1970.

$$C = \sum_i \sum_j T_{ij} c_{ij}$$

Forming the Lagrangean and differentiating, as in the previous chapter, gives:

$$T_{ij} = e^{-\lambda_i^1 - \lambda_j^2 - \beta c_{ij}}$$

$$e^{-\lambda_i^1} = O_i \left(\sum_j e^{-\lambda_j^2 - \beta c_{ij}} \right)^{-1}$$

$$e^{-\lambda_j^2} = D_j \left(\sum_i e^{-\lambda_i^1 - \beta c_{ij}} \right)^{-1}$$

Here $\{\lambda_i^1\}$ and $\{\lambda_j^2\}$ are the Lagrangean Multipliers.

If we set: $K_i = e^{-\lambda_i^1}/O_i$

and $K_j = e^{-\lambda_j^2}/D_j$

we get: $T_{ij} = K_i K_j O_i D_j / e^{\beta c_{ij}}$

$$K_i = \left(\sum_j K_j D_j / e^{\beta c_{ij}} \right)^{-1}$$

$$K_j = \left(\sum_i K_i O_i / e^{\beta c_{ij}} \right)^{-1}$$

This is identical with the naive gravity model, except that the general distance function f_{ij} has become a particular function of the transport cost, $\exp(\beta c_{ij})$. This is obviously a most fortunate finding for all those regional scientists who have been using gravity models, especially those using a distance function incorporating transport cost.

A drawback of this result, though, is that transport frequencies between locations are always presumed to be monotonic decreasing functions of the transport cost, as are all of the derivatives, which may not be a reasonable assumption in many real situations. Webber[43] has tried to

43. M.J. Webber, "Entropy Maximising Models for the Distribution of 'Expenditures'", Papers of the Regional Science Association 37, 1976, p. 185.

overcome this defect in the model by maximising the entropy of the trip expenditure rather than the entropy of the trip frequency. That is:

$$q_{ij} = T_{ij}c_{ij} / \sum_i \sum_j T_{ij}c_{ij}$$

where the model is of the form:

$$\max \; - \sum_i \sum_j q_{ij} \log q_{ij}$$

subject to

$$\sum_i \sum_j q_{ij} = 1$$

$$\sum_i \sum_i p_{ij} = 1$$

$$\sum_j T_{ij} = O_i$$

$$\sum_i T_{ij} = D_j$$

When solved in the normal way this gives T_{ij} as an asymmetrical, unimodal function of transport cost; i.e.,trip frequency can be decreased <u>and</u> increased by suitably increasing transport costs. Whether transport frequencies ever do behave in this way will not be examined here, but at least Webber's technique is available if it should ever be thought that they do so behave.

A recent and rather interesting development in the field of transport models is due to Nijkamp and Paelinck[44], who have shown that Wilson's entropy maximising model is a special case of a dual geometric programming problem. Geometric programming is a development of the simple linear programming technique, and like linear programming has both a primal and dual formulation, but at the same time allows efficient computation of solutions to many non-linear programming problems. The dual geometric programming problem can be written as:

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44. P. Nijkamp and J.H.P. Paelinck, "A Dual Interpretation and Generalisation of Entropy Maximisation Models in Regional Science", <u>Papers of the Regional Science Association</u> <u>33</u>, 1974, p. 13.

$$\max \sum_i \sum_j p_{ij} (\log c_{ij} - \log p_{ij}) + \sum_i (\sum_j p_{ij}) \log(\sum_j p_{ij})$$

subject to
$$\sum_j p_{0j} = 1$$
$$\sum_{k=0} \sum_i a^k_{ij} p_{ki} = 0 \;\; \forall j$$
$$p_{ki} \geq 0 \; ; \; k = 0,..,K \; ; \; i = 1,..,I$$

Clearly the entropy maximising technique is a special case of the dual geometric programming problem, with $c_{ij} = 1 \; \forall \; i, j$, etc. Therefore more general programming models of transportation can be established which are not of the simple entropy maximising type, and which therefore need not give rise to the even more intuitively simple gravity model interpretation.

We have seen, therefore, that the application of the entropy concept in transport models has moved from that of analogy with physical science, as made clear by Wilson[45], through the justification of certain types of gravity model by the minimum information property, to being absorbed into the general programming approach to assignment problems.

Other applications in regional science, though, remain at the level of analogy, or at least mathematical homomorphosism. For example, Arumi[46] sets out to maximise the spatial entropy of a population distribution, using a "quantum of personal area" to define the probability sets, the maximisation being subject to the "conservation of individuals" and the "conservation of area". As we might expect, he finds the probability of occupation of a cell is negative exponentially related to the density of population in that cell. Arumi fits his results to empirical data, obtaining a quantum area for the USA of 650 foot^2. The relevance of this figure to any other social variables is not clear, or expanded upon.

Even stronger use of analogy is demonstrated by Demetrius[47], this time

45. A.G. Wilson, "Notes on Some Concepts in Social Physics", Papers of the Regional Science Association 22, 1968, p. 159.

46. F.N. Arumi, "Entropy and Demography", Nature 243, 1973, p. 497.

47. L. Demetrius, "Measures of Variability in Age-Structured Populations", Journal of Theoretical Biology 63, 1976, p. 397.

in ecology, when he examines measures of variability in age-structured populations and derives a population growth rate which he compares with the Gibbs Free Energy and a generation time analogous to the reciprocal of temperature.

The applications of the entropy concept we have so far examined have all been quite obviously in terms of the information theory measure. Any association of the entropy concept with thermodynamics, as in the case of Demetrius, has been explicitly at the level of analogy. This distinction between the two terms of reference of the entropy concept seems to have unfortunately evaded some writers. For example, Leopold and Langbein[48,49] have applied the entropy concept to the evolution of watershed patterns in the landscape, and although the entropy they define seems to be of the information theory sort, they assume that it will tend to a maximum over time, as for the thermodynamic sort. As no argument is given as to why there should be any correspondence between the entropy formulated and its supposed behaviour, their conclusions as to probable drainage patterns must be considered unproven. Woldenberg[50] seems to be making a similarly unjustified identification of entropy type in his study of the role of energy in the spatial ordering of human settlements.

That information theory and thermodynamics give rise to the same measure must of course stimulate attempts to make statements about the physical nature of systems through estimating the entropy of these systems from an information theory approach, i.e.,by a consideration of possible microstates. As was discussed in the previous chapter, as long as this method involves no more than statements about the uncertainty of the whereabouts of phase points in phase space there would seem to be no

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48. L.B. Leopold and W.B. Langbein, The Concept of Entropy in Landscape Evolution, Geological Survey Professional Paper 500-A, Washington, 1962.

49. W.B. Langbein and L.B. Leopold, "Quasi-Equilibrium States in Channel Morphology", American Journal of Science 262, 1964, p. 782.

50. M.J. Woldenburg, "Energy Flow and Spatial Order", Geographical Review 58, 1968, p. 552.

objection to such thermodynamic estimates. As Brillouin[51] has discussed, in such cases the conversion of information entropy to thermodynamic entropy only needs the use of the scale factor k (Boltzmann's constant). But predating Brillouin's work, and even before the formulation of information theory, Jeans[52] discussed the physical entropy of arrangement (of railway trucks) in a long and rather confusing correspondence with Donnan[53] and Guggenheim[54].

The information-thermodynamics transition has been rigorously pursued in some recent works, in which the thermodynamic entropy of "organised structures" has been examined in an economic context. Berry[55], Allred[56] and Thoma[57] have proposed that the shaping of material has an entropic aspect, an effect which they calculate by estimating the number of ways a machine can be constructed while remaining within the tolerances allowed for its proper functioning. Their estimates indicate that the thermodynamic entropy corresponding to such shaping is negligibly small compared with the entropic contribution of most of the other manufacturing processes.

The field where the information-thermodynamics relationship has been explored (some might say exploited) most prolifically is biology, more

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51. L. Brillouin, Science and Information Theory, Academic Press, London, 1956.
52. J.H. Jeans, "Activities of Life, etc.", Nature 133, 1934, p. 174; p. 612; p.986.
53. F.G. Donnan, "Activities of Life and the Second Law of Thermodynamics", Nature 133, 1934, p.99.
54. F.G. Donnan and E.A. Guggenheim, "Activities of Life, etc.", Nature 133, 1934, p. 530; p. 869; Nature 134, 1934, p. 255.
55. R.S. Berry, "Recycling, Thermodynamics and Environmental Thrift", Bulletin of Atomic Scientists, May 1972, p. 8.
56. J.C. Allred, "Application of Entropy Concepts to National Energy Problems", 1977. (Unpublished paper).
57. J. Thoma, Energy, Entropy and Information, Research Memorandum RM-77-32, IIASA, Laxenberg, Austria, 1977.

specifically the examination of what makes living things different from non-living things. To this end, what seem to have been the first information-type calculations of biological structures were made by Branson[58] and Augenstine, Branson and Carver[59]. The entropy calculation was based upon the set of amino acid frequencies in proteins, i.e. the microstates considered were in terms of the spatial coordinates of a particular group of amino acids. Branson noted that as the number of amino acids in a protein increases the ratio of observed to maximum entropy tends towards one ($H/H_{max} \rightarrow 1$), and he speculated that life became possible when the chance aggregation of more than 450 amino acid residues led to a protein structure with $H/H_{max} > 0.5$, though it is not clear why these particular values should hold the key to life.

Dancoff and Quastler[60] set themselves the even more daunting task of estimating the information content of a living organism. They explicitly restricted their study to the problem of specifying the parts that make up an organism, and make no mention of the interrelation between these parts. Using the frequency of occurrence of the different sorts of atoms in living tissue, they estimated that the average amount of information associated with each atom, in the light of its type and orientation, was approximately 24.5 bits (i.e., questions). This gave approximately 2×10^{28} bits for an adult human, though this ignores all water content, water being regarded as ubiquitous and therefore, they assumed, irrelevant to the structural specification. A similar calculation in terms of molecules gave approximately 5×10^{25} bits per adult human, the saving being due to the internalisation of entropy, caused by the increased size of the phase cells. By ignoring all of the contents of a cell excepting the chromosomes,

58. H.R. Branson, "Information Theory and the Structure of Proteins", in H. Quastler (Ed.), Information Theory in Biology, University of Illinois Press, Urbana, 1953.

59. L. Augenstine, H.R. Branson and E.B. Carver, "A Search for Intersymbol Influences in Protein Structure", In Quastler, op. cit.

60. S.M. Dancoff and H. Quastler, "The Information Content and Error Rate of Living Things", in Quastler, op. cit.

which they adjudged to contain all of the important cell structure, they estimated the information content of a cell at 10^{11} bits, again at the level of molecules. That is, if one had a list of all the types of molecules found in a cell's chromosomes, it would take 10^{11} guesses to assemble a total description from which the chromosomes could be reconstituted. Morowitz[61] has performed a similar calculation, this time ignoring only the water content of the cell, and obtained an estimated information content of 10^{12} bits.

It is instructive to compare these estimates of the information content of a cell with that obtained by Linschitz[62], who used thermodynamic entropy directly. He based his estimate on the assumption that there was an overall energy balance over time between the cell and its surroundings as the cell grew, which allowed him to describe the thermodynamics of the cell activity:

$$\Delta S_{int} = \frac{1}{T} (\Delta F_{ext} - \Delta H_{int})$$

where ΔS_{int} is the entropy change of the cell

ΔF_{ext} is the energy required by the cell for all of its activities other than growth

ΔH_{int} is the difference in heat content between the cell material and its food supply

T is the absolute temperature

By identifying the entropy change with a change in information, using Boltzmann's constant as the scaling factor, Linschitz estimated the information content of one particular sort of bacterium (which consumes hydrogen and carbon dioxide) as approximately 10^{13} bits per cell. This

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61. H.J. Morowitz, "Some Order-Disorder Considerations in Living Systems", Bulletin of Mathematical Biophysics 17, 1955, p. 81.

62. H. Linschitz, "The Information Content of a Bacterial Cell", in Quastler, op. cit.

is at least within shouting distance of the values obtained in the other two studies by the direct information theory approach.

Johnson[63] has used the idea of the information content of organisms to suggest that as a living system ages it loses organizational structure, which he identifies with information. But he notes that animals die at different rates between species, while within a species old animals die at a higher rate than young ones. He suggests that this is due to different species having different levels of informational redundancy, and it is the erosion of this "excess" information by stochastic processes which eventually leaves the organizational level of the animal too low for it to be able to maintain itself, i.e. it dies. From his model he derives a mortality rate curve which he claims is a better fit to animal death rates than the usually assumed negative exponential function.

The *change* in the information content of organisms as they develop has also been examined, by Jacobson[64], by Raven[65] and by Elsasser[66]. There is some disagreement here, Jacobson feeling that the information content of an organism remains constant as it develops from zygote to adult, while Raven and Elsasser propose that the information content increases. As Apter and Wolpert[67] have discussed, this disagreement seems to stem from their using different notions of what constitutes an organism.

For Raven and Elsasser, the fact that an adult contains more cells than a zygote seems to account for the increase in information they note. Jacobson, on the other hand, focusses attention on the "functional"

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63. H.A. Johnson, "Redundancy and Biological Aging", Science 141, 1963, p.910.

64. H. Jacobson, "Information, Reproduction and the Origin of Life", American Scientist 43, 1955, p. 119.

65. C.P. Raven, Oogenesis, Pergamon, London, 1961.

66. W.M. Elsasser, The Physical Foundations of Biology, Pergamon, London, 1958.

67. M.J. Apter and L. Wolpert, "Cybernetics and Development. I: Information Theory", Journal of Theoretical Biology 8, 1965, p. 244.

qualities of the organism, and so seems to assert that as a chicken comes from an egg, it must contain the same amount of "descriptive" information as that egg. What we obviously have here is not only confusion of definition over what constitutes the "essentials" of an organism, we also have a very ill-defined notion of information, which relates but weakly to the notion of thermodynamic entropy.

Perhaps these contradictions can be avoided if one recognises that estimates of the "information content" of organisms are probably made not to serve as estimates of the entropy (or negentropy) of that organism, but to give insights into the much more vexed question of their "complexity". Here the assumption seems to be related to that we noted in ecology, that living things are somehow "more complicated" than non-living things. Johnson obviously saw this complexity as what kept living things alive. The problems that seem to be posed are therefore:

(i) Are "higher" organisms more complex than "lower" organisms? (i.e. does evolution cause complexity to increase?)

(ii) Is an adult organism more "complex" than its zygote?

These problems remain, of course, unanswerable until we have a *definition for this "complexity"*. As it refers to organisms, a defining property of which is that they can reproduce, one path might be to estimate the "complexity" of an organism by the complexity of the "message" it must communicate (to itself and to its environment) to reproduce itself. Von Neumann[68] has shown that self-reproducing automata are theoretically describable, and also suggested that if an automaton has an insufficient number of discrete parts which can relate to one another, then self-replication is not feasible. This is certainly along the lines of the "complexity" ideas we have introduced allowing the self-replicating

68. J. von Neumann, Theory of Self-Reproducing Automata, University of Illinois Press, Urbana, 1966.

to be distinguishable from the non-self-replicating.

Rashevsky[69,70] has therefore approached this problem by examining the information content of a message which is sufficient to allow the reproduction of that message. Cairns-Smith[71] notes that if the receiver is sufficiently sophisticated then the message can be abbreviated to an algorithm; i.e. rather than describing the type and orientation of every molecule in a cell, as attempted by Dancoff and Quastler, one might describe functional relations between moelcules that of themselves produce sufficient constraints upon the other molecules so as to generate a full description of the cell.

In this chapter we have examined some of the formal applications of the entropy concept, where its statistical properties have been utilised at the macro level to provide well formulated tools for the analysis of wholes of related parts. At the micro level, however, the tendency has been towards the mutual substitution of the information and thermodynamic formulations of the entropy concept, giving rise to some confusion both as to the aims of such an approach and the methods by which these aims can be met.

The purpose of this study is to give insight into the physical structuring and functioning of economies, and perhaps some insights can be had if we pursue further the problem of "complexity" or "organisation" and thence examine some of the modes of "complexification" available to physical systems. To this end, the next chapter will explore the applicability of the entropy concept to the problems of "being organised" and of "self-organisation.

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69. N. Rashevsky, "Life, Information Theory and Topology", Bulletin of Mathematical Biophysics 17, 1955, p. 229.

70. N. Rashevsky, "Life, Information Theory, Probability and Physics", Bulletin of Mathematical Biophysics 22, 1960, p. 351.

71. A.G. Cairns-Smith, The Life Puzzle, Oliver and Boyd, Edinburgh, 1971.

CHAPTER FIVE

Self-Organisation and Dissipative Systems

In an earlier chapter we noted Popper's[1] comment that, in apparent opposition to the popular notion of the universe continually increasing its entropy, the universe seems to be well structured and moreover seems to be increasing in structure. In particular we might note that economies seem to "develop", the nineteenth century in Britain often being cited as a period when economic structures were being formed and economic organisation generated.

In this chapter we shall investigate the nature of such structuring in the world, investigating its causes with special reference to the theory of dissipative structures. We shall further examine the general notion of self-organising systems, and particularly explore the inter-relationships between the concepts of "complexity", "order", "structure" and "organisation".

Let us recall that in an earlier chapter the physical variable of entropy was shown to increase to a maximum for an isolated system, an isolated system being one whose boundaries do not allow the entry or egress of matter or energy. If the prohibition on energy exchange with the system's environment is relaxed, the system is said to be "closed". In these circumstances the equilibrium state of the system can be expressed in terms of the "Free Energy" of the system, where the precise definition of this variable depends upon the limitations imposed upon the system's energy relations with its environment. In general, a closed system will tend to a state having a minimum free energy. As an illustrative example let us consider the case of a system in contact with

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1. K.R. Popper, "Time's Arrow and Entropy", Nature 207, 1965, p. 233.

an isothermal environment at temperature T. It can be shown that the relevant descriptive thermodynamic variable for such a system is the Helmholtz Free Energy, F, defined as:

$$F = E - TS \qquad \text{(see e.g. Sears[2])}$$

Here E is the internal energy of the system, S is the entropy, and T is the absolute temperature of the environment (or heat reservoir).

We should immediately note, that, for a given temperature, E and S will in general be mutually dependent. A state of low entropy for a given temperature will correspond to some orderly, close-knit association of the molecules of the system, which will in turn correspond to a high (potential) internal energy; i.e., low S implies high E. On the other hand, a state of low entropy will correspond to a loose association between molecules, implying a lower internal energy. That is, high S implies low E.

This inter-relationship of S and E implies that the minimisation of the Helmholtz free energy by the system will give rise to different entropic states (i.e. different levels of "orderliness") at different temperatures. When T is high a high level of S is indicated, with a corresponding relatively lower contribution of potential energy to E. This combination of values will minimise the positive contribution to F, while maximising the negative contribution. By the same reasoning, a low value for T will imply a low equilibrium value for S. Therefore a closed isothermal system will be in a state of low entropy at low temperatures and high entropy at high temperatures. This is precisely what we observe when a block of ice is removed from a freezer. The low entropy, ordered, ice becomes high entropy, disordered, water.

We see, therefore, that by allowing energy to be exchanged between the system and its environment, order (i.e. a lower entropy state) _can_ be

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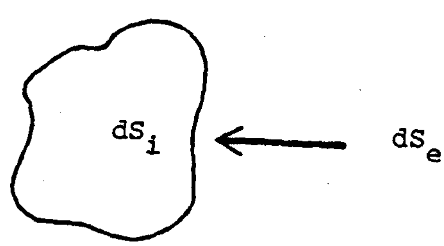
2. F.W. Sears, _Thermodynamics_, Addison-Wesley, London, 1966, p. 159.

generated by reducing the temperature of the system. This process, though, is patently insufficient as an explanation of most of the structure and structuring that we observe in the world. It is common experience that plants do not develop as structured beings simply when the weather gets colder. Quite the contrary. Can we therefore extend our analysis, perhaps by further relaxing the constraints on the systems under examination? Recent work by Prigogine and his colleagues in Brussels and Texas suggest that we can.

If our initially isolated system is not only allowed to exchange energy with its environment, but also to exchange matter, it is termed as "open" system, and it is to such systems that we now turn.

An isolated system has only one source of entropy generation; that source is the internal irreversible processes it undergoes as it approaches its thermodynamic equilibrium state. An open system, on the other hand, has not only an internal component to its entropy increase, dS_i, but also an external component, dS_e, associated with the mass and energy communicated to and/or from its environment, as indicated in Figure 1.

Figure 1



So the total entropy increase in the system during time interval dT can be expressed as:

$$dS = dS_i + dS_e$$

Now the sign of the internal entropy generation term, dS_i, is unambiguous. This is the entropy change corresponding to the isolated system alone, and it is therefore positive in sign. The external entropy generation term may, on the other hand, have positive or negative sign, depending upon whether the system is importing entropy from the environment or exporting entropy to it. This allows the interesting and exciting possibility of an open system behaving in such a way that:

$$-dS_e \geq dS_i \qquad \text{i.e.} \quad dS \leq 0$$

Thus an open system may be maintaining a constant level of entropy, or even decreasing its entropy. Thus if we consider open systems there is no theoretical objection to systems which have decreasing levels of entropy, or levels of entropy maintained below that corresponding to isolated systems. It must have been just such systems that Schroedinger[3] had in mind when he wrote of organisms as consuming "negentropy" from their surroundings. This sentiment would now be more exactly, though less succinctly, expressed in terms of their ingesting low entropy foods and egesting high entropy wastes.

A conceptual framework can therefore be established within which the emergence of structure can be comprehended, whether the systems exhibiting this emergence be usually classified as inorganic, animate to economic. But there remains the problem of the mechanism of such emergence. It may be granted that thermodynamics does not disallow the formation of low entropy open systems, but we must still enquire as to whether it will also predict such formation.

We can immediately note that the consideration of systems at, or tending towards, equilibrium will be of little help here. Quite obviously a system acquiring structure is not at equilibrium, and is probably moving away from it. We should therefore enquire into the behaviour of non-equilibrium systems.

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3. E. Schroedinger, What is Life?, Cambridge University Press, London, 1944.

When a system is not at thermodynamic equilibrium, the lack of equilibrium is revealed by the existence of "gradients" within the system. These gradients may be between unequal concentrations of chemical constituents, unequal pressures, unequal temperatures, etc. In the process of achieving equilibrium these gradients are eliminated by suitable reactions among the chemical constituents, by the flow of matter and energy, etc. These "flows" generated by the "forces" along the gradients constitute irreversible changes in the system, and thereby cause the entropy of the system to increase. It can be shown (see de Groot[4]) that if the flows are termed J_j and the forces, suitably defined, are termed X_j, then the rate of entropy production within the system per unit time can be written as:

$$dS_i = \sum_j J_j X_j$$

A simple example of this effect would be a system composed of a battery connected across a resistor. Here the force is the voltage across the battery (E) and the flow is the electric current (I). The product of the voltage and the corresponding current is the heat dissipated per unit time; i.e., $dQ = IE$. When divided by the ambient absolute temperature this is of the form $dQ/T = dS$. Thus the force times the flow, normalised with respect to temperature, gives the system's internal entropy production rate. (For a generalisation of electrical network theory to the analysis of thermodynamic systems see Oster and Desoer[5]).

In general, the flows within a system are functionally related to the physical configuration of that system. The pertinent aspects of the configuration can themselves be described in terms of the thermodynamic

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4. S.R. de Groot, Thermodynamics of Irreversible Processes, North Holland, Amsterdam, 1951.

5. G.F. Oster and C.A. Desoer, "Tellegen's Theorem and Thermodynamic Inequalities", Journal of Theoretical Biology 32, 1971, p. 219.

forces within the system, so we may write each flow as a function of the forces within the system:

$$J_j = f_j(X_1, X_2, \ldots\ldots) \quad \forall j$$

In particular, close to thermodynamic equilibrium we can expand these functional relations in a power series and ignore all terms higher than the linear. So close to equilibrium we can write:

$$J_j = \sum_k L_{jk} X_k$$

The coefficients L_{jk} are termed the "phenomenological coefficients" and are expressible as functions of the thermodynamic variables relevant to the system.

Therefore near equilibrium the internal entropy production rate of the system can be written as:

$$dS_i = \sum_j X_j \sum_k L_{jk} X_k$$

or in vector notation:

$$dS_i = \underline{x}' \, \underline{L} \, \underline{x}$$

An early result on the nature of the $\underline{L}$ matrix for such near equilibrium systems was derived by Onsager[6]. By considering the micro-reversibility of the interactions within the system he showed that it is always possible to define the flows and corresponding forces such that $\underline{L}$ is symmetric; i.e., $L_{jk} = L_{kj} \; \forall \; j,k$. This result is known as the Onsager Reciprocity Relation.

A second important early result is due to Prigogine[7], though in a simpler form it had been noted by Maxwell as early as 1871. Prigogine considered an open system in a non-equilibrium steady state, i.e., $dS_i = -dS_e$ so $dS = 0$. In these circumstances he showed that if the system

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6. L. Onsager, "Reciprocal Relations in Irreversible Processes", Physical Review 37, 1931, p. 405.

7. I. Prigogine, Etudes Thermodynamiques des Processus Irreversibles, Desoer, Liege, 1947; translated as Thermodynamics of Irreversible Processes, Wiley, London, 1961.

is close to equilibrium then the entropy production per unit time, dS_i, will tend to the minimum value compatible with the boundary conditions placed upon the system. Moreover he showed that the state exhibiting minimum entropy production is stable. This has been termed the Minimum Entropy Production Theorem, and indicates that while isolated systems have equilibrium states characterised by maximum entropy, and closed systems by minimum free energy, open systems near equilibrium attain non-equilibrium stationary states characterised by their having a minimum rate of entropy production.

Both Onsager's Reciprocity Relations and Prigogine's Minimum Entropy Production Theorem are of considerable significance for our understanding of the behaviour of certain classes of "structured" systems.

The former implies that if asymmetry is found in a near equilibrium system, then the thermodynamic flows and forces will evolve in such a way as to eliminate this asymmetry. The latter tells us that near equilibrium systems tend towards states of "minimum work", or "minimum throughput". This would seem particularly applicable to the thermodynamic specification of the maturation process in organisms and ecosystems. In this context work by Stoward[8], and by Ziotin and Zotina[9] has indicated that as organisms develop from the zygote to adulthood their rate of heat dissipation (per unit mass) decreases towards a minimum. One would anticipate that heat dissipation would be the major constituent of internal entropy generation. Further, it has been noted that when organisms are wounded or damaged in some way their rate of entropy production is initially increased, and then falls as healing proceeds.

Both of these results would seem to open to the interpretation of near equilibrium stationary states tending towards their state of minimum

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8. P.J. Stoward, "Thermodynamics of Biological Growth", Nature 194, 1962, p. 977.

9. A.I. Ziotin and R.S. Zotina, "Thermodynamic Aspects of Developmental Biology", Journal of Theoretical Biology 17, 1967, p. 57.

entropy production[10]. Caution must be used here though, as has been noted by Trincher[11]. It is not at all clear that organisms are in fact near equilibrium systems, as required for the application of Prigogine's theorem. An "evolution criterion" for far from equilibrium systems has been developed by Nicolis and Prigogine[12], akin to the minimum entropy production criterion. However, its interpretation is rather ambiguous for the very complicated systems that comprise even the simplest organism. Also, the state towards which a system tends subject to this criterion is not in general a stable one.

We see that the theory of near-equilibrium stationary states allows insights into the possible functioning of certain structured systems, but is inadequate to explain the origin of such structure. The stability alone of such near equilibrium stationary states will ensure that they cannot evolve to further, more structured, states. How then can we explain, or even comprehend the evolution of structures, which we continually observe?

In the realm of biology, one aspect of this evolution of structure is the generation of "life". Since Miller's[13] laboratory synthesis of various amino acids by electrical discharges in a mixture something akin to the Earth's primaeval atmosphere, there has been little disagreement that life could have evolved on Earth from inorganic constituents. However there still seem to be two schools of thought as to whether the generation of life was a chance outcome or was inevitable.

Proponents of the first line (e.g. Monod[14]) argue that to be classified as an organism a system must be self-regulating and self-reproducing.

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10. I. Prigogine and J.M. Wiame, "Biologie et Thermodynamique des Phenomenes Irreversibles", Experientia 2, 1946, p. 451.
11. K.S. Trincher, Biology and Information, Consultants Bureau, New York, 1965.
12. G. Nicolis and I. Prigogine, Self-Organisation in Non-Equilibrium Systems, Wiley, New York, 1977.
13. S.L. Miller, "A Production of Amino Acids Under Possible Primitive Earth Conditions", Science 117, 1953, p. 528.
14. J. Monod, Chance and Necessity, Collins, London, 1972.

This necessitates that the system be subdivided into "organs", and the blueprint for its generation be encoded in DNA, or something similar. This, they suggest, is a far cry from the simple amino acids which nature might provide, and such a system, while being self-maintaining once formed, is extraordinarily unlikely to form by the chance combination of its constituent components. Therefore, they argue, "life" is an extremely happy accident.

Members of the other school (e.g. Schoffeniels[15]) respond that it is ridiculous to expect an organism to leap ready formed from the primaeval sludge. Rather, they argue, one should seek stability of intermediate forms, whereby higher and higher levels of complexity and structure can be attained by the union of substructures which are themselves stable within their environments.

This point can be made clear by Simon's[16] analogy of the watchmaker. Suppose a watch is made of a thousand parts, and the watchmaker has two strategies open to him in assembling the watch. The first strategy is to piece together the watch a unit at a time, taking the risk that an error at any of the thousand stages of construction will ruin the whole and necessitate starting afresh.

The second strategy is to assemble the watch into one hundred stable sub-sub-units, each containing ten elements, these to be then assembled ten at a time into ten stable sub-units which can be brought together to form the whole. An error at any stage of the construction will now require the reassembly of at most ten elements.

If one assumes a one percent probability of making an error at each step of the construction, each step taking one unit of time, then the

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15. E. Schoffeniels, <u>Anti-Chance</u>, Pergamon, Oxford, 1976.

16. H.A. Simon, "The Architecture of Complexity", <u>Proceedings of the American Philosophical Society</u> <u>106</u>, 1962, p. 467.

average time to construct a watch by the first strategy is $\frac{500}{(0.99)^{1000}}$ units, while that of the second is $\frac{5 \times 111}{(0.99)^{10}}$ units. The second strategy will therefore be faster than the first at manufacturing watches by a factor of $\frac{1}{1.11 \times (0.99)^{990}} = 1396.8$.

A second biological problem is, given that life does exist, are the laws of physics sufficient to explain the evolution of further structured self-maintaining systems, or must specifically "biological" laws be invoked? This is, of course, a corollary of our general consideration in this chapter into the nature of structured, self-organising systems. To suggest that more than physics is necessary to explain biological activity is, as Needham[17] has noted, to assert a vitalist approach to the issue. Two proponents of this modern vitalism are Elsasser[18], with his "biotonic laws", and Polanyi[19] with his notion of "life's irreducible structure". For both of them the activities of life may not be contrary to the currently accepted physics, but neither are they fully explained by it.

We can now assert that extra biological or "organismic" modes of explaining the evolution and functioning of organisms are unnecessary. Both the evolution of organic phenomena and the self-maintenance and functioning of such systems can be explained, or at least a conceptual framework for their understanding established, through the investigations of Prigogine and the Brussels school into the behaviour of open systems which are far from thermodynamic equilibrium[20,21]. The remarkable result which they have established is that far from equilibrium systems may exhibit several, or many, modes of behaviour, the mode exhibited depending to some extent upon the boundary conditions imposed upon the system.

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17. J. Needham, "Thoughts on the Problem of Biological Organisation", Scientia 26, 1932, p. 84.
18. W.M. Elsasser, Atom and Organism, Princeton University Press, 1966.
19. M. Polanyi, "Life's Irreducible Structure", Science 160, 1968, p. 1308.
20. I. Prigogine and G. Nicolis, "Biological Order, Structure and Instabilities", Quarterly Review of Biophysics 4, 1971, p. 107.
21. I. Prigogine, G. Nicolis and A. Babloyantz, "Thermodynamics of Evolution", Physics Today 25, 1972, p. 23.

Associated with each state is a characteristic entropy and a characteristic rate of entropy production, and depending upon the nature of the system, alteration of the boundary conditions may cause continuous or abrupt transitions from state to state.

There are several simple classic examples of this behaviour. Perhaps the simplest is the Knudsen effect, where a container of mixed gases of different molecular weights has a temperature gradient imposed upon it. It is observed that gases of higher molecular weight tend to become more concentrated near the colder part of the container, while those of lower molecular weight tend to accumulate at the hotter part. The greater the temperature gradient the greater the separation of the gaseous components.

A greater temperature gradient will of course give rise to a greater flow of heat through the container, while the separation of the gases will reduce the entropy of mixing within the container. One might therefore view the system as somehow becoming more organised because of the greater energy flux through it.

A second example is the formation of Bénard convection cells in liquids subject to a temperature gradient. If a liquid has a temperature gradient imposed upon it, then if the gradient is initially small the liquid will transport energy through its bulk solely by conduction. However, once the temperature gradient exceeds a certain critical level, dependent upon the dimensions of the container and the properties of the liquid in use, then convection cells form spontaneously, considerably increasing the rate of heat transport through the system. The cells which form are invariably hexagonal in shape and regularly arranged.

These convection cells are macro-phenomena, and the long range molecular coordination that they demonstrate is quite outside the bounds of probability for an isolated system. They obviously correspond to a lower entropy state for the system, and again we see an energy flux acting as an ordering agent for an open system.

Further examples of the self-structuring of far from equilibrium systems are to be found among auto-catalytic reactions, where both spatial and temporal ordering of the systems have been observed.

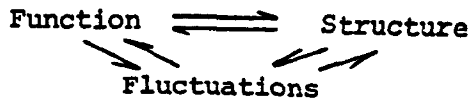
A characteristic of all of the systems noted is that the structure they engender is associated with energy dissipation. As a result such structures have been termed "dissipative structures" to distinguish them from "equilibrium structures" such as crystals.

The theory used to explain the appearance and self-maintenance of such far from equilibrium dissipative structures is basically the analysis of coupled non-linear partial differential equations. In general, analytic solutions to all but the simplest such sets of equations are unavailable, and the analysis used relies upon the resolution and classification of the non-equilibrium stationary states possible for the system. The mathematical techniques that have been employed are complicated, involving bifurcation theory and stability theory; these need not detain us here. (Details are to be found in Nicolis and Prigogine[12].) The physical mechanism behind the behaviour of such systems is interesting and important, though, as it sheds considerable light upon, among other things, the previously mentioned problems of the origin and self-maintenance of organisms.

Within any system above absolute zero temperature, random fluctuations are continually disturbing the system's constituent molecules. The overall effect of such fluctuations will, for a given partitioning of phase space, define a microstate for the system. This, together, with probably many other microstates, will be reflected by an observeable macrostate for the system. Now it is quite conceivable that a random fluctuation within the system will give rise to a microstate markedly different from the normally observed macrostate. For an isolated system the duration of such a fluctuation will be very brief, as we know that the macrostate normally exhibited will be that with the largest number of corresponding microstates. The fluctuation disappears because it is a random occurrence and there is nothing to detain, or maintain, it.

In a far from equilibrium system, on the other hand, the system is maintained far from equilibrium by being in interaction with its environment, the interaction often taking the form of an energy flux. The state that the system is in, and the structure that it exhibits, is crucially dependent upon this interaction. Further, this relationship of the system with its environment is mediated by the structuring of the system. If, therefore, a fluctuation of the system momentarily generates a structuring stable with respect to the new required relationship with the system's environment, then the system may, of itself, continue to exhibit the new configuration, maintaining the new state through that state's new relationship with its environment. We are effectively saying here that if the system spontaneously jumps from one locally stable solution of the defining set of equations to, or near, another locally stable solution, then there is no reason why the system should not stay at, or near, this new solution.

Perhaps the clearest way to understand the relationship between fluctuations and structuring in such systems is to postulate a "function" to the system. One might assert, for example, that the function of a system with an imposed temperature gradient is to transport heat. The appearance of structures facilitating this transfer (e.g. convection cells) is conducive to this function, such that if statistical fluctuations generate such structures, their efficient functioning will aid their own maintenance. Following Prigogine we might express this interplay of function, structure and fluctuations with a diagram:



The attribution of function to what must be regarded as an arbitrary system will give pause here. One might argue that for at least some systems the function might be regarded as diminishing the flow of energy through the system (e.g. roof insulation). Here one's defence must be that the function of the system should be regarded not as what we would <u>like</u> the system to do, but what it is actually <u>observed</u> to do. That is, the notion of function should be <u>ex post</u>.

Another objection, that function implies teleology, which in turn implies volition, can be countered by following Bertalannfy[22]. He has shown that many systems whose behaviour is equifinal or asymptotic to some stationary state can be described in teleological terms, while simultaneously being expressible in a form dependent only upon past and present states, with no mention of future possibilities or system "aims". Certain teleological systems may not be satisfactorily explained in this way (e.g. how one votes) but most simpler systems can be so explained.

We see now that if the discussed techniques of analysis are valid for the extremely complex systems that are living things, then the watch-maker's second method may be available as a means of explaining the origin of life. Random fluctuations establish stable, self-maintaining elements, whose interactions within the open system generate further structures through fluctuation, etc. When put like this the origination of life sounds almost easy and probably inevitable.

So far we have examined one particular mode of structuring available to open systems, that of dissipative structure through fluctuations.

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22. L. von Bertalanffy, "The Theory of Open Systems in Physics and Biology", <u>Science 111</u>, 1950, p. 23.

However, Prigogine[23] has outlined an even more general approach to the problem of structuring, allowing the apparently non-Second Law behaviour of more general classes of systems to be detailed. The approach followed involves a re-examination of Boltzmann's famous H-theorem (outlined in an earlier chapter) and the generalisation of the fundamental inter-particle relation it posits.

We recall that for the H-theorem Boltzmann defined the following function:

$$H = \sum_i p_i \log p_i$$

i.e. H is the negative of the average entropy per particle, as previously defined. Here p_i is the probability of a particle being in the ith cell of phase space. More generally, by using a phase space frequency distribution function $f(\underline{x})$ this can be written as:

$$H = \int_{\underline{x}} f(\underline{x}) \log f(\underline{x})\, d\underline{x}$$

Boltzmann then introduced inter-particle interaction into the analysis through an instantaneous collision operator, G. This acts upon the frequency distribution function via the relation:

$$\partial f/\partial t = G(f)$$

By asserting what seem reasonable attributes of G, Boltzmann showed that $\partial H/\partial t \leq 0$, and thence obtained a probabilitic explanation of the observed increase in the entropy of isolated systems.

The point to notice is that the H-theorem rests upon the nature of the collision operator, G. If the particles in the system interact via very short range effects then the assumption of "instantaneous" interaction is unobjectionable. But obviously if one considers the interaction of, say, stellar objects via gravitational effects, then interactions are not

23. I. Prigogine,"Temps, Structure et Entropie", Bulletin de l'Academie de Belgique: Classe des Sciences 53, 1967, p. 273.

instantaneous, but are temporally extended. Further, if the particles in the system have some initial coordination and the level of random (thermal) disruption is low, then one might expect the system to retain a "memory" of its previous state. Again the assertion $\partial H/\partial t \leq 0$ is thrown into doubt.

Prigogine and his group have shown that the phase space frequency distribution can be subdivided into spatial and velocity components. Representing the velocity component by f_v, they have shown that a more general operator relationship between the frequency distribution and its rate of change is given by:

$$\partial f_v/\partial t = \int_0^t G^* (\tau)\ f_v(t - \tau) + D(\tau)\, d\tau$$

Here G* is the non-instantaneous collision term, the duration of its operation depending upon the reaction mechanism involved. D is the memory term, depending on the initial state of the system. In open dissipative structures it is obviously this memory term which is effective, the functional structure being such as to maintain itself against (most) thermal fluctuations of the system.

Using this more general formulation it can be shown that non-Second Law behaviour might be expected from certain systems, and that indeed to obtain an H-theorem the assumptions necessary are precisely that interactions are instantaneous and the memory term is negligible.

This general approach, if firmly founded, is of great importance, for it allows the two "arrows of time" to be viewed as separate aspects of a single developmental phenomenon. The first arrow is that of the Second Law, where times's direction can be judged by the tendency of isolated systems to increase in entropy, at least in the long run. The second arrow is that of evolution and development, revealed by the tendency of certain systems to become more complicated and more structured. Both may now be thought of as the result of intra-system interactions, where in the first case the collision operator satisfies the H-theorem, while in the second case it does not.

Layzer[24] has recently also attempted to show that the arrows of time are not distinct, though he discusses three arrows. As well as the Second Law and evolutionary arrows he also invokes the cosmological arrow, defined by the observed cosmic expansion.

Layzer's approach is at root that of Prigogine, but it is more general in scope and is expressed in terms of information theory. The key to this analysis is the distinction between internal and external entropies, discussed in a previous chapter. To recapitulate, if p_i is the probability of any of the N particles in the system being in the ith phase cell, then the entropy of the system is given by:

$$S = -N \sum_i p_i \log p_i$$

If the phase space is further subdivided, with new corresponding probabilities p_{ij}, such that $\sum_j p_{ij} = p_i$, then the new entropy is:

$$\begin{aligned} S' &= -N \sum_i \sum_j p_{ij} \log p_{ij} \\ &= S - N \sum_i p_i \sum_j (p_{ij}/p_i) \log (p_{ij}/p_i) \\ &= S + S^* \end{aligned}$$

That is: $S = S' - S^*$

Here S is the original coarse grained entropy, S' is the new fine grained entropy, and S* is the extra entropy contribution derived from making the phase space partitioning finer grained.

Now if the phase space is subject to a partitioning so fine grained that each phase cell contains at most one phase point, then the system

24. D. Layzer, "The Arrow of Time", The Astrophysical Journal 206, 1976, p. 559.

entropy is maximal and cannot be exceeded by any other partitioning. The maximum value that the systems entropy can take we shall term S_{max}, and it will be dependent upon the degree of fine graining necessary to exclude multiple occupancy of any of the phase cells.

Given that we can define a maximal entropy for the system, the entropy of the coarse grained partitioning can be expressed as:

$$S = S_{max} - S^*$$

Or using a clearer notation:

$$S_{ext} = S_{max} - S_{int}$$

That is, the system's external entropy equals the maximal entropy less the internal entropy subsumed within the coarse graining of the phase space.

Let us next recall that we associate "information" with "surprisal". If a message tells us something that surprises us, then it conveys information. Now the most common macrostate exhibited by an isolated system is that corresponding to the greatest number of microstates. Thus when we observe such a macrostate we are not particularly surprised and therefore such an observation conveys little information for us. The information we derive about a system by observing its macrostates Layzer terms "macroinformation".

On the other hand, as the most commonly occurring macrostate is associated with the largest corresponding set of microstates, the revelation of the microstate of a system at a particular time would provide the greatest amount of information when the most common macrostate is extant. This information Layzer terms "microinformation".

We see therefore that low levels of potential macroinformation about the system are associated with high levels of potential microinformation, and vice versa.

We can now relate the concepts of macro- and microinformation to the internal and external entropies we have already discussed. We have seen that the entropy of a system under any given partitioning can be expressed as the difference between the system's maximal entropy under an ultra-fine partitioning and the internal entropy. Now suppose the system is initially in an unlikely macrostate, such that its external entropy is observed to increase over time. Then as the external entropy increases the internal entropy must decrease so as to maintain the sum of the internal and external entropies constant at S_{max}. But we should also note that as the system progresses from an unlikely macrostate to a likely macrostate there is a decrease in the system's potential macroinformation and an increase in its potential microinformation.

We therefore see that low external entropy is associated with high macroinformation and low internal entropy with high microinformation, and vice versa.

This approach is most reminiscent of Brillouin's[25] identification of information with negentropy, as discussed in an earlier chapter.

By choosing suitable units for the measurement of the system's information, the constancy of the sum of a system's internal and external entropies, properly defined, can be translated into the constancy of the sum of its macro- and microinformation.

In these terms, the tendency of an isolated system to increase in (external) entropy can be expressed as a tendency for a system's macroinformation to be transformed into microinformation. If we identify macroinformation with system structure, as Layzer does, then the Second Law decay of such structure corresponds to macroinformation "decaying" into microinformation.

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25. L. Brillouin, Science and Information Theory, Academic Press, London, 1956.

Layzer notes that the transformation of macroinformation into microinformation requires the absence of particle correlations within the system. That is, as Prigogine noted, interactions between particles must be instantaneous and there must be no memory within the system. But he further notes that one can posit a source of increasing information for the universe if one assumes that the universe is expanding.

If the universe is expanding the number of phase cells needed to define the ultra-fine partitioning of the total phase space of the universe will continually increase. This will result in the total information of the universe continually increasing, thus allowing macroinformation to be continually generated while still permitting the decay of macroinformation into microinformation, in accordance with the Second Law. So Layzer is suggesting that cosmic expansion allows the spontaneous generation of structure. This effective source of low entropy for the universe is of course precisely in line with Gal-Or's[26] speculation that an expanding universe can act as an entropy sink, as mentioned in an earlier chapter.

In examining the work of Prigogine and Layzer we have seen that the evolution of structure in the universe need not be regarded as mysterious or requiring the formulation of particular "laws" for its explanation. That it is only recently that such a view has begun to dominate is probably as much due to the strong hold the Second Law has had upon the scientific imagination as to the mathematical difficulty of analysing the behaviour of non-equilibrium states. This problem of structuring, or organisation, is made conceptually more difficult, as we have seen, by the appearance of structuring being apparently self-generated in many systems. The whole problem of the abstract characterisation of self-organising systems, and

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26. B. Gal-Or, "Entropy, Fallacy and the Origin of Irreversibility", Annals of the New York Academy of Science 196, 1972, p. 305.

the analysis of the structural similarities between the mechanisms employed by different systems, has been much studied, particularly with reference to problems in cybernetics and artificial intelligence.

One aspect of the study of self-organising systems is the classification of the properties such systems must exhibit to qualify as self-organising. In an early and perceptive work in this field Gerard[27] suggested that self-organising systems must be in "open dynamic equilibrium", demonstrate "specific synthesis", and be subject to "adaptive amplification". All of these properties we see can be easily recognised as features of open dissipative structures, as analysed by Prigogine. Gerard speculated that systems as diverse as snowflakes and football teams could embrace these qualities, and suggested that all such self-organising systems be termed "orgs".

Fiebleman[28] has seen self-organisation in terms of "integrative levels", where successive levels of organisation within systems have asymmetrical relations with higher and lower levels within the hierarchy of organisational levels. Fiebleman suggests that this asymmetry of relation leads to the highest levels exhibiting structure as a result of the interactions throughout the hierarchy of these relations. Bronowski[29] uses a similar argument to stress the possibility of the evolution of organisms to states of higher complexity through stable intermediate states, implying that the evolutionary arrow of time, like the Second Law arrow, is "barbed". Again, the similarity of these approaches to Simon's "watchmaker" is clear.

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27. R.W. Gerard, "Organism, Society and Science", Scientific Monthly, 1940, p. 340; p. 403, p. 503.

28. J.K. Fiebleman, "Theory of Integrative Levels", British Journal for the Philosophy of Science 5, 1954-5, p. 59.

29. J. Bronowski, "New Concepts in the Evolution of Complexity: Stratified Stability and Unbounded Plans", Zygon 1, 1970, p. 18.

Rashevsky[30,31,32] and Rosen[33] have also sought to unify the organisational principles in physical, biological and social systems, mainly in terms of "organismic relations", which they postulate to be quantitative in physics, and therefore amenable to algebraic analysis, while being qualitative in biological and social systems, and therefore needing treatment by topological methods. Their work is stimulating in that they seek to classify system properties necessary for self-organisation, but must be regarded as unsatisfactory in that the mechanisms generating these properties are not investigated. A similar criticism of incompleteness of treatment might be levelled at Jumarie's[34,35,36] much more abstract discussions of self-organisation.

Von Foerster[37] has noted that Schroedinger[3] suggested two mechanisms for the generation of ordered structures; what he termed "order from order"

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30. N. Rashevsky, "A Note on the Nature and Origin of Life", Bulletin of Mathematical Biophysics 21, 1959, p. 185.
31. N. Rashevsky, "Organismic Sets: An Outline of a General Theory of Biological and Social Organisms", Bulletin of Mathematical Biophysics 29, 1967, p. 139.
32. N. Rashevsky, "Organismic Sets II: Some General Considerations", Bulletin of Mathematical Biophysics 30, 1968, p. 163.
33. R. Rosen, "A Relational Theory of Biological Systems", Bulletin of Mathematical Biophysics 20, 1958, p. 245.
34. G. Jumarie, "Towards a New Approach to Self-Organizing Systems", International Journal of Systems Science 4, 1973, p. 707.
35. G. Jumarie, "Structural Entropy, Information Potential, Information Balance and Evolution in Self-Organizing Systems", International Journal of Systems Science 5, 1974, p. 953.
36. G. Jumarie, "Further Advances on the General Thermodynamics of Open Systems via Information Theory: Effective Entropy, Negative Information", International Journal of Systems Science 6, 1975, p. 249.
37. H. von Foerster, "On Self-Organising Systems and their Environments", in Self-Organizing Systems, Eds. M.C. Yovits and S. Cameron, Pergamon, London, 1959.

and "order from disorder". To these von Foerster[37] adds a third, "order from noise".

An example of the generation of order from order is the increase in structure we notice when an organism grows, taking in relatively ordered foodstuffs as its building materials.

Order from noise we can identify with Prigogine's "order through fluctuation", random disturbances in the system causing the system to pass to a more ordered stable, stationary state.

Order from disorder is the most difficult process to understand, implying as it does that disordering, Second Law processes may, in interaction with the internal structure of the system and the boundary constraints imposed upon it, give rise to even higher levels of order in the system. We can devise an explanation of this behaviour if we analyse the system concerned in terms of its phase space.

Let us suppose that a dissipative system is in a non-equilibrium stationary state, maintaining itself by suitable entropy exchange with its environment. Let us further suppose that this system is spatially structured, and therefore in a relatively low entropy state. We can identify the elements composing its structure as "organs", and devise a partitioning of the system's phase space which is at least partially coincident with these organs. Now the low entropy (structuring) of the system ensures that it is in an unlikely macrostate for an isolated system. Thus the migration of phase points from areas of phase space corresponding to the system's "organs" to other "organs" in the system is in correspondence with Second Law processes. Now if such spontaneously generated "organs" prove, in the new state of interaction of the system with its environment, to be effective at performing the system's function, then the system may be able to further organise itself to an even more structured, lower entropy state than it was in initially. So a Second Law increase in entropy may be more than counteracted by the improved system functioning which it instigates. Hence order may spring from disorder.

In the context of these possible modes of ordering, we may note that Ashby[38a] has suggested that any dynamic system obeying unchanging laws will develop "organisms" adapted to their "environments".

From the above discussion it would seem that we do now have a well defined body of theory and an associated set of concepts dealing with self-organisation, or as Haken[38] terms it, "Synergetics". Thus equipped we can now approach the problem we have set ourselves, viz. the study of the relationship between the organisation of economies and their use, that is dissipation, of energy. To concretise our ideas about self-maintaining, far from equilibrium, dissipative structures we shall introduce a simple example.

Consider an electric refrigerator, the motor of which is of special construction, being entirely made of low melting point materials, such that the motor will maintain its integrity only if kept at a temperature below 0°C. For example, the conducting components might be made of mercury or sodium alloys, and the insulating components of a suitable paraffin wax. Quite obviously the refrigerator will only function if the motor is kept sufficiently cold, which we can achieve by mounting the motor in the freezer box of the refrigerator.

As long as the motor continues to work the refrigerator will function and the freezer box will stay cold; and as long as the freezer box stays cold the motor will continue to work. This novel refrigerator is therefore acting as a self-maintaining, far from equilibrium, dissipative structure. It continues to work only because there is a continual flux of energy

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38. H. Haken (Ed.), Synergetics; Cooperative Phenomena in Multi-Component Systems, Teubner, Stuttgart, 1973.

38a W.R. Ashby, "Principles of the Self-Organizing System" in Principles of Self-Organization. Ed. H. von Foerster and G.W. Zopf, Pergamon, London, 1962.

through the system. This energy enters as low entropy electrical energy and leaves as high entropy waste heat.

This refrigerator is not, unfortunately, self-organising, but it will serve nevertheless as an illustration of many of the abstract concepts discussed above, such that when we later come to the detailed analysis of economies in terms of their thermodynamic attributes we will always have a simple behavioural model ready to hand.

In the above example of the refrigerator, the functioning of the system is dependent upon the low temperature of the motor. If this low temperature is not maintained then the motor would melt and cease to work. As the motor must remain cold it is unambiguous that the entropy of this open system is lower when the system is functioning than when it has ceased to function, say after a lengthy power cut. That is, we might identify the functioning of the system via its energy dissipation with the system being in a low entropy state. Indeed, within limits the faster we run the motor the colder it becomes, thus simultaneously reducing the entropy level of the system and increasing its energy dissipation. We might, in this case, assert an inverse relation between the "complexity" of the system (i.e. its low entropy) and its energy dissipation.

Just such as inverse relation between entropy and dissipation has been demonstrated by Fox[39], for systems which are not too far from equilibrium. Morowitz[40] has gone even further and asserted that it is almost invariably the case that open systems subject to an energy flux will generate order within themselves as reflected by a lower entropy state. This is a tempting hypothesis, subsuming as it does the self-organisation observed in the weather, organisms, ecosystems and economies under the effect of

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39. R.F. Fox, "Entropy Reduction in Open Systems", Journal of Theoretical Biology 31, 1971, p. 43.

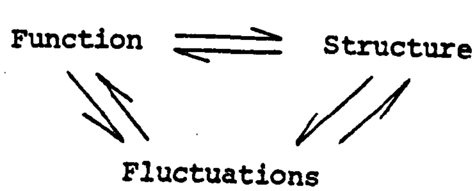
40. H.J. Morowitz, Energy Flow in Biology, Academic Press, London, 1968.

the flux of solar energy which bathes the Earth. However this very general hypothesis has been criticised by Prigogine[41] as ignoring much of the sublety of self-organisation.

There seem to be two problems here, both of which must be scrutinised. The first is whether an energy flux of itself is sufficient to generate greater system complexity. The second is whether greater organisation of the system is necessarily reflected by reduced system entropy, and vice versa.

For the first question, Fox's result indicates that for systems not too far from equilibrium the answer is in the affirmative and increased dissipation does lead to increased complexity. But for systems far from equilibrium one can quite easily envisage cases where increased flux might act as a disordering influence upon the system. For example, if the temperature applied at the hot end of a temperature gradient imposed upon a liquid system exhibiting convection cells becomes significantly greater than the boiling point of the liquid, then at least some of the system material will be transformed into high entropy, disordered gas.

Even in cases where such extreme disruptive influence is not envisaged, and one can unambiguously state that increased entropy flux and increased system structuring go together, as to whether one can state that the energy flux causes the appearance of the complexity must be open to question. We noted for the case of convection cells that the appearance of structure facilitates the flow of energy through the system. Let us recall Prigogine's diagrammatic representation of system functioning:



41. I. Prigogine in Irreversible Thermodynamics and the Origin of Life, Eds. G.F. Oster, I.L. Silver and C.A. Tobias, Gordon and Breach, New York, 1974.

We note that function and structure are shown as mutually interacting, structure being generated by and also facilitating function. We therefore cannot assert an unambiguous direction of causation, though it might well be that an initial increase of the energy flux through the system _initiated_ the generation of the new state of the system. But this initiation might alternatively have been the spontaneous generation of effective structure through a self-amplifying system fluctuation. So in the analysis of change in a dissipative system, the direction of causation between function and structure will be ambiguous, and the initiation of the change will depend upon the particular circumstances of the specific system.

With respect to the second question, that of the relationship between organisation and entropy, we should first note that the low entropy contribution of functional organisation might be very slight. For example, if our special refrigerator motor were allowed to melt until it formed an unstructured lump, and were then refrozen back to its normal working temperature, calorimetric measurement of its entropy change in freezing would probably give a value indistinguishable from that of its entropy change in melting. But as far as the refrigerator's function is concerned, the refrozen mixture is a disaster. It could not possibly allow the refrigerator to function as a self-maintaining system, as the essence of this function is the motor's structural organisation, not the motor's entropy.

Therefore, statistically expressed, the structure of the functional motor is unlikely, but not so unlikely as to be entropically significant. Calorimetric measurements of the entropic changes that occur when animals die and decompose have been made, though the results are difficult to assess because of the complicated chemical changes that take place. However, no entropy increase has been observed which can be unambiguously associated with the loss of functioning organic structure. Too close an identification of structure with low entropy may therefore be misleading, both analytically and in terms of the functioning of economies.

If physically defined entropy is unavailable to us as a significant explanatory variable in the analysis of dissipative systems, we must seek some surrogate function. Here, of course, we must move carefully to avoid tautology. We might suggest a variable measuring the "organisation" of a system which is defined to increase as the system's rate of dissipation increases. Using this measure, all speculations which equate the growing "organisation" of economies with greater demand for energy would then be necessarily true, but totally vacuous. Let us therefore proceed by closely examining certain concepts which we have so far used rather loosely, but which must henceforth be used with more discipline.

The concepts we shall examine are "complexity", "order", "structure" and "organisation". The <u>Concise Oxford Dictionary</u> gives the following meanings:

Complexity: "Consisting of parts; composite; complicated"

Order: "..... Sequence, Arrangement"

Structure: "Manner in which a building or organism or other complete whole is constructed, supporting framework of the essential parts of something, ..."

Organisation: "....., form into an organic whole (...); give orderly structure to, frame and put into working order, ..."

We see here a hierarchy of concepts. To say a system is "complex" is to say that it is composed of distinguishable components.

To assert that a system has "order" is to say that these components are arranged in some recognisable pattern.

The notion of "structure" is rather stronger than that of "order", implying some unity to the arrangement of components.

Finally, to say a system is "organised" implies that the system's "structure" is in some sense a realisation of inter-relations, perhaps a realisation in terms of function.

The distinction most frequently recognised in the literature is that between complexity and organisation, the consensus being that a system is organised if it is complex and there exist strong inter-relations between its parts. Needham[42] has further noted that entropy seems to be a quantitative expression of the concept of order, which is of course distinct from organisation. We say a system has high entropy not because it is "disorganised", but rather because it is "disordered", or to use Gibbs's term, because it is "mixed-up". In this context Needham has suggested that organisation requires a "patterned mixed-upness".

The notion that organisation requires function has also been expressed by several authors, Schlegel[43] seeing the concept of "organisation" as being normative and by implication teleological. But as has already been discussed, we need not be too upset by teleology.

If one recognises entropy as having some correspondence with the concept of "order", we must ask if it is possible to generalise the idea of entropy to the more embracing, and for our study more useful, concept of "organisation". Denbigh[44,45] suggests that this generalisation is possible, and for this wider application has coined the term "integrality".

Denbigh speculates that while entropy is the variable that is maximised,

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42. J. Needham, "Evolution and Thermodynamics: A Paradox with Social Significance", Science and Society 6, 1942, p. 352.

43. R. Schlegel, "Time and Entropy", in Time in Science and Philosophy, Ed. J. Zeman, Elsevier, London, 1975.

44. K.G. Denbigh, "A Non-Conserved Function for Organised Systems", in Entropy and Information, Eds. L. Kubat and J. Zeman, Elsevier, London, 1975.

45. K.G. Denbigh, An Inventive Universe, Hutchinson, London, 1975.

in the long run, in isolated systems, it is integrality which is maximised, in the long run, by open systems. In this speculation he was proceeded by Lotka[46].

Denbigh has noted that complexity is a necessary, though not a sufficient condition for organisation. Thus one element of the measure of the integrality of any system must be some measure of the number of its distinguishable components. The further condition for organisation is the "meaningful" or "operational" coordination of these components via identifiable inter-relationships. Therefore a further element of integrality is the number of these relations and a measure of their absolute and relative intensities.

As defined, integrality has only a weak theoretical foundation, especially in comparison with the formidable rigour to which the entropy concept is amenable. We should not let this deter us, though. We have seen that the entropy concept serves well at or near equilibrium, perhaps because this is the region of experimental consideration for which the variable was formulated. We have also seen that the entropy concept serves less well far from equilibrium, and it would seem to be on precisely this region that we should concentrate our analysis of economic structures and their use of energy. We shall therefore later devote considerable space to devising measures applicable to economic systems which adequately reflect Denbigh's notion of integrality, while attempting to avoid the trap of tautology already mentioned.

So far in this chapter we have dealt with the theory of self-organisation and dissipation in very general terms. Let us recall that in an earlier chapter we noted the hypothesis that the "organisation" of an economy was positively related to the energy dissipation by that economy. The obvious step now is to assert that such a relationship need not be

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46. A.J. Lotka, "The Law of Evolution as a Maximal Principle", Human Biology 17, 1943, p. 167.

seen as some special property of economies, but rather that the physical functioning of economies can be imbedded in the wider conceptual framework of dissipative systems. The plan of argument would then be:

(i) Demonstrate that one might anticipate the degree of "organisation" of a dissipative system to be positively correlated with its rate of entropy production (i.e. energy dissipation).

(ii) Argue plausibly that the physical aspects of economies can be regarded as those of self-organising dissipative systems, as discussed above.

(iii) Suggest meaningful measures of "organisation" and "dissipation" that not only reveal the nature of the interrelations within economies in a physical sense, but also are amenable to quantification using available economic statistics.

(iv) Test whether there is evidence to support the hypothesis that energy dissipation by economies increases with "organisation".

Elements (ii), (iii) and (iv) of this sequence are left to later chapters. Here, though, we can easily propose a model of self-organising systems with the required property.

First let us note that many self-organising systems dissipate energy merely in the process of keeping themselves warm, i.e. maintaining the conditions necessary for the functioning of the system. The economic corollary would be the use of space heating by the people in economies to keep themselves comfortable, and able to produce and consume. Such dissipation for the maintenance of the conditions for the system should be distinguished from the dissipation for the maintenance, growth and change of the system itself. Dissipation for the maintenance of conditions will be ignored in all subsequent discussions, where possible.

Let us consider a self-organising system in a steady state. That is, its degree of organisation is constant and its level of entropy production (energy dissipation) per unit time is constant. In as much as its internal state is constant, its entropy must be constant, i.e., $dS = 0$.

We recall that the entropy increase of an open system can be represented by:

$$dS = dS_i + dS_e$$

Here S_i and S_e are the internally and externally generated entropy contributions. We see that here;

$$dS_i = -dS_e$$

i.e. Entropy is being expelled into the environment at exactly the same rate as it is internally generated.

The relation between the level of organisation and energy dissipation (entropy expulsion) now becomes that between the level of organisation and the rate of internal entropy generation.

The internal generation of entropy by the system is symptomatic of the Second Law tendency of the system towards "disorder". Such disorder will of necessity destroy the "organisation" of the system, and expulsion of this entropy is therefore necessary for the self-maintenance of the system.

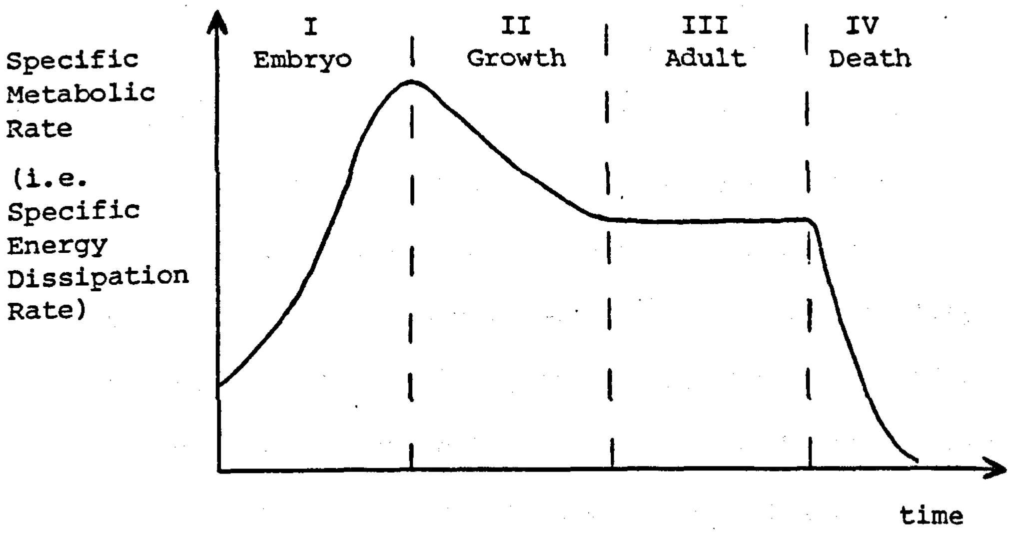
Obviously if the system grows by duplication, keeping a constant organisation per unit "size" (i.e. a constant "specific organisation") then the entropy dissipation will be simply multiplied up in proportion to size. Thus neglecting scale effects, a given specific organisation for a system will generate a constant corresponding specific entropy production rate.

The interesting case is where the "size" of the system is constant, but its degree of organisation increases. Now a reasonable supposition might be that as a system becomes gradually more organised, the nature of the processes leading to its internal decay will be approximately constant.

That is, the greater the degree of organisation of the system the greater is its tendency to "break down"; i.e. the greater is the internal rate of entropy production. Therefore, increasing the organisation of the system in a gradual fashion might be expected to produce an increase in the rate of internal entropy production; that is, entropy expulsion; that is, energy dissipation.

Evidence for this phenomenon has been offered by Trincher[11], in terms of the specific metabolic rates of developing chickens. His findings are schematically represented in Figure 2.

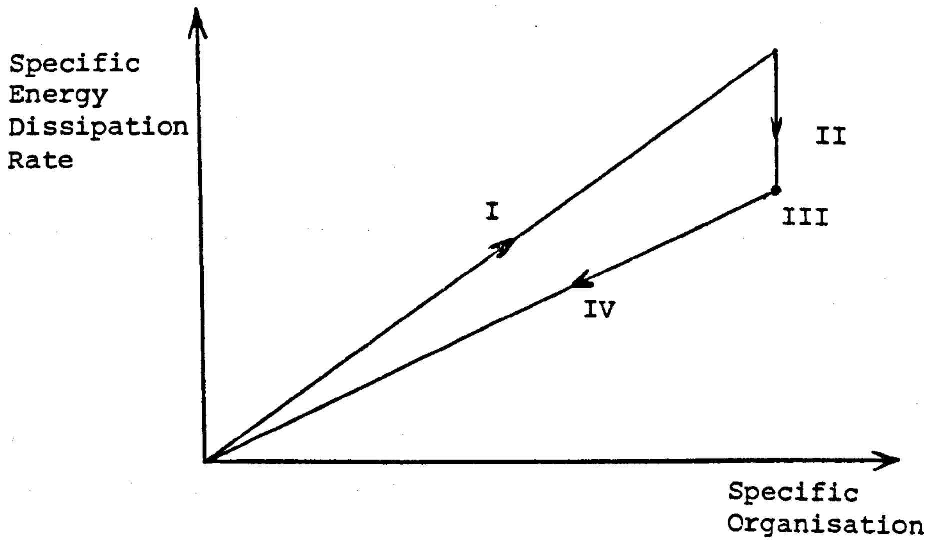
Figure 2



If we make the very reasonable assumption that specific organisation increase in Stage I, decreases in Stage IV and is constant in Stages II and III, we get the relationship shown in Figure 3.

One presumes that the decrease in specific energy dissipation in Stage II is largely a scale effect on the dissipation requirements to maintain a constant internal temperature, i.e. to maintain conditions. As such, Stage II is of little interest to us. What is important to note is that Stages I and IV both have positive slopes in Figure 3.

Figure 3



One should stress here that the changes in organisation that can be reasonably assumed in this model are that they are small and gradual. This is because one might envisage that once a system reaches a certain level of organisation the relations between its components might undergo a qualitative change, and improve the systems internal stability. This will lead to a reduced rate of internal entropy production and consequently a reduced rate of dissipation in the steady state. We might therefore anticipate that there are two competing effects taking place in a system undergoing self-organisation. The hypothesised effect of dissipation increasing with organisation, and an "efficiency" effect, where dissipation <u>decreases</u> with organisation.

CHAPTER SIX

Economic Paradigms and Physical Insights

In this chapter we shall examine how, and to what extent, economies can be viewed as self-organising dissipative systems. But before entering into a detailed discussion of this correspondence, we should first consider why anyone would wish to view the world in such a way. Further, we should examine whether such an approach is simply a modification of one of the prevailing modes of thought in economic analysis, or whether it involves a new way of looking at the nature of economic activity.

The motivation for this approach came through considerations of energy use by economies. Now, as will be discussed in more detail later, the four major "paradigms" of economics make no explicit mention of energy use, although each can be modified to cope with the observed use of energy by economies. In each case, though, energy use, or more accurately energy dissipation, is seen as a secondary element in economic functioning. Energy is simply another "consumption good", or another "factor of production"

When pressed, the use of energy in such paradigms can be expressed in more detail by invoking the Second Law of Thermodynamics, and its application to the productive process. This approach, though, is really answering the question "How do economies use energy?", and to a large extent this question has been answered through the detailed studies by energy analysts, as discussed in an earlier chapter. The question that motivates this work, though, is "Why do economies use energy?". That is:

> "What is the nature of the physical functioning of economies that requires the dissipation of energy?"

We see that the simple invocation of the Second Law does not answer this question, as the explanation of the productive process itself is not, under the normal paradigms, expressed in terms of physical functioning. "Economists" normally assert that production is for profit, or the satisfaction of producer and/or consumer wants, the explanation of production being in terms of individual or institutional behaviour within a social framework. The most celebrated analysis of this kind, which invokes the Second Law, is by Georgescu-Roegen[1]. He asserts that "purpose" and "enjoyment" are the motivating forces behind economic activity, viz:

> "Without the concepts of purposive activity and enjoyment of life we cannot be in the economic world. And neither of these concepts corresponds to an attribute of elementary matter or is expressible in terms of physical variables."[2]

In this chapter it will be argued that although the explicit avoidance of "purpose", "happiness" and other social concepts may take one outside "economics", it need not necessarily deter one from making possibly fruitful statements about "economies". In particular it will be argued that physical models of economies do allow us to answer the question "Why do economies use energy?".

As the distinction between the nature of "economies" and the subject matter of "economics" is crucial here, we shall start by examining "economics" as an intellectual pursuit.

We need not dwell here on the "definition" of economics; instead let us pass directly to a consideration of what economists *do*; that is, what, and how, they think.

Intuition would suggest that how an economist views the "economic world", that is, the area of economic theory and discourse, will depend upon what elements of that world he sees as productive of insight, and

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1. N. Georgescu-Roegen, The Entropy Law and the Economic Process, Harvard University Press, Cambridge, Mass., 1971.

2. *Ibid.* p. 282.

of novelty (in moderate quantities). That is, an economist's mode of thought will probably be structured in terms of economic "points at issue". The analysis of these "points at issue" will generate a vocabulary, both verbal and mathematical, and a clustering of interested scholars into a "school of thought".

Kuhn[3] has used such an analysis in his examination of scientific revolutions, where he postulates that in a revolution schools of thought supersede each other, following the alteration of the scientific points at issue.

Now here we need not pursue the issue as to whether economics is or is not a science. What is important is that the Kuhnian approach to "Science" has been immensely influential in the "Social Sciences" (see e.g. Mehta[4]). Briefly, Kuhn has proposed that most of the time scientists are engaged in "puzzle-solving" within a "paradigm". That is, they are trying to confirm what they already feel sure of, because their view of the world, their paradigm, indicates what structure should be used to find the answer. Occasionally circumstances force a new paradigm upon the scientific community, and after a period of initial re-alignment of ideas by the younger scientists, and the expiry of the older, the "normal science" activity of puzzle solving reasserts itself within the new paradigm.

Kuhn's approach to the methodology of science has been criticised, particularly by Lakatos[5]. Lakatos has suggested that a better approach to scientific activity is to be had by replacing Kuhn's "paradigms" with the notion of the "Methodology of a Scientific Research Programme" (MSRP),

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3. T.S. Kuhn, <u>The Structure of Scientific Revolutions</u>, University of Chicago Press, London, 1962.

4. G. Mehta, <u>The Structure of the Keynesian Revolution</u>, Martin Robertson, London, 1977.

5. I. Lakatos, "Falsification and the Methodology of Scientific Research Programmes", in <u>Criticism and the Growth of Knowledge</u>, Eds. I. Lakatos and A. Musgrave, Cambridge University Press, Cambridge, 1970.

arguing that an MSRP is followed, and should be followed, if it is fruitful of novelty and (potentially) open to falsification. The distinction between an MSRP and a paradigm seems to be that an MSRP stresses the normative appraisal of scientific activity, while a paradigm is more concerned with the positive aspect of the observed scientific attitudes of scientists. We can recognise this distinction, and continue to talk in terms of paradigms.

Many authors[6] have stressed that although physics and chemistry may progress by the process of paradigm replacement, in economics there seems to be a rule that "a new paradigm is a gained paradigm." That is, several paradigms, with their embodying schools of thought can exist and flourish contemporaneously.

Further, economic thought does not seem to progress linearly, but rather in a spiral. New economic world views often seem to embody one or more older views, with the work of Keynes being seen as foreshadowed by the Mercantilists, and by Marx[7], and Sraffa's[8] critique of Neo-classical analysis being most Ricardian in its approach.

It has been argued by Stigler[9] that, in essence, economics since Adam Smith had had only one paradigm, that of "constrained maximisation", and that in the various extant schools of economic thought we are seeing the exploration of themes upon this paradigm.

Masterman[10] has identified twenty-one distinct uses of the word "paradigm" by Kuhn, and among these are "a source of tools", and "a

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6. T.W. Hutchison, On Revolutions and Progress in Economic Knowledge, Cambridge University Press, Cambridge, 1978.

7. P. Mattick, Marx and Keynes, Merlin, London, 1974.

8. P. Sraffa, Production of Commodities by Means of Commodities, Cambridge University Press, Cambridge, 1960.

9. G.J. Stigler, "The Influence of Events and Policies on Economic Theory", in Essays in the History of Economics, University of Chicago Press, Chicago, 1965.

10. M. Masterman, "The Nature of a Paradigm", in Lakatos and Musgrave, op. cit.,

machine tool factory". In these senses constrained maximisation may pass as an overall paradigm. But in the weaker sense of "a whole tradition", and "a textbook or classic work" it will not suffice. Using this weaker sense, most economists recognise four economic paradigms, these "modes of thought" having their corresponding "points at issue".

The oldest is the Classical paradigm, mainly due to Adam Smith, Ricardo and Malthus, where the point at issue seems to be the role of the market in long term growth of national economies.

The Marxian paradigm concentrates upon the social reaction to technical change in the context of the institutions of ownership, and the resultant exploitation of workers by capitalists.

The Neo-classical paradigm takes the problems of exchange, efficiency and rational behaviour as central.

The point at issue for the Keynesian paradigm is permanent unemployment.

Each of these paradigms is still with us in the current economic literature. As Georgescu-Roegen[11] has pointed out, none of these paradigms explicitly account for the use of raw materials. In particular none offer an integrated "way of seeing" the physical dissipation of energy by economies as a matter of consequence, or indeed of seeing it at all. This should not surpise us, for two reasons.

First, the easily observed aspects of economic activity are those of interpersonal interaction and physical transformation. As physicists, chemists, biologists and engineers all claim physical transformation as within their writ, economists have not unnaturally concentrated upon the aspects of personal and institutional behaviour in their theorising, taking physical transformation as either given or irrelevant.

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11. N. Georgescu-Roegen, "Energy and Economic Myths", Southern Economic Journal 41, 1975, p. 347.

Second, it is the intellectual division of labour of defining physics as distinct from chemistry, biology as distinct from geology, and even economics as distinct from politics, that allows the specialisation likely to produce prolific research. Further, the success of such specialisation will surely tend to strengthen the divisions between areas of discourse.

Thus once economics defines itself as a subject exterior to most physical phenomena then it is unlikely of itself to reintroduce physical considerations into its "modes of thought" or paradigms.

Recently the general unease over energy supplies has stimulated some research into the role of energy in economies. Much of this has been undertaken by non-economists, and as discussed in an earlier chapter, many economists have found this intrusion unwelcome. Some have reacted more positively, actively enquiring into the role of energy in production via expanded production functions, or at a more aggregated level through elasticities of energy "consumption" with price and national income.[12] But we should note that even when involved in such research, the "world view" of these economists still seems to be in terms of "social" phenomena.

Economics, as an intellectual discipline has been moderately successful in giving insights into certain aspects of the world. Relations between individuals, institutions and classes are all explored in one or more of the four paradigms, and although these explorations have been relatively unproductive in terms of accurate prediction, they have provided some conceptual coherence for the analysis of society's behaviour, as well as guides for the formulation of government policy. That is, as an intellectual and practical scheme economics has had some success.

However, let us recall that economics is defined not only by its subject matter, viz. "economies", but also by its conceptual apparatus, its paradigms. What is suggested here is that "economies" can also be

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12. W.D. Nordhaus (ed.), International Studies in the Demand for Energy, North Holland, Oxford, 1977.

viewed from a framework other than these paradigms. As Georgescu-Roegen noted, this is then not "economics", but we should not let this deter us. This route had already been partially traversed by modern energy analysts, whose main premise seems to be that energy is not just another "good", but is somehow centrally important to the understanding of how economies function. For their part, from within their paradigms, economists are equally correct to reject this dogma. The role of energy, and the entropy concept, do not appear and <u>need</u> not appear within these paradigms.

In the light of the above remarks it is evident that what is being presented here is not "economics", as presently understood, though that is not to say that it may not be of interest to some economists. What is being presented is an alternative perspective on how economies function, where individual and social behaviour are explicitly excluded, the entire focus being upon the physical nature of economic activity.

In particular the case argued for here is that aspects of the physical processes of economic activity can be imbedded into the theory of dissipative structures. The case is not being made for an analogy between economies and physical systems; i.e. an economy is "like" a hive of bees, a balanced system of weights, a set of interconnected tubes. Instead it is suggested that an economy is, when viewed from a certain perspective, the "same sort of thing" as an organism, a flame or a convection cell. It is felt that only through such a physical approach can one hope to deal with the point at issue of "Why do economies use energy?". Whether there is evidence to support the validity of this perspective will be examined in a later chapter. Prigogine has certainly made it clear that he feels economies <u>can</u> be viewed in such a way.

> "An appropriate example (of a dissipative structure) would be a town that can only survive as long as it

> is a centre of inflow of food, fuel and other commodities and sends out products and wastes."[13]

> "While sociocultural evolution has, of course, its own very specific characteristics, the recent advances of the natural and mathematical sciences points to the view that it is one of the many aspects of the evolution of our physical universe, in which nonlinear processes and nonequilibrium conditions play a significant role in self-organization."[14]

We now proceed to the second part of the programme outlined in the previous chapter. That is, to argue plausibly that the physical aspects of economies can be regarded as those of self-organising dissipative structures. A full argument would include three steps:

1. An enumeration of the properties of general dissipative structures.
2. A demonstration that economies possess these properties.
3. A description of the coming about of these properties in economies in terms of appropriate "primitive" physical relationships (e.g. convection, chemical interaction, fluctuation, etc.).

Step 1 has been attempted in the previous chapter, though as noted there the theory is still far from complete.

Step 3 is conceptually and technically extremely difficult, and will not be attempted. The problem is that, as noted earlier, self-organisation seems likely to proceed by progression up a hierarchy, lower "elements" interacting to give rise to the "emergence" of higher elements. At present only the elements on the "bottom rung" are accessible

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13. G. Nicolis and I. Prigogine, Self-Organisation in Non-Equilibirum Systems, Wiley, New York, 1977, p. 4.

14. I. Prigogine, P.M. Allen and R. Herman, "Long Term Trends and the Evolution of Complexity", in Goals in a Global Community, Eds. E. Laszlo and J. Bierman, Pergamon, New York, 1977, p. 60.

to physical theory. The complete description of the emergence of even simple organisms is still not available, and the emergence of "social organisms" is at present only accessible through descriptions utilising social variables, which here are eschewed. We are unable therefore at present to describe suitably the coming about of self-organisation and dissipation in economies. To reject this approach, however because such a description is unavailable would seem unnecessarily harsh. If such a criterion had been applied to biology even recently then it too would have been considered to have failed as a soundly based and coherent discipline. That such a criterion was not applied is perhaps due to the tremendous overlap of the physical and biological paradigms. Most "scientists", though not all, feel sure that biology is potentially describeable in physical terms.

The remaining step, Step 2, now requires us to demonstrate that economies possess the properties of self-organising dissipative systems.

Dissipation is the simplest property to begin with. Casual observation shows us that production dissipates energy. In Adam Smith's pin factory the cutting, the sharpening, even the moving around of the pins, all cause energy to be dissipated.

But dissipation, as described in this way, is simply the effect of activity. This is the view of the physical functioning of economies "from the inside". A fuller understanding of economies from a physical viewpoint is, perhaps, to be had "from the outside". Let us therefore take a "Martian's-eye view" of the Earth's surface.

We shall suppose that the Martian can only see the atmosphere and the surface of the Earth, and these only remotely. With respect to the atmosphere, it will notice temperature, velocity and humidity fluctuations and relationships. If its physics is no less advanced than our own it might conclude that these quasi-cyclic phenomena are generated by the energy flux from the sun, and might, propose to itself that they are understandable as self-maintaining dissipative systems. If it supposed that these systems were "not too far" from equilibrium then it might think that these atmospheric phenomena were predictable by invoking the Martian equivalent of Prigogine's Minimum Dissipation Law.

Looking more closely, at the surface of the Earth, the Martian would notice that large parts of the globe were covered in a thin layer of green substance, which expanded and contracted slightly with the seasons. Spectrographic analysis of the atmosphere would indicate that oxygen is emitted by this substance during the daytime while carbon dioxide is absorbed, the reverse process taking place at night.

The physical reasoning the Martian might use is that the green material constitutes dissipative systems, which absorb low entropy sunlight and structure themselves by using the carbon from atmospheric carbon dioxide.

A more detailed study would show another remarkable phenomenon. Starting from localised regions, all over the planet, the green material was in places being altered, in other places disappearing altogether. The Martian would also note the appearance of structures, some of which generated further structures. It would notice linking structures appearing between the localised regions of alteration. Infra-red surveillance would also reveal that all this activity was accompanied by massive dissipation of energy. Supposing Martians are very dissimilar from Earthmen, so that their Areocentricity does not find outlet in supposing these physical phenomena reflect social behaviour, the Martian might well conclude that again it is observing self-organising dissipative processes.

That is, we assume the Martian has no knowledge of our social institutions, modes of interaction, or even our size and shape. But direct observation of man's artefacts, their interaction, and their dissipative nature would still be accessible to the Martian, and provide some information as to the nature of the activities on the Earth's surface, in the framework of physical, rather than social, functioning.

The model suggested here is that the perceived self-organisation and dissipation of man's economic activity has, at root, the same nature as the perceived behaviour of the atmosphere and the biosphere, the only difference being that the latter two are directly driven by solar energy.

In the previous chapter, for the example of the special refrigerator it was noted that although the motor had low entropy, and was also organised, entropy and organisation are not the same concept. It was necessary for the motor to have low entropy for it to be organised, but it was by no means sufficient. We should explore the relationship of entropy and organisation for economies.

Now if an economy is functioning, and producing the things economies produce, it will do so with organised structures. In Lokta's[15] terminology these are man's "exosmatic instruments". In usual economists' language they constitute capital equipment. These organised structures will invariably have been constructed by the transformation of high entropy ore into low entropy metals, ceramics, etc. We can therefore suggest that the physical objects constituting an economy will have lower (specific) entropies than their surrounding material environment, i.e. they will be far from equilibrium, as we would anticipate for the elements of a self-organising system.

If we accept that capital equipment has low (specific) entropy then one might posit that increasing the capital stock will reduce the overall entropy of the economy, by replacing high entropy ore with low entropy machines, while the functioning of these machines will increase the rate of

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15. A.J. Lotka, Elements of Physical Biology, Baltimore, 1925.

dissipation by the economy. This relation has been confirmed by Frank[16], who has analysed time series data for the USA (1880-1948) and the UK (1865-1914), and found a remarkably close correlation between capital stock and energy use (R^2 = 0.9990 for the USA and R^2 = 0.9787 for the UK). These impressive figures have brought the comment from Landes[17] that "... the story of power is the story of industrialisation".

But objects do not constitute an economy, for an economy is not so much a set of objects as a function, which presupposes organisation. A system with objects but without organisation is no more an economy than is a fish-finger a fish. We should therefore not seek to identify the physical functioning of an economy with the fact of the low entropy of its constituent objects; we must also include the functional interrelations between the objects, that is, the economy's organisation.

In summary, economies are dissipative in nature, and need organisation as well as low entropy for their functioning. That economies can generate new structure through interaction with their environment is also evident when we take a Martian's-eye view. Therefore, if we discard social descriptions and make the change of perspective to that of viewing economies from the "outside", then the argument that economies can be viewed as self-organising dissipative systems does not seem ridiculous. If economies are viewed in this way then the "point at issue" embodied in the question can be recognised as meaningful and answerable within such a conceptual framework.

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16. A.G. Frank, "Industrial Capital Stocks and Energy Consumption", Economic Journal 69, 1959, p. 170.

17. D.S. Landes, The Unbound Prometheus, Cambridge University Press, Cambridge, 1970, p. 293.

CHAPTER SEVEN

Energy Intensities and the Energy Coefficient

In earlier chapters we have examined the relationship between thermodynamics and economic analysis, the entropy concept, with its application to social and biological studies, and particularly discussed the more general concept of "organisation". The previous chapter proposed a physical perspective on the functioning of economies.

We shall now attempt to confront the substantive issue in this inquiry, the analysis of economic systems as complex dissipative structures. As the techniques for such an analysis are not in common use we shall spend this chapter in formulating and describing them. Let us begin by discussing the energy used by economies.

One aspect of the production of goods and services by economies is the concommitant use of energy that this production entails. In recent years there has been a good deal of work done into finding out how the energy that is used by an economy can be "charged" to the goods produced. The motivation for these studies has been largely practical, and received a special impetus from the alarm felt in the industrial world over the oil price rises of 1973. However, in the context of our inquiry into the thermodynamic nature of dissipative structures, such as economies, it will be seen that the energy "costing" of goods is also of great theoretical interest. This is because such costing gives insights into the relationship between energy dissipation and the degree of interrelationship between the various producing sectors in the economy. That is, between the way energy is dissipated by an economy and its "organisation".

The energy dissipated by an economy in the production of a certain good (or service) can be charged to a unit of a physical property of that good (e.g. tonnes, litres, etc.) or to the monetary unit involved in its exchange (e.g. £, \$, etc.). If a monetary unit is used, as will be the case in this section, then we talk of the "Energy Intensity" of that good, in units of kWh/£ (say).

To analyse the various methods used to derive energy intensities we shall use two tools; the Mathematical tool of Matrix Algebra and the Economic tool of Input-Output (I-O) Analysis.

The notation we shall use in the matrix algebra is as follows:

Matrices are represented by upper case Roman letters, underlined e.g. $\underline{A}$, $\underline{Q}$.

Vectors are represented by lower case Roman letters, underlined. e.g. $\underline{x}$, $\underline{e}$.

Elements of matrices and vectors are represented by their corresponding lower case letter, with one subscript for a vector and two subscripts for a matrix, e.g. a_{ij}, q_{ij}, x_i, e_j.

The Identity matrix will be represented by $\underline{I}$, where:

$$i_{jj} = 1$$

$$i_{jk} = 0 \qquad j \neq k$$

$\hat{\underline{s}}$ represents the matrix with the vector $\underline{s}$ on its diagonal, zeros elsewhere, i.e.:

$$\hat{s}_{ii} = s_i$$

$$\hat{s}_{ij} = 0 \qquad i \neq j$$

Input-Output theory was introduced by Wassily Leontieff,[1,2]

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1. W. Leontieff, The Structure of the American Economy, 1919-1939, Oxford University Press, London 1951.

2. W. Leontieff, Input-Output Economics, Oxford University Press, London, 1966.

and since its inception has been recognised as a powerful tool for the unravelling of the complex interrelationships within real economies, as opposed to their simpler theoretical siblings.

The essence of the I-O method is the abstracting, from the complexity of economic production and consumption, of a finite (and usually small) number of interrelationships between certain "sectors".

A relationship is constituted between two sectors when an Output of one sector is an Input of the other. This tells us that three sorts of sector can be involved in an economy:

(1) Sectors which absorb inputs but produce no outputs.

(2) Sectors which produce outputs but absorb no inputs.

(3) Sectors which absorb inputs and produce outputs.

Sectors of the first sort are the eventual repository of all the outputs of the economy, and are therefore known as "Final Demand" sectors. The various types of domestic consumption by the man in the street are of this sort, as are exports.

Sectors of the second type contribute outputs, but as there are no *corresponding inputs* to these sectors, the production of the outputs is exogenous to the particular physical economic structure, and they are therefore known as non-produced outputs. When these outputs are absorbed as inputs by other sectors they are renamed non-produced inputs. Such non-produced inputs are also known as "Value Added". Value added can usually be classified under one or other of the headings Land, Labour and Capital.

Sectors of the third sort can be called "Producing Sectors", their inputs being used to produce their outputs. Agriculture, the steel industry and the advertising industry are all examples of such sectors.

Let us suppose that we have a particularly simple economy, with one final demand sector, one value added sector and n producing sectors.

Let the outputs to final demand by the n producing sectors be y_i (i=1,n), the value added inputs to the producing sectors be v_i and the output of producing sector i absorbed as an input by producing sector j be x_{ij}.

We can now calculate the total output by producing sector i. This will be $\sum_j x_{ij} + y_i$. That part of total output which does not go to final demand (i.e. $\sum_j x_{ij}$) is usually called "Intermediate Demand". The total input to producing sector i is $\sum_j x_{ji} + v_i$. If we impose the condition that, within each producing sector, no input or output can be unaccounted for, then the total output of each sector must equal its total input. This amount we call x_i. i.e.

$$x_i = \sum_j x_{ij} + y_i \qquad \text{Output Equation} \qquad (1)$$

and

$$x_i = \sum_j x_{ji} + v_i \qquad \text{Input Equation} \qquad (2)$$

These equations are customarily displayed for any selected economy by means of an I-O table. i.e.:

Table 1

		OUTPUT						
		Intermediate Demand x_{ij}					Final Demand	Total Demand
		1	2	3	n		y_i	x_i
INPUT	1	x_{11}	x_{12}	x_{13}	x_{1n}		y_1	x_1
	2	x_{21}	x_{22}	x_{23}	x_{2n}		y_2	x_2
	3	x_{31}	x_{32}	x_{33}	x_{3n}		y_3	x_3
	.	.			.		.	.
	.	.			.		.	.
	n	x_{n1}	x_{n2}	x_{n3}	x_{nn}		y_n	x_n
Value Added	v_i	v_1	v_2	v_3	v_n		-	-
Total Demand	x_i	x_1	x_2	x_3	x_n		-	-

The formulation of such an I-O table requires that the inputs and outputs be in compatible units, and to enforce this condition I-O tables are usually constructed in price units (£).

The power of I-O theory lies, though, not in this simple preamble, but in its descriptive conciseness due to the assumption of a linear relationship between the several distinct inputs to a sector and the total output of that sector. The reasoning behind this assumption runs as follows:

> "If the motor car industry produces 1,000 cars per year with inputs of 1,000 tons of steel, 10,000 tons of oil and 50,000 man-hours of labour, then the production of 1,100 cars per year will require inputs of 1,100 tons of steel, 11,000 tons of oil and 55,000 man-hours of labour".

That is, economies and diseconomies of scale with respect to marginal changes are ignored. This is a reasonable approach, as long as the changes under consideration in the real world are marginal.

Using this assumption we can write:

$$x_{ij} = a_{ij}\, x_j$$

The a_{ij} are known as the Technical Coefficients for the economy.

Using this assumption our output equation can be rewritten as:

$$x_i = \sum_j a_{ij}x_j + y_i$$

Or in matrix notation we have:

$$\underline{x} = \underline{A}\,\underline{x} + \underline{y} \qquad (3)$$

Solving for $\underline{x}$ gives us:

$$\underline{x} = (\underline{I}-\underline{A})^{-1}\underline{y} \qquad (4)$$

This equation relates the structure of the total output of the economy ($\underline{x}$) to the structure of output to final demand ($\underline{y}$) and to the technical relations of production ($\underline{A}$).

The matrix $(\underline{I}-\underline{A})^{-1}$ is known as the Leontieff Inverse, and is of fundamental importance in the analysis that follows.

Let us now introduce energy dissipation into this I-O methodology. Suppose our economy uses E (kWh) of energy in a given year. This energy will mostly be dissipated, due to the production of entropy in the irreversible manufacturing processes that furnish the economy. Very little energy will be stored.

We should distinguish between two sorts of energy dissipation which take place in the economy.

First, there is the direct consumption of energy by final demand as a good itself. This will correspond to the petrol used by private motorists, the oil, gas, coal and electricity used for home heating and the operation of domestic appliances, etc. We shall call this quantity E_{dom} ("dom" for "domestic").

Second, and for us more important, there is the energy dissipated by manufacturing industry in the production of goods and services. This quantity we shall call E_{ind} ("ind" for "Industry"). We can obviously combine these two quantities to give:

$$E = E_{dom} + E_{ind} \qquad (5)$$

E_{ind} can be further subdivided into the energy dissipated by each of the industrial sectors. We shall call these quantities e_i $(i = 1,n)$. i.e.:

$$\sum_i e_i = E_{ind} \qquad (6)$$

Within each sector energy is dissipated for the production of outputs to other sectors (e_{ij}) and for the production of output to final demand (e_i^Y). So for each industrial sector we can represent the structure of its energy dissipation by:

$$e_i = \sum_j e_{ij} + e_i^Y$$

This is of the same form as the earlier monetary I-O output equation, and we shall similarly assume that there is a linear relationship between the energy dissipated to produce the total output from a sector, and the energy dissipated to produce the inputs to the sector, at least for marginal changes in the total output. i.e. We can write:

$$e_{ij} = q_{ij}\, e_j$$

So we get:

$$e_i = \sum_j q_{ij}\, e_j + e_i^y$$

Or in matrix notation:

$$\underline{e} = \underline{Q}\,\underline{e} + \underline{e}^y \qquad (7)$$

i.e.

$$\underline{e} = (\underline{I}-\underline{Q})^{-1}\underline{e}^y \qquad (8)$$

Equations (4) and (8) between them give us a complete description of the market transactions within an economy and the concommitant energy dissipation which takes place. Using these equations we can now proceed to a discussion of energy intensities.

We define the energy intensity for the output of sector i as k_i. But for the intensity to be well defined we must also know to which part of the output of sector i it refers. Further, if we suppose that this output is some quantity q_i, then we must ensure that for the whole economy the energy intensities we are using account for all industrial energy dissipation, but no more than this. i.e.:

$$\sum_i k_i\, q_i = E_{ind}$$

Or in matrix notation:

$$\underline{k}\,\underline{q} = E_{ind}$$

So before we can set about defining the energy intensities, k_i, we must first decide what the output q_i is to be. There are two obvious choices available. First there is total output (x_i), and second there is output to final demand (y_i). So we may define two energy intensities by:

$$\underline{c}\ \underline{x} = E_{ind} \tag{9}$$

and

$$\underline{c}^*\underline{y} = E_{ind} \tag{10}$$

We can see that $\underline{c}^*$ can be derived from $\underline{c}$ by recalling that:

$$\underline{x} = (\underline{I}-\underline{A})^{-1}\underline{y}$$

so

$$\underline{c}\ \underline{x} = \underline{c}\ (\underline{I}-\underline{A})^{-1}\underline{y}$$

$$= E_{ind}$$

So we see that $\underline{c}\ (\underline{I}-\underline{A})^{-1}$ will serve as $\underline{c}^*$. This means that we can define a linear mapping from a vector of energy intensities referring to total output to one referring to output to final demand. But there still remains the definition of $\underline{c}$. The elements of the vector $\underline{c}$ must have units kWh/£, so we can construct $\underline{c}$ from any vector of energies $\underline{f}$, where $\sum_i f_i = E_{ind}$, by simply setting $c_i = f_i/x_i$. The condition on $\underline{c}$ of $\underline{c}\ \underline{x} = E_{ind}$ is then automatically satisfied. But what should we choose for f_i? Our energy dissipation scheme allows us two approaches.

First, for f_i we can use e_i, the energy dissipated in production by sector i.

A second, and less obvious possibility, is the energy dissipated in production <u>for</u> sector i, where we define this as the energy dissipated in producing inputs to sector i (e_{ji}) plus the energy dissipated by sector i in production for final demand (e_i^y).

These two approaches give:

$$f_i = e_i$$

and

$$f_i' = \sum_j e_{ji} + e_i^y \tag{11}$$

But how do we decide which gives a more reasonable and useful energy intensity? We shall decide this by supposing that there is a simple linear relationship between the structure of the energy dissipation taking place within the economy, and the market transactions which occur. In particular, we shall suppose we have a single "fuel" sector, sector g (gas) say. This assumption is not a restrictive one, as the analysis that follows can be easily extended to allow for multiple fuel sectors, as well as permitting secondary fuel sectors such as electricity generation.

For a single fuel sector (g), the total value of the output of this sector in one year (x_g) will correspond to the total quantity of fuel dissipated by the economy in that year (E). In the particular case of there being a linear relation between the physical and financial aspects of the economy, the fuel sold by sector g will be at the same price for all purchasers, this price being E/x_g (kWh/£). That is, fuel pricing will be non-differential.

The quantity of energy dissipated by sector i (e_i) will simply be the quantity of the fuel purchased, which gives us:

$$e_i = (E/x_g)\ x_{gi}$$

Similarly, under the assumption of the linear relationship, the amount of energy dissipated by sector i in producing output to sector j is given by:

$$e_{ij} = e_i\ (x_{ij}/x_i)$$

$$= (E/x_g)\ x_{gi}\ (x_{ij}/x_i)$$

Finally, the energy dissipated by sector i in production for final demand is:

$$e_i^y = e_i\ (y_i/x_i)$$

$$= (E/x_g)\ x_{gi}\ (y_i/x_i)$$

We can substitute these values in f_i and f'_i, which give us the two energy intensities as:

$$c_i = f_i/x_i$$

$$= (E/x_g)\ x_{gi}\ x_i$$

$$= (E/x_g)\ a_{gi}$$

And similarly:

$$c'_i = f'_i/x_i$$

$$= (\sum_j e_{ji} + e_i^y)/x_i$$

$$= (E/x_g)\ x_{gi}\ (\sum_j (x_{ji}/x_i) + y_i/x_i)/x_i$$

$$= (E/x_g)\ a_{gi}\ (\sum_j a_{ji} + y_i/x_i)$$

We see that while $\underline{c}$ depends only upon the technical coefficients, and therefore only upon the interrelations of the producing sectors of the economy, $\underline{c}'$ also depends upon the structure of the output to final demand. As final demand is here regarded as exogenous to the structural economic relations, $\underline{c}'$ must be regarded as the less satisfactory measure. The energy intensity vector $\underline{c}$ is often seen in the literature, and has been tabulated for selected manufacturing industries by Chesshire and Buckley[3] for the U.K. and Makhijani and Lichtenberg[4] for the U.S.A.

Now without invoking non-differential fuel pricing we can represent $\underline{c}$ by:

$$c_i = e_i/x_i$$

or $$\underline{c} = \underline{e}\ \hat{\underline{x}}^{-1}$$

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3. J. Chesshire and C. Buckley, "Energy Use in U.K. Industry", Energy Policy, Sept. 1976, p.237.

4. A.B. Makhijani and A.J. Lichtenberg, "Energy and Well Being", Environment 14 (5), 1972, p.10.

So for $\underline{c}^*$ this gives us:

$$\underline{c}^* = \underline{c}\ (\underline{I}-\underline{A})^{-1}$$

$$= \underline{e}\ \hat{\underline{x}}^{-1}\ (\underline{I}-\underline{A})^{-1}$$

If we do invoke the simplifying condition of non-differential fuel pricing then, as above:

$$c_i = (E/x_g)\ a_{gi}$$

To express this in vector notation we can follow either of two courses. Both will be useful later, so we shall examine them in turn.

The first approach is to define a vector $\underline{b}^i$, which has 1 in the i th position and zeros elsewhere. i.e.:

$$b^i_i = 1$$

$$b^i_j = 0 \qquad i \neq j$$

When a matrix is left-multiplied by b_i the i th row of the matrix is left as a vector. Using $\underline{b}^g$ we can represent $\underline{c}$ by:

$$\underline{c} = (E/x_g)\ \underline{b}^g\ \underline{A} \tag{12}$$

The second approach is to define the matrix $\underline{A}^i$, which has the i th row of matrix $\underline{A}$ on its diagonal, zeros elsewhere. i.e.:

$$a^i_{jj} = a_{ij}$$

$$a^i_{jk} = 0 \qquad j \neq k$$

We also define the vector $\underline{u}$, which has every element equal to 1. i.e.:

$$u_i = 1 \qquad \forall\ i$$

Left-multiplication of a matrix by $\underline{u}$ gives a vector of the column sums of that matrix. Using $\underline{A}^g$ and $\underline{u}$ we can write:

$$\underline{c} = (E/x_g)\ \underline{u}\ \underline{A}^g \tag{13}$$

So our two representations of $\underline{c}$ lead us to the following two ways of expressing $\underline{c}^*$, under the assumption of non-differential fuel pricing.

$$\underline{c}^* = (E/x_g)\ \underline{b}^g\ \underline{A}\ (\underline{I}-\underline{A})^{-1} \tag{14}$$

and
$$\underline{c}^* = (E/x_g)\ \underline{u}\ \underline{A}^g\ (\underline{I}-\underline{A})^{-1} \tag{15}$$

Let us note that $\underline{c}^*$ is defined by using the matrix $(\underline{I}-\underline{A})^{-1}$, and this matrix can be expanded as follows:

$$(\underline{I}-\underline{A})^{-1} = \underline{I} + \underline{A} + \underline{A}^2 + \underline{A}^3 + \ldots$$

A sufficient condition for the convergence of this series is that the eigenvalues of $\underline{A}, \lambda_i$, satisfy $0<|\lambda_i|<1$. This condition will be satisfied if $0<a_{ij}<1$ and $0<\sum_i a_{ij}<1\ \forall\ i,j$ (see Bellman)[5]. In economics texts this is often referred to as the Hawkins-Simon[6] condition. From the definition of a_{ij} we see that this will always be the case, so the series will always converge. Using the expanded version of $(\underline{I}-\underline{A})^{-1}$ we can write $\underline{c}^*$ as:

$$\underline{c}^* = \underline{c} + \underline{c}\ \underline{A} + \underline{c}\ \underline{A}^2 + \underline{c}\ \underline{A}^3 + \ldots$$

Now as $a_{ij} \geqslant 0\ \forall i,j$ we see that $c^*_i \geqslant c_i\ \forall i$. This result is intuitively obvious, and can be expressed by saying that $\underline{c}$ is the vector of direct energy requirements per £ of output, and $\underline{c}^*$ is the vector of direct plus indirect energy requirements per £ of output.

This distinction between $\underline{c}$ and $\underline{c}^*$ has been mentioned by Chapman[7], Slesser[8] and Hannon[9], though they did not use I-O terminology, preferring instead to talk of "systems" and "system boundaries". Chapman[10] makes

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5. R. Bellman, Matrix Analysis, McGraw-Hill, New York, 1961.
6. D. Hawkins and H.A. Simon, "Note: Some Conditions of Macroeconomic Stability", Econometrica 17, 1949, p.245.
7. P.F. Chapman, "The Energy Costs of Materials", Energy Policy, March 1975, p.245.
8. M. Slesser, "Accounting for Energy", Nature 254, 1975, p.170.
9. B. Hannon, "Bottles, Cans, Energy", Environment 14, (2), 1972, p.170
10. P.F. Chapman, "Energy Costs: A Review of Methods", Energy Policy, June 1974, p.91.

this point for a bakery, where he notes that the energy intensity of the bread increases as more previously external factors are included within the "baking system". The initial position of only considering direct fuel inputs gives a "baking" (b) energy intensity of c_b, the final position where all external features are included within the system boundary gives c_b^*, and intermediate levels of inclusion of externalities gives c_b^I, where $c_b < c_b^I < c_b^*$.

In the literature one quite often sees $\underline{c}^*$ approximated by the finite series $\underline{c}\ (\underline{I} + \underline{A} + \underline{A}^2 + \ldots + \underline{A}^n)$, though this formulation is usually not explicit. This method is normally applied to only one or two sectors at a time, and as it concentrates on the technical details of production it is usually referred to as "Process Analysis". Examples of such an approach are to be found in the work of Chapman[11,12], Leach and Slesser[13], and Berry, Long and Makino[14], for metals and chemicals, Barnes and Rankin[15], Gartner and Smith[16], and Brown and Stellon[17] for housing, and Berry and Fels[18] for car manufacture.

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11. P.F. Chapman, "The Energy Costs of Producing Copper and Aluminium from Primary Sources", Metals and Materials, Feb. 1974, p.107.

12. P.F. Chapman, "The Energy Costs of Producing Copper and Aluminium from Secondary Sources", Research Report ERG002, Open University, 1972.

13. G. Leach and M. Slesser, Energy Equivalent of Network Inputs, Strathclyde University, 1972.

14. R.S. Berry, T.V. Long and H. Makino, "An International Comparison of Polymers and their Alternatives", Energy Policy, June 1975, p.144.

15. D. Barnes and L. Rankin, "The Energy Economics of Building Construction", Building International 8, 1975, p.31.

16. E.M. Gartner and M.A. Smith, "Energy Costs of Home Production", Energy Policy, June 1976, p.144.

17. G. Brown and P. Stellon, "The Material Account", Built Environment Aug. 1974, p.415.

18. R.S. Berry and M.F. Fels, "The Energy Cost of Automobiles", Science and Public Affairs Bulletin of Atomic Scientists, Dec. 1973, p.11.

We shall now turn to the examination of some energy intensities arrived at by other techniques, and compare them with $\underline{c}$ and $\underline{c}^*$.

One method that might prove fruitful, but does not yet seem to have been fully implemented, involves the use of the $(\underline{I}-\underline{Q})^{-1}$ matrix of eqn.8. The use of this matrix only presupposes that the physical aspects of production are linearly interrelated, but does not invoke either non-differential fuel pricing or a linear relationship between the financial and physical aspects of the economy.

The $(\underline{I}-\underline{Q})^{-1}$ matrix relates the vector of total energy dissipation by the economy ($\underline{e}$) to the vector of energy dissipation required for the production of output to final demand ($\underline{e}^y$). Therefore the column sums of $(\underline{I}-\underline{Q})^{-1}$ will give a vector of the total direct plus indirect energy dissipation required per unit of direct energy dissipation to produce output to final demand. This vector ($\underline{c}^e$) will have units of kWh/kWh, and behaves in an analogous manner to $\underline{c}^*$, in that $\underline{c}^*\underline{y} = E_{ind}$ and $\underline{c}^e\ \underline{e}^y = E_{ind}$. We shall examine $\underline{c}^e$ in more detail later, with special attention to its use as a tool for currency free inter-economy comparison.

To convert the vector $\underline{c}^e$ to a measure for the total energy dissipation requirements per monetary unit of final demand (an energy intensity) we need to multiply each element c_i^e by the corresponding $r_i = e_i^y/y_i$. Using the vector $\underline{u}$ defined above we see that this approach gives the vector of energy intensities as:

$$\underline{c}^q = \underline{u}\ (\underline{I}-\underline{Q})^{-1}\ \hat{\underline{r}} \qquad (16)$$

To compare $\underline{c}^q$ with $\underline{c}^*$ we shall have to assume that the physical system is linearly related to the financial system, when we have:

$$r_i = e_i/x_i$$

$$= (E/x_g)\ a_{gi}$$

i.e.: $$\hat{\underline{r}} = (E/x_g)\ \underline{A}^g$$

This assumption also implies that $\underline{Q}$ has elements:

$$\begin{aligned} q_{ij} &= e_{ij}/e_j \\ &= ((x_{ij}/x_i)E(x_{gi}/x_g))/((x_{gj}/x_g)E) \\ &= (x_{ij}x_{gi})/(x_i x_{gj}) \\ &= (a_{ij}a_{gi})/a_{gj} \end{aligned}$$

In matrix notation this becomes:

$$\underline{Q} = \underline{A}^g \ \underline{A} \ \underline{A}^{g-1} \qquad (\underline{A}^{g-1} \text{ is } (\underline{A}^g)^{-1})$$

So we see that:

$$\begin{aligned} (\underline{I}-\underline{Q})^{-1} &= (\underline{I} - \underline{A}^g \ \underline{A} \ \underline{A}^{g-1})^{-1} \\ &= (\underline{A}^g \ \underline{I} \ \underline{A}^{g-1} - \underline{A}^g \ \underline{A} \ \underline{A}^{g-1})^{-1} \\ &= (\underline{A}^g \ (\underline{I}-\underline{A}) \ \underline{A}^{g-1})^{-1} \\ &= \underline{A}^g \ (\underline{I}-\underline{A})^{-1} \ \underline{A}^{g-1} \end{aligned}$$

Substituting for $(\underline{I}-\underline{Q})^{-1}$ and $\hat{\underline{r}}$ in (16) we get:

$$\begin{aligned} \underline{c}^q &= \underline{u} \ \underline{A}^g \ (\underline{I}-\underline{A})^{-1} \ \underline{A}^{g-1} \ (E/x_g) \ \underline{A}^g \\ &= (E/x_g) \ \underline{u} \ \underline{A}^g \ (\underline{I}-\underline{A})^{-1} \\ &= \underline{c}^* \qquad \text{from (15).} \end{aligned}$$

So we see that an approach which would, given perfect information about the structure of energy dissipation in an economy, provide a very good direct method for defining energy intensities, will reduce to $\underline{c}^*$ when both $\underline{c}^*$ and $\underline{c}^q$ are restricted to an economy that has linearly related physical and financial aspects and non-differential fuel pricing.

The approach taken to $\underline{c}^q$ seems to be similar to the method outlined by Chapman, Leach and Slesser[19], in their analysis of the energy costs of fuel production.

19. P.F. Chapman, G. Leach and M. Slesser, "The Energy Cost of Fuels" Energy Policy, Sept. 1974, p.231.

Another method of deriving energy intensities is that used by Herendeen[20], Bullard and Herendeen[21], and by Casper, Chapman and Mortimer[22]. The basis of this approach is the assumption of the principle of the "Conservation of Embodied Energy", as represented by an "Energy Balance Equation". This equation reads:

Embodied Energy In + Fuel In = Embodied Energy Out.

The "In" and "Out" refer to the inputs and outputs of the various producing and absorbing sectors, as discussed above. "Embodied Energy In" is the "Energy Cost" to the economy of the inputs to a sector, and for each type of good this is simply the product of a suitable energy intensity for that good (kWh/£) and the quantity of that good absorbed (£).

This energy is not physically embodied in the good, but has been dissipated in the manufacturing of the good, so the energy is only "Embodied" in a formal, accounting sense. "Embodied Energy Out" is similarly defined.

The "Fuel In" is the quantity (kWh) of fuel purchased, and therefore the quantity of energy dissipated by a sector in the time period under consideration. Thus "Fuel In" is identical with e_i, as defined above.

Let us suppose that this methodology produces an energy intensity vector $\underline{c}^b$. We can therefore rewrite the Energy Balance Equation in I-O notation, giving:

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20. R.A. Herendeen, "An Energy Input-Output Matrix for the United States, 1963: Users Guide", CAC Document No.69, Center for Advanced Computation, University of Illinois, March 1973.

21. C.W. Bullard and R.A. Herendeen, "The Energy Costs of Goods and Services", Energy Policy, Dec. 1975, p.268.

22. D.A. Casper, P.F. Chapman and N.D. Mortimer, "Energy Analysis of the 'Report of the Census of Production, 1968'", Research Report ERG006, Open University, Aug. 1974.

$$\sum_j x_{ji} c_j^b + e_i = x_i c_i^b \qquad (17)$$

Now if we sum both sides of this equation over i we get:

$$\sum_i \sum_j x_{ji} c_j^b + \sum_i e_i = \sum_i x_i c_i^b$$

We recall that:

$$x_i = \sum_j x_{ij} + y_i$$

Substituting for x_i in (17) we get:

$$\sum_i \sum_j x_{ji} c_j^b + \sum_i e_i = \sum_i \sum_j x_{ij} c_i^b + \sum_i y_i c_i^b$$

Now: $$\sum_i \sum_j x_{ji} c_j^b = \sum_i \sum_j x_{ij} c_i^b$$

So: $$\sum_i y_i c_i^b = \sum_i e_i$$

Now: $$\sum_i e_i = E_{ind}$$

So: $$\sum_i y_i c_i^b = E_{ind}$$

i.e.: $$\underline{y}\, \underline{c}^b = E_{ind}$$

So we see that this approach gives rise to an energy intensity vector which imputes all energy dissipated in production to final demand.

Now we know:

$$e_i = c_i x_i$$

So substituting for e_i in (17) we get:

$$\sum_j x_{ji} c_j^b + c_i x_i = x_i c_i^b$$

i.e.: $$\sum_j a_{ji} c_j^b + c_i = c_i^b$$

In matrix notation this becomes:

$$\underline{c}^b \underline{A} + \underline{c} = \underline{c}^b$$

i.e.: $$\underline{c} = \underline{c}^b (\underline{I} - \underline{A})$$

Solving for $\underline{c}^b$ we get:

$$\underline{c}^b = \underline{c}\ (\underline{I}-\underline{A})^{-1}$$

$$= \underline{c}^*$$

So the embodied energy approach gives rise to an energy intensity vector identical to $\underline{c}^*$.

A third approach is taken by Wright[23,24], Krenz[25], and Tummala and Connor[26]. This assumes that all the necessary information to establish a vector of energy intensities is available in the financial matrix $\underline{A}$. The $(\underline{I}-\underline{A})^{-1}$ matrix is used directly, both for details of the technical relations of production and for fuel supply data.

The g th row of matrix $(\underline{I}-\underline{A})^{-1}$ contains the monetary value of the total energy required for the production of £1 worth of each of the types of goods output to final demand. This method necessarily assumes that all fuel is purchased at the same price, which is E/x_g (kWh/£). So this vector of energy intensities is defined by:

$$\underline{c}^w = (E/x_g)\underline{b}^g\ (\underline{I}-\underline{A})^{-1} \qquad (18)$$

Wright[23], and also Chapman[10], have outlined an iterative approach to the production process to obtain this result. Wright states "... if the vector $\underline{x}$ represents an amount x_i of the i th commodity, $i = 1,..,n$, then the production of final output $\underline{x}$ requires a direct input of commodity vector $\underline{y} = \underline{A}\ \underline{x}$, where the n matrix $\underline{A}$ is the input/output table". "... the secondary requirements are $\underline{z} = \underline{A}\ \underline{y} = \underline{A}^2\underline{x}$, so

23. D.J. Wright, "Goods and Services: An Input-Output Analysis", Energy Policy, Dec. 1974, p.307.

24. D.J. Wright, "The Natural Resource Requirements of Commodities", Applied Economics 7, 1975, p.31.

25. J.H. Krenz, "Energy per Dollar Value of Consumer Goods and Services", IEEE Trans. on Systems, Man and Cybernetics SMC-4 (4), 1974, p.386.

26. R.L. Tummala and L.J. Connor, "Mass-Energy Based Economic Models", IEE Trans. on Systems, Man and Cybernetics SMC-3 (6), 1973, p.548.

the total requirements input/output matrix $\underline{B} = \underline{A} + \underline{A}^2 + \underline{A}^3 + \dots$ $= (\underline{I}-\underline{A})^{-1}-\underline{I}$". This implies that for output to final demand vector $\underline{y}$, the total output vector $\underline{x}$ is given by:

$$\underline{x} = ((\underline{I}-\underline{A})^{-1}-\underline{I})\ \underline{y}$$

$$= (\underline{I}-\underline{A})^{-1}\ \underline{y} - \underline{y}$$

$$= \underline{x} - \underline{y}$$

i.e.: $\underline{o} = \underline{y}$ $(o_i = 0\ \forall\ i)$

This contradiction arises from Wright's formulation of the I-O process. Let us more properly state the case.

For an economy with a matrix of technical coefficients $\underline{A}$, the direct requirements for the production of output to final demand $\underline{y}$ is simply $\underline{y}$.

The secondary requirement for the output of $\underline{y}$ is $\underline{A}\ \underline{y}$.

The tertiary requirement is $\underline{A}(\underline{A}\ \underline{y}) = \underline{A}^2\ \underline{y}$, etc..

So the total output required is $\underline{x} = \underline{y} + \underline{A}\ \underline{y} + \underline{A}^2\ \underline{y} + \underline{A}^3\ \underline{y} + \dots = (\underline{I}-\underline{A})^{-1}\underline{y}$. This is, of course, identical with the result obtained by the more conventional analysis.

Fortunately, however, Wright identifies the published tables for the $(\underline{I}-\underline{A})^{-1}$ matrix with his $((\underline{I}-\underline{A})^{-1}-\underline{I})$, so his results are reasonable, though his theoretical justification of these results is not.

Let us now check that $\underline{c}^w$ corresponds to the output to final demand, $\underline{y}$.

$$\underline{c}^w\ \underline{y} = (E/x_g)\ \underline{b}^g\ (\underline{I}-\underline{A})^{-1}\ \underline{y}$$

$$= (E/x_g)\ \underline{b}^g\ \underline{x}$$

$$= (E/x_g)\ x_g$$

$$= E$$

That is, $\underline{c}^w$ imputes all energy dissipation to final demand, and not just that dissipated by the producing sectors, E_{ind}. We can remedy this defect by subtracting out the output to final demand of the fuel sector, which we can do by replacing $\underline{b}^g(\underline{I}-\underline{A})^{-1}$ with $(\underline{b}^g(\underline{I}-\underline{A})^{-1} - \underline{b}^g)$. If we refer to this new "reduced" vector of intensities as $\underline{c}^{wr}$, then we see that:

$$\underline{c}^{wr}\underline{y} = (E/x_g)(\underline{b}^g(\underline{I}-\underline{A})^{-1} - \underline{b}^g)\ \underline{y}$$

$$= (E/x_g)\ \underline{b}^g(\underline{x} - \underline{y})$$

$$= (E/x_g)(x_g - y_g)$$

$$= E - E(y_g/x_g)$$

Now under the assumptions necessary for this approach we know that:

$$E_{con} = E(y_g/x_g)$$

So substituting E_{con} we have:

$$\underline{c}^{wr}\underline{y} = E - E_{con}$$

$$= E_{ind} \qquad \text{from (5)}$$

So $\underline{c}^{wr}$ now imputes only the industrial energy dissipation to the output to final demand. We can now examine $\underline{c}^{wr}$ more closely. We have:

$$\underline{c}^{wr} = (E/x_g)(\underline{b}^g(\underline{I}-\underline{A})^{-1} - \underline{b}^g)$$

$$= (E/x_g)\ \underline{b}^g\ ((\underline{I}-\underline{A})^{-1} - \underline{I})$$

Now: $$(\underline{I}-\underline{A})^{-1} - \underline{I} = \underline{A}\ (\underline{I}-\underline{A})^{-1}$$

So: $$\underline{c}^{wr} = (E/x_g)\ \underline{b}^g\ \underline{A}\ (\underline{I}-\underline{A})^{-1}$$

$$= \underline{c}^* \qquad \text{from (14).}$$

So $\underline{c}^{wr}$ is identical with one of the forms taken by $\underline{c}^*$ when it is formulated under the same restrictions as apply to $\underline{c}^{wr}$.

The expression for $\underline{c}^{wr}$ involves $((\underline{I}-\underline{A})^{-1} - \underline{I})$, and therefore resembles the result Wright obtained by using the faulty summation $\underline{A} + \underline{A}^2 + \underline{A}^3 \ldots$. However, as Wright's tabulated results do not correspond to $((\underline{I}-\underline{A})^{-1} - I)$, but rather to $(\underline{I}-\underline{A})^{-1}$, this resemblance is fortuitous rather than informative as to Wright's methodology.

So far we have examined four energy intensity vectors which impute all industrial energy dissipation to output to final demand. Let us list them, along with the assumptions necessary for their formulation.

Energy Intensity Vector		Assumptions for Formulation
$\underline{c}^q = \underline{u}\,(\underline{I}-\underline{Q})^{-1}\,\hat{\underline{r}}$	(i)	Linear physical relations of production. $[e_{ij} = q_{ij}e_j]$
$\underline{c}^* = \underline{e}\,\hat{\underline{x}}^{-1}(\underline{I}-\underline{A})^{-1}$	(i)	Linear physical relations of production.
	(ii)	Linear mapping available from $\underline{Q}$ to $\underline{A}$. $[(\underline{I}-\underline{Q})^{-1} = \underline{A}^g(\underline{I}-\underline{A})^{-1}\underline{A}^{g-1}]$
$\underline{c}^b = \underline{e}\,\hat{\underline{x}}^{-1}(\underline{I}-\underline{A})^{-1}$		As for $\underline{c}$*.
$\underline{c}^{wr} = (E/x_g)\;\underline{u}\,\underline{A}^g(\underline{I}-\underline{A})^{-1}$	(i)	Linear physical relations of production.
	(ii)	Linear mapping available from $\underline{Q}$ to $\underline{A}$.
	(iii)	Non-differential fuel pricing in force. $[e_i = (E/x_g)x_{gi}]$

The ordering of this list establishes a hierarchy of "goodness", with $\underline{c}^q$ as the best energy intensity vector and $\underline{c}^{wr}$ the worst. The measure of "goodness" we are using is the number of assumptions necessarily made in the formulation of the energy intensity, the fewer assumptions the "better". Taking $\underline{c}^q$ as our standard, let us now test $\underline{c}^b$ and $\underline{c}^{wr}$ against it. Ideally this test should be performed on real economic data with a large number of sectors involved. Unfortunately this is not possible, as data suitable to construct the matrix $\underline{Q}$ are not available, so we shall have to make do with a simple three sector imaginary economy. We hope that this imaginary economy bears at least a passing resemblance to economies in the real world, though it is necessarily a caricature.

The financial transactions in this economy are described in Table 2.

Table 2

I-O Table (£)

INPUT \ OUTPUT	1	2	3	y_i	x_i
1	5.0	10.0	15.0	20.0	50.0
2	20.0	15.0	10.0	30.0	75.0
3	15.0	25.0	20.0	40.0	100.0

Let us next suppose that sector 1 is the fuel sector, and that 150 units (kWh) are used by the economy in one year, i.e. $E = 150$ units. Let us further suppose that a differential fuel pricing system is in operation, which is normally the case in the real world. We shall use the pricing system:

50p per unit up to 5 units

25p per unit from 5 to 10 units

12½p per unit for over 10 units

We shall also suppose that the output of the fuel sector to final demand is to a multitude of households, which must therefore purchase fuel on the highest tariff. Using these tariffs and the I-O table we find the quantities of fuel purchased by the various sectors (e_i) are as in Table 3.

Table 3

Fuel Purchased (kWh)

Sector	1	2	3	Final Demand	Total
	10.0	30.0	70.0	40.0	150.0

So for this economy E can be subdivided into $E_{ind} = 110$ units and

E_{dom} = 40 units.

We recall that $c_i = e_i/x_i$, so $\underline{c}$ is given by:

$$\underline{c} = (0.2, 0.4, 0.7)$$

Let us further suppose that the outputs of the various sectors are not homogeneous with respect to their energy intensiveness. i.e.

$$(\underline{I}-\underline{Q})^{-1} \neq \underline{A}^g \; (\underline{I}-\underline{A})^{-1} \; \underline{A}^{g-1}$$

For example, a heavy manufacturing sector may produce tempered metals, as well as the less energy intensive untempered metals. Let us use the energy intensiveness data for the outputs of the various sectors shown in Table 4.

Table 4

Relative energy intensiveness of sectoral outputs

	1	2	3	Final Demand
1	1.0	2.0	1.0	1.0
2	3.0	2.0	2.0	1.0
3	1.0	1.0	3.0	2.0

i.e.: The output of sector 1 to sector 2 is twice as energy intensive as the output of sector 1 to sector 3; and the output of sector 3 to sector 3 is three times as energy intensive as the output of sector 3 to sector 2, etc..

The combined effect of the differential fuel pricing and the non-homogeneity of the outputs of the various sectors is the energy dissipation by the economy as in Table 5.

Table 5

Energy Dissipation Table (kWh)

	1	2	3	Final Demand	Total
1	0.8333	3.3333	2.5000	3.3333	10.0
2	12.8570	6.4285	4.2857	6.4285	30.0
3	5.8333	9.7222	13.3333	31.1111	70.0

From the original I-O table we can find the matrix $\underline{A}$, which gives $(\underline{I}-\underline{A})^{-1}$ as:

$$(\underline{I} - \underline{A})^{-1} = \begin{pmatrix} 1.3684 & 0.3534 & 0.3008 \\ 0.7895 & 1.5226 & 0.3383 \\ 0.8421 & 0.7669 & 1.5038 \end{pmatrix}$$

Similarly, from the energy dissipation table we can find $\underline{Q}$, which gives $(\underline{I}-\underline{Q})^{-1}$ as:

$$(\underline{I} - \underline{Q})^{-1} = \begin{pmatrix} 1.5202 & 0.2583 & 0.1052 \\ 2.6932 & 1.7805 & 0.3078 \\ 2.6393 & 1.0916 & 1.7416 \end{pmatrix}$$

The vector $\underline{r}$, giving the ratio of the energy dissipated to final demand ($\underline{e}^y$ kWh) to the value of output to final demand ($\underline{y}$ £) is derived from Table 2 and Table 5, which gives $\underline{r}$ as:

$$\underline{r} = (0.1667,\ 0.2142,\ 0.7778)$$

We recall that $\underline{c}^q$ is found by multiplying the column sums of $(\underline{I}-\underline{Q})^{-1}$ by $\hat{\underline{r}}$, $\underline{c}^b$ we find by multiplying $\underline{c}$ by $(\underline{I}-\underline{A})^{-1}$, and $\underline{c}^{wr}$ we find directly from the first row of $(\underline{I}-\underline{A})^{-1}$ and E. These vectors are shown in Table 6, with percentage deviations from $\underline{c}^q$ in brackets for each element.

Table 6

Table of Energy Intensities

	1	2	3	$\underline{c}^y\ \underline{y}$
$\underline{c}^q$	1.1421 (0%)	0.6708 (0%)	1.6758 (0%)	110.0
$\underline{c}^b$	1.1789 (3%)	1.2165 (79%)	1.2481 (-34%)	110.0
$\underline{c}^{wr}$	1.1052 (-3%)	1.0602 (58%)	0.9024 (-45%)	90.0

We see that most of the elements of $\underline{c}^{wr}$ are too small, as the assumption of non-differential fuel pricing leads to the under-estimation of E_{ind} by 20 units. As fuel is almost invariably sold to industrial users on a lower tariff than to households, the use of $\underline{c}^{wr}$ will almost invariably lead to an understimation of E_{ind}. On the other hand $\underline{c}^{b}$ does correctly estimate E_{ind}, although the individual elements of $\underline{c}^{b}$ are in error here by up to 58%

So we have satisfied ourselves that $\underline{c}^{*}$, $\underline{c}^{b}$ and $\underline{c}^{wr}$ are all approximations to $\underline{c}^{q}$. But now we should ask ourselves the general question:

> "Given a vector of energy intensities $\underline{c}^{y}$, which imputes all industrial energy dissipation to output to final demand, what are the procedures for which $\underline{c}^{y}$ can and cannot be used?".

Let us try to answer this question by looking at some procedures for which $\underline{c}^{y}$ <u>can</u> be used:

(1) As $\underline{c}^{y}\,\underline{y} = E_{ind}$, if one can predict, or guess, the structure of final demand for some future date, then one can predict E_{ind}, the future industrial energy needs. This prediction must be subject to the proviso that the future date for the prediction must not be so distant that $\underline{A}$, or $\underline{Q}$ is likely to change substantially in the intervening period. If one can also estimate E_{dom}, then one can predict the overall energy needs, E. In particular, if one has energy intensities for each of the individual fuel types, which requires only a simple extension of the above theory, then we can also predict the "mix" of fuels required.

Such predictions have been attempted by O'Neill[27] for the UK and

27. P.G. O'Neill, "The Income-Elasticity of Demand for Primary Energy", <u>Institute of Fuel/Operations Research Society Conference</u>, 15/16 April 1975.

by Hannon[28], Herendeen[29], and Bullard and Herendeen[30] for the USA, where they have examined the energy requirements of these countries in terms of the structure of consumer spending ($\underline{y}$). It seems that their results are contradictory, in that the results of Bullard and Herendeen for the USA indicate that a less uneven distribution of disposable income would have little effect upon the total energy requirement of the nation, while the UK study by O'Neill indicates that such an income levelling would significantly reduce the gross energy requirement.

Bezdek and Hannon[31] have also used $\underline{c}^y$ like data to examine the energy consumption implications of changes in public and private final expenditure.

(2) The I-O model used to derive a $\underline{c}^y$ vector is a static one, capital accumulation being therefore included in final demand. This means that any expansion of the economy can be costed in terms of the necessary dissipation of energy using $\underline{c}^y$. e.g. One can legitimately calculate the new energy cost of a new nuclear reactor, or a Severn Barrage, using $\underline{c}^y$. Such studies have been outlined for reactor systems by Chapman[32], and Walford, Atherton and Hill[33], for wave and wind power systems by Musgrove[34] and for solar-power systems by Slesser and Hounam[35].

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28. B. Hannon, "Energy Conservation and the Consumer", Science 189, 1975, p.95.
29. R.A. Herendeen, "Affluence and Energy Demand", Mechanical Engineering, Oct. 1974, p.18.
30. C.W. Bullard and R.A. Herendeen, "Energy Impact of Consumption Decisions" Proc. IEEE 63 (3), 1975, p.484.
31. R. Bezdek and B. Hannon, "Energy, Manpower and the Highway Trust", Science 185, 1974, p.669.
32. P.F. Chapman, "Energy Analysis of Nuclear Power Stations", Energy Policy, June 1974, p.166.
33. F.J. Walford, R.S. Atherton and K.M. Hill, "Energy Costs of Inputs to Nuclear Power", Energy Policy, Dec. 1975, p.285.
34. P.J. Musgrove, "Energy Analysis of Wave-Power and Wind-Power Systems", Nature 262, 1976, p.206.
35. M. Slesser and I. Hounam, "Solar Energy Breeders", Nature 262, 1976, p.244.

(3) $\underline{c}^y$ can be used to compare the energy intensities of the outputs to final demand of different economies, with the proviso that the same I-O scheme of sectoral classification must have been used in the description of the economies to be compared.

Also, as exports are a component of final demand, the energy intensities of the exports of various countries can be compared, and an assessment of the energy being indirectly imported and exported can be made. Such a study of the indirect energy imports and exports of the USA has recently been published by Fieleke[36], while Denton[37] has performed a similar calculation for W. Germany.

Now let us examine some of the procedures for which $\underline{c}^y$ has been, but should not, be used.

(1) $\underline{c}^y$ cannot be used as a means of calculating the indirect (non-fuel) energy inputs to an industrial sector. This process would entail finding $\sum_j x_{ji} c_j^y$ for a given sector j. But we should note that if instead of performing this calculation for one or two sectors, as has been done by Leach[40] for agriculture, for example, we perform this calculation for all of the producing sectors simultaneously, then the total energy dissipation which we impute to intermediate demand is:

$$\sum_i \sum_j x_{ji} c_j^y = \sum_i \sum_j a_{ji} x_i c_j^y$$

$$= \underline{c}^y \underline{A} \underline{x}$$

But $\underline{c}^y \underline{y} = E_{ind}$ is already imputed to final demand, implying that the total energy dissipated by the industrial sectors is:

$$E_{ind} = \underline{c}^y \underline{A} \underline{x} + E_{ind}$$

$$> E_{ind}$$

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36. N.S. Fieleke, "The Energy Trade: The United States in Deficit", New England Economic Review, May/June 1975, p.25.

37. R.V. Denton, "The Energy Costs of Goods and Services in the Federal Republic of Germany", Energy Policy, Dec. 1975, p.279.

This contradicts our definition of E_{ind} as the total industrial energy dissipation.

An approach often seen in the literature, normally only for one or two sectors at a time, is to calculate the "direct energy input" as the fuel purchased, the "indirect energy input" as $\sum_j x_{ji} c_j^y$, and the "total energy input" as the sum of these two components, as for example in papers by Steinhart and Steinhart[38], and Pimentel et al[39]. This, of course, involves double counting on a grand scale. As an example, consider the outcome of such a calculation using the I-O table and $\underline{c}^b$ described above:

Table 7

Sector	Direct Input (e_i)	Indirect Input ($\sum_j x_{ji} c_j^b$)	Direct plus Indirect Input
1	10.0	5.90+12.17+18.72 = 36.78	46.78
2	30.0	11.79+18.25+31.20 = 61.24	91.24
3	70.0	17.68+12.17+24.96 = 54.81	112.81
Total	110.0	152.83	262.83

In the drawing up of this table two errors have been made. First, this approach ignores the fact that the indirect energy inputs are imputed direct energy inputs, so to calculate them separately and then add them must lead to double counting. Second, if one is to calculate some measure of indirect energy input to a sector one must recognise that intermediate demand is involved, and therefore one must use an energy intensity which refers to total output, not output to final demand; i.e. one can use $\underline{c}$ but not $\underline{c}^*$.

38. J.S. Steinhart and C.E. Steinhart, "Energy Use in the U.S. Food System", Science 184, 1974, p.307.

39. D. Pimentel, L.E. Hurd, A.C. Bellotti, M.J. Forster, I.N. Oka, O.D. Sholes and R.J. Whitman, "Food Production and the Energy Crisis", Science 182, 1974, p.307.

(2) We cannot "extend" our vector of energy intensities by using previously omitted sectors in conjunction with the incomplete set of intensities. For example, our previous I-O table gave us $\underline{c}^b$ = (1.1789, 1.2165, 1.2481). Now suppose we were to "lose" producing sector 3, so that $\underline{c}^b$ became a 2-vector, $\underline{c}^{br}$. If we were then to "find" sector 3 we might attempt to use our $\underline{c}^{br}$ with the "new" I-O data to construct a 3-vector, $\underline{c}^b$. Performing this operation, the I-O table for the reduced two sector economy will be:

Table 8

2 Sector I-O Table (£)

		OUTPUT 1	2	y_i	x_i
	1	5.0	10.0	35.0	50.0
INPUT	2	20.0	15.0	40.0	75.0

The outputs from sectors 1 and 2 which originally went to sector 3 have been included in final demand, so the total outputs of the two sectors has been left unaltered.

Using the Energy Balance Equation method we obtain:

$$5.0\ c_1^{br} + 20.0\ c_2^{br} + 10.0 = 50.0\ c_1^{br}$$

$$10.0\ c_1^{br} + 15.0\ c_2^{br} + 30.0 = 75.0\ c_2^{br}$$

These two equations have solution:

$$\underline{c}^{br} = (0.5052,\ 0.6373)$$

We can now "discover" sector 3, and use $\underline{c}^{br}$ in the Energy Balance Equation to find c_3^{br}. i.e.:

$$15.0\ c_1^{br} + 10.0\ c_2^{br} + 20.0\ c_3^{br} + e_3 = 100.0\ c_3^{br}$$

This has solution:

$$c_3^{br} = 1.0581$$

So the three component energy intensity vector obtained by this extension is:

$\underline{c}^{br}$ = (0.5052, 0.6373, 1.0580)

We note that each element of $\underline{c}^{br}$ is considerably smaller than the corresponding element of $\underline{c}^{b}$. As a result $\underline{c}^{br}\ \underline{y}$ = 75.54, an underestimation of the true value of E_{ind}, which is 110.0. Thus $\underline{c}^{br}$ is not only dissimilar in detail to $\underline{c}^{b}$, but does not even correctly estimate E_{ind}. In the case of a loosely connected economy, and particularly for a sector which is only tenuously bound to the other producing sectors, such a method may by acceptable, but caution must be used and the limitations of this approach stressed. Leach[40] has performed such a calculation for the agricultural sector, using the $\underline{c}^{b}$ vector of Chapman and the $\underline{c}^{w}$ vector of Wright. The agricultural sector is indeed loosely connected to the rest of the economy, but Leach makes no mention of the restrictions necessarily imposed upon his method, and indeed seems unaware of them.

Having established the existence and properties of the vectors $\underline{c}$, $\underline{c}^{*}$, and $\underline{c}^{e}$, our next step is to apply these vectors to the examination of the important question: "What is the nature of the correlation between a country's overall energy use and its Gross Domestic Product?". By using $\underline{c}$, $\underline{c}^{*}$ and $\underline{c}^{e}$ we can break down the relationship between the gross variables of energy use and GDP to a relationship which recognises that an economy is composed of many industrial sectors which are interrelated to allow the efficient output of goods and services to a structured, and changing, final demand. Let us begin by examining the question as it is usually formulated.

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40. G. Leach, Energy and Food Production, IIED, London, 1975.

The relationship between the amount of energy used by an economy in a given period of time, and the corresponding gross quantity of goods and services produced by that economy has been an object of study for several years, Felix,[41,42] Dilloway[43] and Darmstadter [44,45] being three of the major authors. At first sight a cross-section plot of GDP against gross energy use seems to give a good correlation between these two variables. However, closer examination shows that there is considerable scatter in such plots, as Makhijani and Lichtenberg[4] have pointed out, though time series data for individual countries gives a better correlation, as has been noted by Smil and Kuz[46]. In particular, if the GDP (G) and gross energy consumption (E) are closely related, one would anticipate that the Output Elasticity of energy consumption, otherwise known as the Energy Coefficient (R), would be a smoothly changing variable. We define this elasticity by:

$$R \equiv \frac{\Delta E/E}{\Delta G/G} \quad \left(= \frac{\Delta E/\Delta G}{E/G} \right) \qquad (19)$$

Adams and Miovic[47] have pointed out that the energy coefficients for many countries do not in fact seem to change smoothly, and following Turvey and Nobay[48] have suggested that to understand this behaviour one

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41. F. Felix, World Markets of Tomorrow, Harper and Row, London, 1972.
42. F. Felix, "Annual Growth Rate on Downward Trend", Electrical World, July 1970, p.36.
43. A.J. Dilloway, "Energy and Economic Growth: How Close the Relation?", Energy International, Aug. 1970, p.31.
44. J. Darmstadter, Energy in the World Economy, Resources for the Future, London, 1971.
45. J. Darmstadter, "Energy and the Economy", Energy International, Aug. 1970, p.31.
46. V. Smil and T. Kuz, "European Energy Elasticities", Energy Policy, June 1976, p.171.
47. F. Adams and P. Miovic, "On Relative Fuel Efficiency and the Output Elasticity of Energy Consumption in Western Europe", Journal of Industrial Economics 7, 1968, p.41.
48. R. Turvey and A. Nobay, "On Measuring Energy Consumption", Economic Journal 75, 1968, p.300.

must remember that gross energy consumption involves the use of several different fuel types, which may not be equally "efficient" in use, either from the thermodynamic or economic point of view.

For example, the thermodynamic efficiencies of energy use by rail transport are 5% for coal, 35% for oil and 75% for electricity.

Adams and Miovic set out to discover relative weightings between fuels, such that GDP could be related to "effective" energy consumption. They did not attempt to treat the thermodynamic and economic aspects separately, but incorporated both of them into a single efficiency measure. They assumed a production function linearly relating "Output" (G) to effective energy consumption. Three energy types (i) were used; coal, oil and electricity. Coke and gas were subsumed under coal. The equation used by Adams and Miovic was:

$$G = \phi_1 t_1 + \phi_2 t_2 + \phi_3 t_3$$

Here t_i are the quantities of the fuels used. Several empirical series of data were fitted to this equation and the ϕ_i giving the best fit were found. The coefficients for the "efficiency" of electricity were the largest, followed by oil and then coal. Such an analysis, being non-causal, must not be regarded as giving a good indication of the actual relative roles played by the different fuel types (as Adams and Miovic acknowledge). Unfortunately Brookes[49,50,51] does not seem to have exercised

49. L. Brookes, "Energy and Economic Growth", Atom 83, 1972, p.7.

50. L. Brookes, More on the Output Elasticity of Energy Consumption", Journal of Industrial Economics 21, 1972, p.83.

51. L. Brookes and P.F. Chapman, Energy and the World Economy, Open University Press, Milton Keynes, 1975.

sufficient caution in this respect, and has used these efficiency coefficients as relative weightings for the fuel types, to obtain "effective" energy consumptions, from which he obtains "corrected" Energy Coefficients.

Brookes argues that the likely behaviour of the Energy Coefficient for a given country is that it starts from a value greater than 1, due to the introduction of energy intensive production into an initially low energy intensive economy. Then, as the country develops, the coefficient tends to 1 from above, though his argument as to why the limit of this tendency should be 1, rather than (say) zero, is rather hazy. Unadjusted energy coefficients do not obviously behave like this, and indeed O'Neill[27] has suggested that for the UK there is good evidence, based upon an analysis of changing consumption patterns, that the Energy Coefficient is tending to 1 from below.

Chapman[51] has pointed out that Brookes's analysis is circular; to understand why this is so we shall need some definitions:

$G \equiv$ G.D.P.

$E \equiv$ Gross Energy Consumption

$E^* \equiv$ "Corrected" Gross Energy Consumption

Now $$E^* = \theta \sum_{i=1}^{3} \phi_i t_i \qquad (\theta \equiv \text{constant}) \qquad (20)$$

But Adams and Miovic's method requires that:

$$G = \gamma \sum_{i=1}^{3} \phi_i t_i \qquad (21)$$

Here the constant γ is near to or far from 1 depending on whether the ϕ_i used give a good fit to that particular value of G.

Combining (20) and (21) we get:

$$E^* = (\theta/\gamma)\, G \qquad (22)$$

i.e. $E^*/G = \theta/\gamma$

Differentiating (22) gives:

$$\Delta E^*/\Delta G = \theta/\gamma$$

So the energy coefficient is:

$$R = \frac{\Delta E^*/\Delta G}{E^*/G}$$

$$= \frac{\theta/\gamma}{\theta/\gamma}$$

$$= 1$$

So the uniform tendency for the Energy coefficients to be close to 1, discovered by Brookes when using this method, is an artefact of the statistical fitting, and only reassures us that the countries used by Adams and Miovic have similar fuel substitution preferences to those selected by Brookes.

Brookes's results also confirm his intuitive statement that R will decrease towards 1, but this finding is also spurious. It is spurious because developing countries will usually install modern capital equipment, which will mostly use oil and electricity. As oil and electricity are given large weightings, this alone will almost certainly ensure that $\Delta E^*/\Delta G$ will be larger than E^*/G.

So we see that any attempts to make Energy Coefficients conform to our preconceptions by using efficiency weightings must be tautological, and are therefore to be regarded as methodologically disreputable.

Smil and Kuz[46] have suggested that while searching for a smoothly changing Energy Coefficient is equivalent to fitting a curve of the form $E = aG^R$, perhaps a more realistic curve to fit would be $E = a + bG$. This, of course, requires that there be a certain "residual" energy requirement for an economy, even when no goods are being produced, a supposition that runs counter to both intuition and to normal economic thinking. Further, when they attempted such fits for several European countries, some of their values for this residual (a) were NEGATIVE, implying that submerged in some economic systems are technologies able to produce goods and services at no energy cost.

Smil and Kuz's only justification for this empirical method is that good correlations are produced, which may be a useful finding but not an illuminating one. Therefore, to attempt to obtain insights into the GDP - Energy relationship (if any), we shall persevere with the Energy Coefficient, but first recast it into a form more closely corresponding to our intuitive theoretical requirements.

Let us first note that we are seeking a relation between the energy used by an economy and the economic output of that economy. However, the gross energy consumption is not all involved in manufacturing processes, a good proportion being delivered direct to domestic consumption for home heating and personal transport. We should therefore split our gross energy consumption up into the two sections mentioned above i.e.:

$$E = E_{ind} + E_{dom}$$

Here E_{ind} is that energy used by manufacturing and service industries and E_{dom} that used by domestic consumers. We might anticipate that, having rid our relation from the vagaries of the weather, a more suitable, and useful, Energy Coefficient would be:

$$R = \frac{\Delta E_{ind}/E_{ind}}{\Delta G/G}$$

But even this may still be subject to extraneous fluctuations, due to the "cold winter" effect of increasing E_{dom}, which can then effect GDP and E_{ind} at a secondary level and thereby alter R in an unwanted manner. To understand the details of this effect, and to be able to compare various alternative energy coefficients that might arise, we must resort to more definitions.

We have already defined E, E_{ind} and E_{dom}, with $E = E_{ind} + E_{dom}$. So we can also write $E_{ind} = \beta E$ $(\beta \leq 1)$.

The GDP is equal to the sum of the final demands, y_i, for the n industrial sectors that make up the economy. i.e.

$$G = \sum_{i=1}^{n} y_i$$

We shall assume that the domestic energy consumption is proportional to the final demand for energy, y_g, where sector g is the one which sells fuel. i.e. $E_{dom} = \alpha y_g$. ($\alpha \equiv$ constant)

We shall use $\underline{c}^*$ as a vector of energy intensities which imputes all industrial energy use to the output to final demand, as discussed above. Let us allow that these energy intensities remain constant during the period of interest, as this makes the following analysis less tedious, but does not materially alter its conclusions.

Suppose that over a period of time the sum of the final demands, the GDP, alters from G to G + ΔG. Further, let us suppose that we have a "cold winter", such that there is an additional exogenous alteration in the output of the "energy" sector from y_g to $y_g + \Delta y_g$. So we see that the total change in GDP will be from G to $G + \Delta G + \Delta y_g$.

Similarly, the change in industrial energy use will be from E_{ind} to $E_{ind} + \Delta E_{ind} + c_g^* \Delta y_g$.

The change in domestic energy use will be from E_{dom} to $E_{dom} + \alpha \Delta y_g$. The "best" Energy Coefficient will ignore both the primary "cold winter" effect by leaving out E_{dom} altogether, and also ignore the secondary effect of Δy_g on G and $\Delta y_g c_g^*$ on E_{ind}. i.e. we define:

$$R_o = \frac{\Delta E_{ind}/E_{ind}}{\Delta G/G}$$

The next best Coefficient will still ignore E_{dom}, but will not be temperature insensitive enough to ignore the secondary effect. i.e.:

$$R_1 = \frac{(\Delta E_{ind} + \Delta y_g c_g^*)/E_{ind}}{(\Delta G + \Delta y_g)/G}$$

The worst coefficient is that mostly found in the literature, where both the primary and secondary effects are included. i.e.:

$$R_2 = \frac{(\Delta E_{ind} + \Delta y_g c_g^* + \alpha \Delta y_g)/(E_{ind} + E_{dom})}{(\Delta G + \Delta y_g)/G}$$

Obviously R_1 and R_2 will, in general, be biased estimates of R_o, and it would be useful to know whether this bias will generally be positive or negative. Let us first compare R_1 and R_o.

$$\frac{R_1}{R_o} = \frac{(\Delta E_{ind} + \Delta y_g c_g^*)\ \Delta G}{\Delta E_{ind}\ (\Delta G + \Delta y_g)}$$

$$= \frac{(1 + \Delta y_g c_g^*/\Delta E_{ind})}{(1 + \Delta y_g/\Delta G)}$$

$$\approx (1 + \Delta y_g c_g^*/E_{ind})(1 - \Delta y_g/\Delta G)$$

$$\approx (1 + \Delta y_g c_g^*/E_{ind} - \Delta y_g/\Delta G)$$

So $R_1 > R_o$ iff $\Delta y_g c_g^*/\Delta E_{ind} > \Delta y_g/\Delta G$

i.e. iff $c_g^* > \Delta E_{ind}/\Delta G$

Now by definition $\Delta E_{ind}/\Delta G = R_o E_{ind}/G$

i.e. $R_1 > R_o$ iff $\dfrac{c_g^*}{E_{ind}/G} > R_o$

As R_o is involved in both inequalities, let us resort to a practical test. For the UK (1968), using a 10 producing sector model, we find that:

$c_g^* = 5.1$ and $E_{ind}/G = 2.8$ (in compatible units)

i.e. $R_1 > R_o$ iff $5.1/2.8 = 1.82 > R_o$

We would normally be surprised by such a large coefficient, so we would anticipate that R_1 is usually an overestimate of R_o.

Let us now compare R_o and R_2.

$$\frac{R_2}{R_o} = \frac{(\Delta E_{ind} + \Delta y_g c_g^* + \alpha \Delta y_g) E_{ind}\ \Delta G}{\Delta E_{ind}(E_{ind} + E_{dom})(\Delta G + \Delta y_g)}$$

Now $E_{ind} + E_{dom} = E = (1/\beta)E_{ind}$

i.e.: $$\frac{R_2}{R_o} = \frac{\beta(\Delta E_{ind} + \Delta y_g c_g^* + \alpha \Delta y_g)\ \Delta G}{(\Delta G + \Delta y_g)\ \Delta E_{ind}}$$

$$= \frac{\beta(1 + \Delta y_g c_g^*/\Delta E_{ind} + \alpha \Delta y_g/\Delta E_{ind})}{(1 + \Delta y_g/\Delta G)}$$

$$\simeq \beta(1 + \Delta y_g c_g^*/\Delta E_{ind} + \alpha \Delta y_g/\Delta E_{ind} - \Delta y_g/\Delta G)$$

$$= \beta(1 + \frac{1}{R_o}(\Delta y_g/\Delta G)(G/E_{ind})(\alpha + c_g^* - R_o(E_{ind}/G))$$

Let us put: $P = E_{ind}/G$

$S = \Delta y_g/\Delta G$

and $1 + \delta = R_2/R_o$ i.e. $R_2 > R_o$ iff $\delta > 0$

i.e. $1 + \delta = \beta(1 + (S/PR_o)(\alpha + c_g^* - PR_o))$

So $$R_o = \frac{(\alpha + c_g^*)\ S}{P((1 + \delta)/(\beta - 1 + S)} \qquad (23)$$

We see that R_o is a decreasing function of δ.

We wish to know the size of R_o at which $\delta = 0$. i.e. at which R_2 is an unbiased estimate of R_o. If this R_o is unrealistically low, then we would expect a reasonable practical outcome to be $\delta < 0$; i.e. R_2 would underestimate R_o. On the other hand, if R_o is large, then we would expect $\delta > 0$ and R_2 would overestimate R_o.

Using UK (1968) data again we find that:

$\alpha = 129.4$ $c_g^* = 5.1$ $\beta = 0.43$ $P = 2.8$

We recall that $S = \Delta f_e/\Delta G$. Now $f_e/G = 0.03$, so $S = 0.03$ seems reasonable. (Remember, $\delta = 0$)

Substituting these values in (23) we get $R_o = 2.74$. This is rather large, so in this case we would anticipate that $\delta > 0$ and R_2 is also an overestimate of R_o.

Our next problem is to assess whether even our "best" energy coefficient will, on its own, tell us much of use or interest about the evolving way energy is used by economies. We should be aware that the ΔE_{ind} and ΔG we examine may depend upon changes in the technology of manufacture, or on changing consumer choices, or both, so any energy coefficient may be reflecting several sorts of change. A full understanding of the Energy Consumption - GDP relationship can only be had by the analysis of these individual changes.

At this stage we shall fully introduce I-O theory and our energy intensity vectors into the formulation of the energy coefficient. We recall that:

$$E_{ind} = \underline{c}^* \underline{y}$$

and that:

$$R_o = \frac{\Delta E_{ind}/E_{ind}}{\Delta G/G}$$

i.e.
$$R_o = \frac{\Delta(\underline{c}^*\underline{y})/E_{ind}}{\Delta G/G}$$

We can expand ΔE_{ind} as:

$$\Delta(\underline{c}^*\underline{y}) \simeq \Delta\underline{c}^*\underline{y} + \underline{c}^*\Delta\underline{y}$$

Now
$$\underline{c}^* = \underline{c}(\underline{I} - \underline{A})^{-1} \qquad (\text{put } (\underline{I} - \underline{A})^{-1} \equiv (\underline{I} + \underline{B}))$$
$$= \underline{c}(\underline{I} + \underline{B})$$

So
$$\Delta\underline{c}^* \simeq \Delta\underline{c}(\underline{I} + \underline{B}) + \underline{c}\Delta\underline{B}$$

This gives
$$\Delta E_{ind} \simeq \Delta\underline{c}(\underline{I} + \underline{B})\underline{y} + \underline{c}\Delta\underline{B}\,\underline{y} + \underline{c}^*\Delta\underline{y}$$
$$= \Delta\underline{c}\,\underline{x} + \underline{c}\Delta\underline{B}\,\underline{y} + \underline{c}^*\Delta\underline{y}$$

i.e.
$$R_o \simeq (\Delta\underline{c}\,\underline{x} + \underline{c}\Delta\underline{B}\,\underline{y} + \underline{c}^*\Delta\underline{y})(G/E_{ind}\Delta G)$$

Our energy coefficient now contains terms expressing three sorts of change in our economic structure:

(i) That containing $\Delta\underline{c}\,\underline{x}$. $\Delta\underline{c}$ reflects the change in the energy input to each sector required to produce a fixed value of output. This will correspond to changing technology within industries. We shall call this the Intra-Industry term.

(ii) That containing $\underline{c}\Delta\underline{B}\ \underline{y}$. $\Delta\underline{B}$ reflects changes in the relationships between the industrial sectors, so we shall call this the Inter-Industry term.

(iii) That containing $\underline{c}^*\Delta\underline{y}$. This corresponds to changes in the pattern of consumer behaviour and we shall call this the Final Demand term.

Now in the above expansion we have differentiated $E_{ind} = \underline{c}\ (\underline{I}-\underline{A})^{-1}\underline{y}$ and used a difference approximation to give:

$$\Delta E_{ind} \simeq \Delta\underline{c}\ (\underline{I} + \underline{B})\ \underline{y} + \underline{c}\Delta\underline{B}\ \underline{y} + \underline{c}\ (\underline{I} + \underline{B})\Delta\underline{y} \qquad (24)$$

But, supposing we are using a central difference approximation, how does ΔE_{ind} derived from the above estimate differ from the true value of ΔE_{ind}? Let us suppose that we are examining a transition between an initial state described by:

$$E_{ind1} = \underline{c}_1\ (\underline{I} + \underline{B}_1)\ \underline{y}_1$$

and a final state described by:

$$E_{ind2} = \underline{c}_2\ (\underline{I} + \underline{B}_2)\ \underline{y}_2$$

Now the true value of ΔE_{ind} is given by:

$$\Delta E^t_{ind} = E_{ind2} - E_{ind1}$$

$$= \underline{c}_2\ (\underline{I} + \underline{B}_2)\ \underline{y}_2 - \underline{c}_1\ (\underline{I} + \underline{B}_1)\ \underline{y}_1$$

We can express the differences between the variables in the two cases by:

$$\Delta\underline{c} = \underline{c}_2 - \underline{c}_1$$

$$\Delta\underline{B} = \underline{B}_2 - \underline{B}_1$$

$$\Delta\underline{y} = \underline{y}_2 - \underline{y}_1$$

So we can write:

$$\Delta E^{t}_{ind} = (\underline{c}_1 + \Delta\underline{c})(\underline{I} + \underline{B}_1 + \Delta\underline{B})(\underline{y}_1 + \Delta\underline{y}) - \underline{c}_1(\underline{I} + \underline{B}_1)\underline{y}_1$$

$$= \Delta\underline{c}(\underline{I} + \underline{B}_1)\underline{y}_1 + \underline{c}_1\Delta\underline{B}\,\underline{y}_1 + \underline{c}_1(\underline{I} + \underline{B}_1)\Delta\underline{y}$$

$$+ \Delta\underline{c}\Delta\underline{B}\,\underline{y}_1 + \Delta\underline{c}(\underline{I} + \underline{B}_1)\Delta\underline{y} + \underline{c}_1\Delta\underline{B}\Delta\underline{y}$$

$$+ \Delta\underline{c}\Delta\underline{B}\Delta\underline{y}$$

Using these same difference definitions we can evaluate our approximation to ΔE_{ind}. Using a central difference technique implies that

$$\underline{c} = \tfrac{1}{2}(\underline{c}_1 + \underline{c}_2) = \underline{c}_1 + \tfrac{1}{2}\Delta\underline{c}$$

$$\underline{B} = \tfrac{1}{2}(\underline{B}_1 + \underline{B}_2) = \underline{B}_1 + \tfrac{1}{2}\Delta\underline{B}$$

$$\underline{y} = \tfrac{1}{2}(\underline{y}_1 + \underline{y}_2) = \underline{y}_1 + \tfrac{1}{2}\Delta\underline{y}$$

Substituting these values in (24) we get:

$$\Delta E_{ind} = \Delta\underline{c}(\underline{I} + \underline{B}_1 + \tfrac{1}{2}\Delta B)(\underline{y}_1 + \tfrac{1}{2}\Delta\underline{y})$$

$$+ (\underline{c}_1 + \tfrac{1}{2}\Delta\underline{c})\Delta\underline{B}(\underline{y}_1 + \tfrac{1}{2}\Delta\underline{y})$$

$$+ (\underline{c}_1 + \tfrac{1}{2}\Delta\underline{c})(\underline{I} + \underline{B}_1 + \tfrac{1}{2}\Delta\underline{B})\Delta\underline{y}$$

$$= \Delta\underline{c}(\underline{I} + \underline{B}_1)\,\underline{y}_1 + \underline{c}_1\Delta\underline{B}\,\underline{y}_1 + \underline{c}_1(\underline{I} + \underline{B}_1)\Delta\underline{y}$$

$$+ \underline{c}_1\Delta\underline{B}\,\underline{y}_1 + \underline{c}_1(\underline{I} + \underline{B}_1)\Delta\underline{y} + \underline{c}_1\Delta\underline{B}\Delta y$$

$$+ \tfrac{3}{4}\Delta\underline{c}\Delta\underline{B}\Delta\underline{y}$$

$$= \Delta E^{t}_{ind} - \tfrac{1}{4}\Delta\underline{c}\Delta\underline{B}\Delta\underline{y}$$

So this central difference approximation will always misestimate ΔE^{t}_{ind}, and therefore misestimate R_o. This defect can be overcome by reducing the number of variables used in the approximation from three to two. We can perform this reduction by recalling that:

$$\underline{c}^{e}\,\underline{e}^{y} = E_{ind}$$

where $\underline{c}^{e} = \underline{u}(\underline{I} - \underline{Q})^{-1}$

i.e. $E_{ind} = \underline{u}(\underline{I} - \underline{Q})^{-1}\underline{e}^{y}$

$$= \underline{u}(\underline{I} + \underline{W})\ \underline{e}^{y} \quad (\underline{u} \text{ is constant, } u_i = 1\ \forall i)$$

A central difference approximation for ΔE_{ind} will therefore be:

$$\Delta E_{ind} = \underline{u}(\underline{I} + \underline{W})\Delta\underline{e}^{Y} + \underline{u}\Delta\underline{W}\ \underline{e}^{Y}$$

Using the two states defined by:

$$E_{ind1} = \underline{u}(\underline{I} + \underline{W}_1)\ \underline{e}_1^{Y}$$

and $$E_{ind2} = \underline{u}(\underline{I} + \underline{W}_2)\ \underline{e}_1^{Y}$$

Then, as above, the true value of ΔE_{ind} is given by:

$$\begin{aligned}\Delta E^{t}_{ind} &= \underline{u}(\underline{I} + \underline{W}_2)\underline{e}_2^{Y} - \underline{u}(\underline{I} + \underline{W}_1)\underline{e}_1^{Y} \\ &= \underline{u}(\underline{I} + \underline{W}_1 + \Delta\underline{W})(\underline{e}_1^{Y} + \Delta\underline{e}^{Y}) - \underline{u}(\underline{I} + \underline{W}_1)\underline{e}_1^{Y} \\ &= \underline{u}(\underline{I} + \underline{W}_1)\Delta\underline{e}_1^{Y} + \underline{u}\Delta\underline{W}\ \underline{e}_1^{Y} + \underline{u}\Delta\underline{W}\Delta\underline{e}^{Y}\end{aligned}$$

The central difference approximation becomes:

$$\begin{aligned}\Delta E_{ind} &= \underline{u}(\underline{I} + \underline{W}_1 + \tfrac{1}{2}\Delta W)(\underline{e}_1^{Y} + \tfrac{1}{2}\Delta\underline{e}^{Y}) \\ &= \underline{u}(\underline{I} + \underline{W}_1)\Delta\underline{e}^{Y} + \underline{u}\Delta\underline{W}\ \underline{e}_1^{Y} + \underline{u}\Delta W\Delta\underline{e}^{Y} \\ &= \Delta E^{t}_{ind}\end{aligned}$$

So the use of this method of analysis will produce no error in R_o. The economic relevance of the $\Delta\underline{e}^{Y}$ and $\Delta\underline{W}$ terms is less obvious than the corresponding terms derived from the first expansion, but they do have a physical significance, which will be discussed later.

In this chapter we have attempted to introduce structural concepts into the analysis of the energy use by economies. We have done this by introducing the vectors $\underline{c}$, $\underline{c}^*$ and $\underline{c}^{e}$, which refer to the quantity vectors $\underline{x}$, $\underline{y}$ and $\underline{e}^{Y}$ respectively, such that:

$$\underline{c}\ \underline{x} = \underline{c}^*\ \underline{y} = \underline{c}^{e}\ \underline{e}^{Y} = E_{ind}$$

Here E_{ind} is the quantity of energy used in industrial production. These vectors have been applied to an analysis of the energy coefficient, indicating that a fuller understanding of the behaviour of this function can be had in terms of more disaggregated variables than the overall energy use and GDP normally used in its formulation.

In the next chapter we shall attempt to apply further our structural analysis of the energy dissipation by economies to the problem of defining physical measures of economic organisation. In this process we shall use both the energy intensities and energy coefficients we have already discussed, as well as examining the energy I-O matrix $(\underline{I}-\underline{Q})^{-1}$ in some detail.

CHAPTER EIGHT

Organisation, Dissipation and Economies : An Empirical Analysis

In an earlier chapter a case was presented for thinking of economies as self-organising dissipative systems. It was also noted earlier that several authors have hypothesised that the more "organised" an economy becomes the more energy it must "use". A simple model of the behaviour of dissipative systems has been proposed which meets this suggestion. In this chapter we shall attempt to meet parts (iii) and (iv) of the argument sketched in Chapter Five. That is, to suggest meaningful measures of organisation and dissipation for economies, and use them to test whether, for economies, organisation and dissipation are positively related.

For measures of organisation and dissipation, a prerequisite is a satisfactory definition of "organisation", or "integrality", as Denbigh[1] has termed it. As was mentioned earlier, physical theory is as yet unable to give a full and well defined meaning to this intuitively attractive concept.

We recall that for a system to be "organised" it must be constituted of parts, and those parts must be interrelated. The parts recognised as constituting the system should not be arbitrary, but preferably defined in terms of functional elements within the system. For an economy these might be seen as individuals, or producing and consuming groups, or as we shall use, industrial sectors. These are used because they are the smallest functional elements for which extensive data are available, and therefore allow the greatest resolution of detail in the analysis.

In the best of all possible worlds the number of sectors necessarily identified in an economy would be one constituent of that economy's degree of organisation, as Denbigh has recommended. Unfortunately this

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1. K.G. Denbigh, An Inventive Universe, Hutchinson, London, 1975.

identification of necessary sectoral division has not yet been undertaken, though it should, in principle, be feasible. As a result all of the economies considered will be represented by a constant number of sectors.

Another aspect of the organisation of an economy is the interaction between the sectors, so we would like a measure of this interaction which was both accessible and physically relevant. That is, the properties of the system used to measure interaction between its elements should be in terms of physical variables rather than social variables. In the previous chapter we introduced the energy dissipation table for an economy, and it might prove feasible to use this to allow us to define a suitable measure of economic organisation. We recall that this table takes the form:

$$
\begin{aligned}
e_1 &= e_{11} + e_{12} + \cdots\cdots + e_{1n} + e_1^Y \\
e_2 &= e_{21} + e_{22} + \cdots\cdots + e_{2n} + e_2^Y \\
&\vdots \\
e_n &= e_{n1} + e_{n2} + \cdots\cdots + e_{nn} + e_n^Y
\end{aligned}
$$

Here e_i is the total energy dissipated (per unit time) by sector i. Hence $\sum_i e_i = E_{ind}$, the total industrial energy use. e_{ij} is the energy dissipated by sector i due to activity through its relation with sector j. e_i^Y is the energy dissipated by sector i directly attributable to its relation with final demand.

One can therefore think of this table as a map detailing the dissipation of energy in an economy because of the functional relations between sectors required to meet the system's function.

For ease of analysis we can reduce the (n + 1) xn table of dissipation to an n x n table, simply by adding the e_i^Y to the diagonal elements, e_{ii}.

This can be justified by noting that as we are essentially interested in inter-sectoral relations, the intra-sectoral dissipations represented by the e_{ii} are outside our interests. Indeed, this principle is frequently applied in the construction of standard financial Input-Output tables, where the x_{ii} elements are often suppressed.

Given our modified map of energy dissipation $\{e_{ij}\}$ the problem of assessing its organisation can be presented in terms of the "spread-outness" of the sizes of the components of the map. If half of the e_{ij} were small and the other half were large, we would surely consider the economy less interconnected and organised than if most of the e_{ij} were of a similar size, with only a few large and a few small elements. We would therefore want a measure of organisation which reflected the frequency distribution of the relative sizes of the elements, the more peaked the distribution the more even in size will be the elements, and the greater the organisation of the economy. As has already been discussed, the entropy, or information, measure has just such a property.

We recall that for a set of probabilities (or more accurately here, frequencies) $\{p_i\}$ (i.e. $\sum_i p_i = 1$), the entropy of the set is defined as:

$$H = -\sum_i p_i \log p_i$$

It was shown earlier that H is maximised when $p_i = 1/n \ \forall\ i$, when:

$$H_{max} = -n\ (1/n \log 1/n) = \log n$$

H is minimised when $p_i = 1,\ p_j = 0 \ \forall\ j \neq i$, when:

$$H_{min} = -(1 \log 1) = 0$$

However, our n x n table is not a set of frequencies, but it can be converted into one by the normalisation:

$$p_{ij} = \frac{e_{ij}}{\sum_i \sum_j e_{ij}}$$

The definition of the entropy of the <u>table</u> $\{p_{ij}\}$ is then:

$$H^t = -\sum_i \sum_j p_{ij} \log p_{ij}$$

$$= -\sum_i \sum_j (e_{ij}/\sum_i \sum_j e_{ij}) \log(e_{ij}/\sum_i \sum_j e_{ij})$$

This is maximised when $e_{ij} = \sum_i \sum_j e_{ij}/n^2 \ \forall \ i, j$

when $H^t_{max} = -n^2(1/n^2 \log 1/n^2) = 2 \log n$

Similarly H^t is minimized when $e_{kl} = E_{ind}$, $e_{ij} = 0 \ \forall \ i \neq k, j \neq l$, when $H^t_{min} = 0$.

We see that if H^t is defined on $\{e_{ij}\}$ as above then H^t is a maximum when all sectors are equally connected with all others, and a minimum when only one sector is connected with one other. This would certainly be in line with our notion of organisation. But we must also be sure that if we transform the energy table in some way, then the change in the measure H^t must conform with out intuition about the change in the degree of organisation exhibited by the economy. That is, any measure of organisation should be at least a quantification of our intuitive expectations, though the usefulness of the measure must also be judged by its ability not only to conform with, but also surpass intuition.

We should first note that H^t has a maximum value which depends upon the number of sectors into which the economy can be, or needs to be, subdivided. If it makes no sense to divide an economy into more than agriculture and smithying, our intuition does indeed suggest that such an economy has less potentiality for organisation than one which needs, for its full description, division into agriculture, smithying and boat building. The example is extreme, and is not meant to suggest that economic progress consists only in adding sectors to an economy. Sectors are also lost or replaced e.g. parchment making. The point remains, though, that intuition would suggest that an economy with one thousand sectors has greater scope for becoming organised than one with ten sectors.

Here the measure H^t is in accord with intuition, H^t_{max} being a monotonic increasing function of n. Similarly, the simplest possible notion of an economy is where only one good is produced by one sector, and here H^t is a minimum, again in accord with intuition.

Even though these maximum and minimum properties of H^t seem reasonable, what of transformations of the e_{ij} when H^t is away from its extreme points? For example, what happens to the "organisation" of the energy dissipation of an economic system when another sector is added to the system? Also, if an extra sector is gained by disaggregation, or lost by aggregation, what is the effect on the system's organisation? And what is the effect on the organisation if a sector autonomously changes its level of activity? In Appendix A detailed proofs are given of the following four theorems.

Theorem 1 If a sector is added to an n sector economy, causing an increase in its dissipation by δE, then a sufficient condition that H^t increases is that the increase in energy dissipation as a proportion of the original energy dissipation is less than $1/n^2$. i.e. $\delta E/E < 1/n^2$.

Theorem 2 On disaggregating one sector of an economy into two sectors, H^t for that economy can never decrease, the maximum increase depending upon the relative amount of energy dissipated by the disaggregated sector to the total energy dissipation of the economy.

Theorem 3 On aggregating two sectors of an economy into one sector, H^t for that economy can never increase, the maximum decrease depending upon the relative amount of energy dissipated by the aggregated sector to the total energy dissipation of the economy.

Theorem 4 If one sector of an economy decreases (increases) its energy dissipation, proportionally across its functions, then H^t for that economy is increased (decreased) if the proportion of total energy dissipation attributable to that sector is greater (less) than its proportional contribution to H^t. Otherwise H^t is decreased (increased).

Our intuition might suggest that adding another sector to an economy will make it more organised. Theorem 1 indicates that the most stringent condition upon H^t increasing is that $\delta E/E < 1/n^2$, which is indeed quite a harsh condition. As shown in Appendix A, if the conditions on the initial state of the economy are relaxed somewhat the condition becomes $\delta E/E < 1/n$; i.e., the added sector must not be more energy dissipating than the average sector. The assumptions needed for this result are still much harsher than one would anticipate needing in practice. One might therefore conclude that adding a sector will "almost always" increase H^t, in accord with intuition.

Another property we might anticipate for a measure of organisation is that if one sector of the economy were disaggregated into two sectors, greater detail would be exposed, and we would therefore regard the new map of the economy as reflecting more organisation. Similarly, aggregation would reduce detail and reduce the observed organisation. Theorems 2 and 3 show that H^t satisfies both of these requirements.

The final problem here is that of changing the relative "importance" of a sector in an economy. We would anticipate that if a sector dissipates a larger than average amount of energy in the economy, reducing the scale of its activities will cause interrelationships between the other sectors in the economy to become proportionally more important. The "strengths" of interrelation between sectors will therefore become more even in the economy, causing one to see the economy as more organised. Similarly, increasing the scale of activity of a less important sector will also make the elements of the table more even in size, and increase organisation. Theorem 4 shows that H^t exactly satisfies this requirement.

So far, then, the H^t measure would seem to fulfil our requirements as a measure of organisation as applied to a table of energy dissipations. Further insights into the behaviour of H^t are to be had by considering the value of H^t generated by its application to a table of dissipations, the elements of which have a known frequency distribution. In Appendix B it is shown that the expected value of H^t can be expressed in terms of the expected values of its elements; i.e:

$$\langle H^t \rangle = 2 \log n + \log \langle z \rangle - \langle z \log z \rangle / \langle z \rangle$$

The function $\langle H^t \rangle$ has been evaluated for four distributions. The results are displayed in Table 1.

The estimated value for $\langle H^t \rangle$ has been tested by simulation for the rectangular distribution, and this comparison is displayed in Graph 1. There is evidently quite a good fit for a five sector economy and a very good fit for a ten sector economy.

It will be noted that the Rectangular and Negative Exponential distributions depend on only one parameter and are therefore of fixed "shape". As H^t depends only upon the normalised values of the elements, any Rectangular or Negative Exponential distribution of elements will produce the same value for H^t, respectively. Now the Negative Exponential distribution runs from 0 to $+\infty$, while the Rectangular distribution is contained within finite limits. That is, the elements generated by a Rectangular distribution might tend to be "more similar" than those generated by a Negative Exponential distribution. We would therefore anticipate that H^t for a Rectangular distribution is greater than for a Negative Exponential distribution. We see that this is indeed the case, as $\Gamma' + \log 2 < 3/2$.

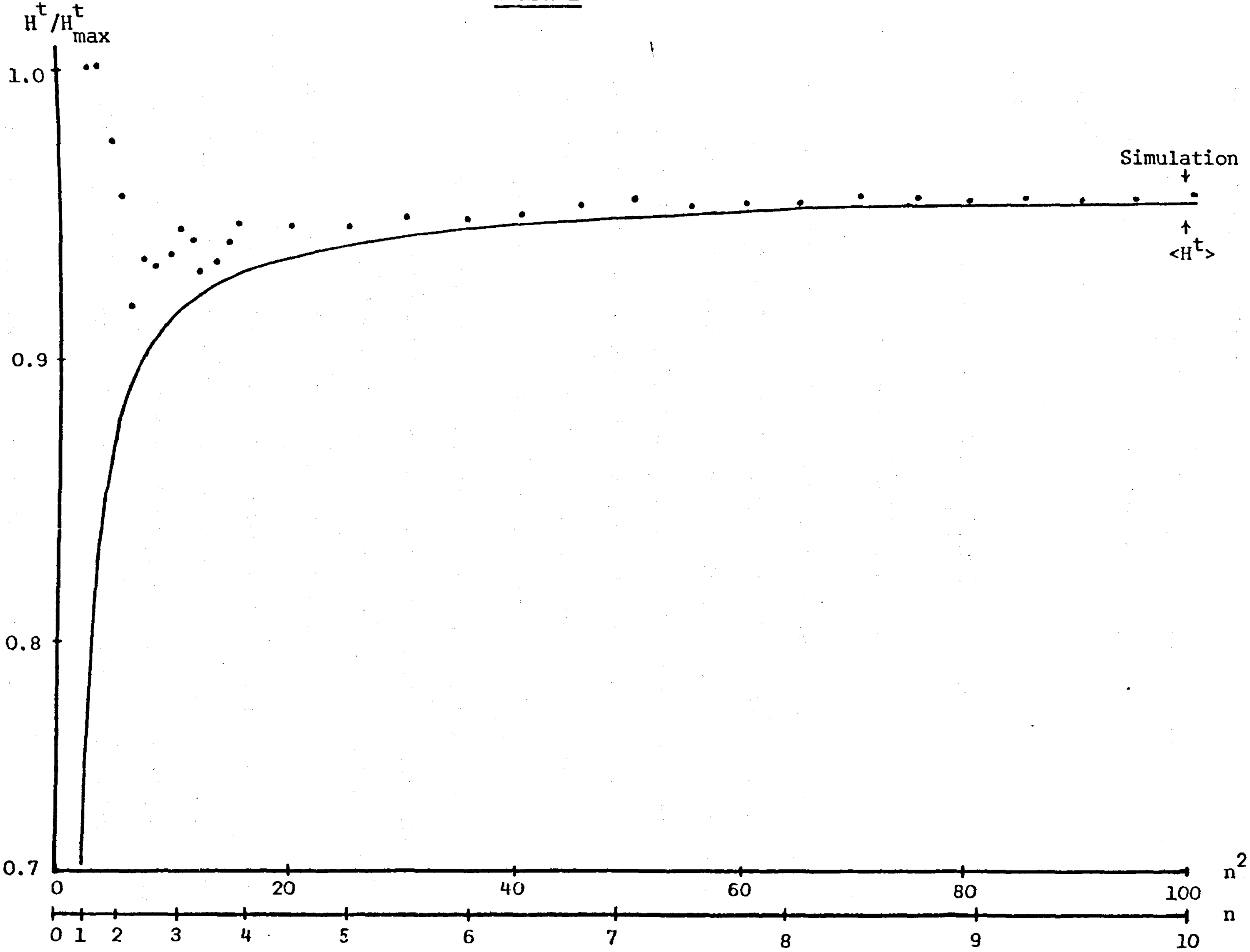
When a distribution depends upon more than one parameter, its shape will be determined by the relative sizes of the parameters. In the case of the Gamma and Lognormal distributions the parameters governing the size of H^t can both be expressed in terms of the mean, μ, and the variance, v, of the distribution. For the Gamma distribution $p = \mu^2/v$, while for the Lognormal distribution $s^2 = \log(\mu^2 + v) - 2 \log \mu$. For example, using $\mu = 1$ and $v = 1/3$, we find that in this case $\langle H^t \rangle$ for the Gamma distribution

TABLE 1

Distribution	Function	$\langle H^t \rangle$
Rectangular	$\begin{cases} f(z) = 1/c, & 0 \leq z \leq 2c \\ f(z) = 0 & \text{otherwise} \end{cases}$	$2 \log n - \log 2 + 1/2$
Negative Exponential	$f(z) = \alpha e^{-\alpha z}$	$2 \log n - 1 + \Gamma'$
Gamma	$f(z) = (\alpha z)^{p-1} e^{-\alpha z}/(p-1)!$	$2 \log n + \log p - \sum_{i=1} 1/i + \Gamma'$
Lognormal	$f(z) = \dfrac{\exp(-(\log z-m)^2/2s^2)}{zs(2\pi)}$	$2 \log n - s^2/2$

($\Gamma' \equiv 0.57721 \ldots\ldots.$ Euler's number).

GRAPH 1



H^t/H^t_{max}
1.0
0.9
0.8
0.7
Simulation
$<H^t>$
0
20
40
60
80
100
n^2
0 1 2 3 4 5 6 7 8 9 10
n

is approximately 1½% larger than for the Lognormal. That is, for this combination of mean and variance the Gamma distribution is more "peaked" than the Lognormal distribution.

We can also examine the behaviour of $\langle H^t \rangle$ as a function of n when compared with the maximum possible value of H^t.

Now $H^t_{max} = 2 \log n$

so:

$$\frac{\langle H^t \rangle}{H_{max}} = \frac{2 \log n + \log \langle z \rangle - \langle z \log z \rangle / \langle z \rangle}{2 \log n}$$

$$= 1 + \frac{\log \langle z \rangle - \langle z \log z \rangle / \langle z \rangle}{2 \log n}$$

Now the numerator of the element on the right is constant for a given distribution. Thus the ratio $\langle H^t \rangle / H^t_{max}$ tends to unity as the number of the sectors in the economy tends to infinity. So increasing the number of sectors in an economy by accretion of "similar" sectors will cause organisation to increase <u>and</u> increase the level of organisation as a proportion of the maximum possible organisation. So growth by accretion has positive returns to scale for organisation.

We should now enquire whether the evolution of interrelations we might anticipate seeing in an economy are such as to increase the organisation of that economy, as measured by H^t, and simultaneously increase the energy dissipation by the economy.

Let us consider a model that involves previously zero elements in the table becoming non-zero. An example would be the manufacture of a modern motor car requiring the input of plastics, artificial rubber and electrical instrumentation which were not available in, say, 1920. Suppose that in an n x n table there are q zero elements, and that over time these are filled with non-zero elements, from the same distribution as the other already non-zero elements, It is easy to show that in this case:

$$\langle H^t \rangle = \log (n^2 - q) + \log \langle z \rangle - \langle z \log z \rangle / \langle z \rangle$$

In this model the number of zero elements is decreasing over time; i.e. $\partial q/\partial t < 0$. Then $\partial \langle H^t \rangle/\partial t = -(n^2 - q)^{-1}\, \partial q/\partial t > 0$. Also, the industrial energy dissipation by the economy will be given by: $E_{ind} = (n^2 - q)\langle z \rangle$. So $\partial E_{ind}/\partial t = - \langle z \rangle \partial q/\partial t > 0$. We see that such a model would predict increasing H^t corresponding to increased energy dissipation. Of course, other models of increasing H^t can be postulated which would lead to reduced energy dissipation. For example, if all the elements of the table were reduced in size by an amount proportional to the square of their size, then the distribution of the elements would become more even, while representing less dissipation. Thus H^t would increase while E_{ind} would decrease. Indeed, it might well be the case that economies are simultaneously becoming more "connected", with $\partial E_{ind}/\partial H^t > 0$, and also becoming more "efficient" at performing their functions, with $\partial E_{ind}/\partial H^t < 0$.

The measure H^t is also rather unsatisfactory in that the "complexity" measured is dependent upon the relative sizes of the final demands for goods, as reflected by the energy dissipated directly to meet final demand, $\underline{e}^y$. Thus H^t is dependent on consumer taste. One might argue that this is no bad thing, as it is through the altering structure of final demand that economies change and evolve, and if one were to eliminate references to final demand then one would only see part of the process of economic evolution. The contrary argument is that the "inter-connectedness" of an economy should be held as conceptually distinct from the "interrelationships" that occur, these interrelationships being dependent upon the conjunction of inter-connectedness and the particular structure of final demand at that time. Inter-connectedness will obviously be technologically determined, and as such may better reflect the underlying nature of the economy, and its evolution over time, than a measure such as H^t, which involves in its formulation the consuming public's relative preferences for bananas and brown boots. We shall therefore endeavour to devise a measure of economic organisation which is independent of the structure of final demand. To proceed we recall the energy dissipation equation from the previous chapter:

$$e_i = \sum_j e_{ij} + e_i^y$$

i.e. $$e_i = \sum_j q_{ij} e_j + e_i^y$$

i.e. $$\underline{e} = \underline{Q}\,\underline{e} + \underline{e}^y$$

i.e. $$\underline{e} = (\underline{I} - \underline{Q})^{-1}\,\underline{e}^y$$

i.e. $$\underline{e} = \underline{V}\,\underline{e}^y$$

The matrix $\underline{V}$ contains elements which correspond to the e_{ij} elements from the dissipation table, except that while the e_{ij} refer to a given, though probably changing, vector $\underline{e}^y$, the elements v_{ij} refer to the unit vector. Let us consider the elements of $\underline{V}$. The simplest configuration of the v_{ij} will occur when the only energy dissipated by the economy is that dissipated directly due to production of output directly to final demand. In this case: $v_{ii} = 1 \ \forall\ i$, $v_{ij} = 0 \ \forall\ j \neq i$.

The most complicated configuration will be where all of the elements of the matrix take the same value, i.e. v_{ij} = const $\forall$ i,j.

If we define a normalised entropy function, H^m, on the elements of this <u>matrix</u> we have:

$$H^m = -\sum_i \sum_j (v_{ij}/\sum_i \sum_j v_{ij}) \log (v_{ij}/\sum_i \sum_j v_{ij})$$

We see that the simplest configuration gives:

$$\begin{aligned} \min H^m &= -\sum_i (1/n) \log (1/n) \\ &= -n((1/n) \log (1/n)) \\ &= \log n \end{aligned}$$

The most complicated configuration gives:

$$\max H^m = -\sum_i \sum_j (\alpha/n^2\alpha) \log (\alpha/n^2\alpha)$$

$$= -n^2 (\alpha/n^2\alpha) \log (\alpha/n^2\alpha)$$

$$= 2 \log n$$

We see from this that, once again, the maximum organisation of the economy depends upon the number of sectors involved, in accordance with our intuitive requirements.

Unfortunately it is clearly not possible to derive theorems for H^m, similar to those for H^t, under aggregation, marginal changes of sectors, etc.. The elements of an inverted matrix are not susceptible to such treatment. The lack of such theorems must certainly be regarded as a point against H^m.

The expected value of H^m can be calculated, though. If we write $(\underline{I} - \underline{Q})^{-1} = \underline{V} = (\underline{I} + \underline{W})$, and assume the w_{ij} are elements from a given distribution function, then it is shown in Appendix C that the expected value of H^m is given by:

$$\langle H^m \rangle = \log (n^2\langle z \rangle + n) - \frac{(n-1)\langle z \log z \rangle + \langle z \rangle + \langle z^2 \rangle}{n\langle z \rangle + 1}$$

If we assume that of the n^2 elements, q are initially zero, then the expression becomes:

$$\langle H^m \rangle = \log ((n^2 - q)\langle z \rangle + n) - \frac{(n^2-q-n)\langle z \log z \rangle + \langle z \rangle + \langle z^2 \rangle}{(n^2-q)\langle z \rangle + n}$$

We might again suggest that the initially zero elements will tend to become non-zero over time, i.e. $\partial q/\partial t < 0$.
By differentiating $\langle H^m \rangle$ with respect to t we see that the necessary and sufficient condition that $\partial H^m/\partial t > 0$ is that $n > \langle z \log z \rangle / \langle z \rangle$. We would normally expect $\langle z \log z \rangle / \langle z \rangle$ to be $O(1)$, so this condition would normally be met.

So we would anticipate that H^m would increase as more interconnections were established in the economy. If we assume that the vector $\underline{e}^y$ is constant for the economy, recalling that $E_{ind} = \underline{u}(\underline{I} + \underline{W})\ \underline{e}^y$, infilling of zero elements of $\underline{W}$ will cause E_{ind} to increase for a given $\underline{e}^y$. So causing H^m to increase by this method will also increase E_{ind}. However, it is very likely that over time $\underline{e}^y$ will in fact change, so the relationship between increasing H^m and increasing E_{ind} may be absent, even though there is infilling of $\underline{W}$.

This problem can be avoided by simply setting $\underline{e}^y = \underline{u}$; i.e.,by normalising the effects of final demand. The corresponding normalised energy dissipation will then be $E_n = \underline{u}'(\underline{I} + \underline{W})\ \underline{u}$, which is simply the sum of all of the elements of $(\underline{I} + \underline{W})$. The relationship between H^m and E_n will then, under infilling, again be increasing.

Once more we should note that infilling is only one method of causing H^m to increase, and methods can be envisaged which would simultaneously <u>decrease</u> E_n. A positive relationship between H^m and E_n may therefore appear likely, but it is not necessary. The identification of such a relationship in a real economy will therefore not be merely tautologous, but will give insight into the mode of organisational change exhibited by that economy.

So far we have noted two possible measures of economic organisation, H^t and H^m, and two possible measures of energy dissipation, E_{ind} and E_n. We might therefore test the hypothesis that the energy dissipation by economies increases with organisation by testing whether we can support the relationships:

$$\partial H^t / \partial E_{ind} > 0$$

$$\partial H^m / \partial E_{ind} > 0$$

$$\partial H^m / \partial E_n > 0$$

These methods will be used, but because the formulation of H^t and H^m requires the use of Input-Output tables, and since suitable comparable tables are available for only a limited number of countries over a limited period, it would be useful if we could construct a proxy for H which depended only upon aggregated data, and which would therefore be available for many more countries over longer periods. The proxy we shall use is the ratio of industrial energy use to Gross Domestic Product; i.e., E_{ind}/GDP. Let us recall that $E_{ind} = \underline{c}\ (\underline{I} - A)^{-1}\underline{y}$ and GNP $= \underline{u}\ \underline{y}$. So the proxy measure is $\underline{c}\ (\underline{I} - \underline{A})^{-1}\ \underline{y}/\underline{u}\ \underline{y}$.

Let us also recall that if an economy is linearly related then:

$$(\underline{I} - \underline{Q})^{-1} = \underline{A}^g(\underline{I} - \underline{A})^{-1}\underline{A}^{g-1}$$

i.e. $$\underline{A}^{g-1}(\underline{I} + \underline{W})\underline{A}^g = (\underline{I} - \underline{A})^{-1}$$

So the proxy measure can be written: $\underline{c}\ \underline{A}^{g-1}(\underline{I} + \underline{W})\underline{A}^g\ \underline{y}/\underline{u}\ \underline{y}$. Now $\underline{A}^g$ has only diagonal elements, so for constant $\underline{c}$ and $\underline{y}$ infilling of $\underline{W}$ will cause this ratio to increase. Now we know that infilling of $\underline{W}$ will also increase H^m, so we would expect the ratio E_{ind}/GNP to increase with H^m if the structure of final demand does not change too rapidly.

One further complication to note is that we are seeking the relationship between economic organisation and energy dissipation. Now it is obvious that the energy dissipation by an economy can increase simply by expanding the size of the economy. We would therefore wish to normalise the energy dissipation with respect to economic size in some way. This is automatically accomplished for E_n. We shall normalise E_{ind} in terms of the population of the economy, P. So we shall be seeking a positive relationship between E_{ind}/GNP and E_{ind}/P.

In this study, with rather meagre data available, we can only hope to identify the signs of interactions between the variables. We shall therefore posit a model for statistical analysis which is relatively insensitive to the particular functional forms describing the underlying relationship between the variables, but still unambiguously reflects the sign of the slope of the relationship. This model is that of constant elasticity, i.e. : $E = e^a H^b$. This can be written more conveniently as:

$$\log E = a + b \log H$$

The sign of the coefficient b will indicate whether E tends to increase or decrease as H increases, i.e., whether the rate of energy dissipation increases or decreases with organisation.

Two groups of data were used initially. The first is the UK Input-Output tables for 1963, 1968 and 1970[2]. The second group was composed of fifteen Input-Output tables for six countries, made available for this study by the team working under Prof. Bottomley at Bradford University. The method of extraction from these tables of the variables used below is explained in Appendix D. The values of the relevant variables for the two groups are shown in Tables 2 and 3.

The I-O tables for the data in Table 2 were 70 x 70, while those used for Table 3 were 10 x 10. This means that, as noted earlier, the values of H^t and H^m are not fully comparable, even though they have all been expressed as the proportions of the corresponding maximum value.

We shall start by examining what should, in theory, be the most robust relationship. This is between $\log E_n$ and $\log H^m$. Here all final demand effects have been eliminated, and the energy dissipation has been normalised in an unambiguous fashion.

With only three observations the data in Table 2 cannot be asserted to give any statistically very significant results. They may act as useful pointers, though, as the errors in the variables associated with the 70 x 70 UK tables are probably far smaller than those associated with the other 10 x 10 tables. Here we note that for the UK, 1963 - 1970, $\partial \log E_n/\partial t < 0$ and $\partial \log H^m/\partial t > 0$; i.e. $\partial \log E_n/\partial \log H^m < 0$. This

2. Input-Output Tables for the United Kingdom, HMSO, London, 1968; 1973; 1975.

TABLE 2

(UK)

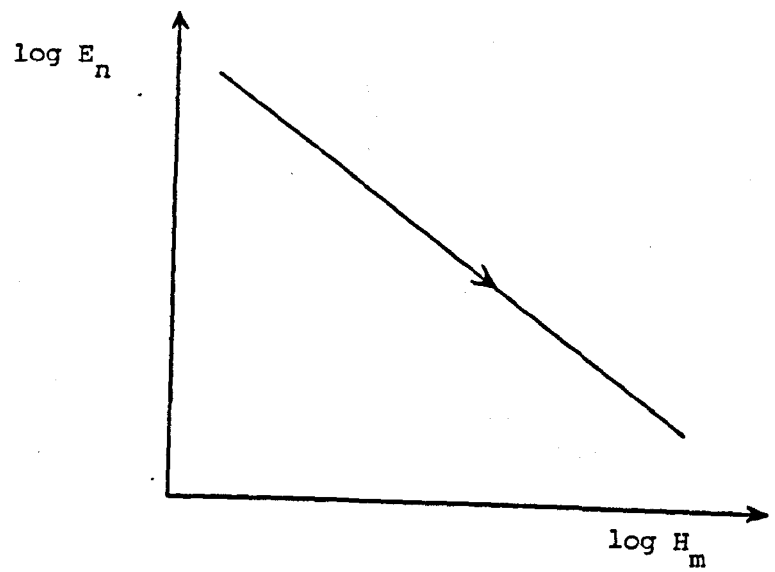
	log H^t	log H^m	log E_n	log (E_{ind}/P)	log (E_{ind}/G)
1963	-0.519	-0.327	5.794	9.793	2.597
1968	-0.507	-0.322	5.667	9.845	2.574
1970	-0.496	-0.313	5.649	9.876	2.572

TABLE 3

	Country	Date	log H^t	log H^m	log E_n	log (E_{ind}/P)	log (E_{ind}/G)
1.	Phillipines	1956	-0.646	-0.377	3.005	6.699	1.559
2.	Phillipines	1965	-0.508	-0.323	2.806	6.482	1.579
3.	India	1951-2	-0.789	-0.393	2.862	6.305	2.246
4.	India	1960-1	-0.439	-0.291	2.945	6.245	2.010
5.	India	1964-5	-0.453	-0.333	2.845	6.762	2.318
6.	Netherlands	1948	-0.577	-0.290	2.929	8.662	2.332
7.	Netherlands	1953	-0.594	-0.291	2.945	8.977	5.102
8.	Netherlands	1957	-0.477	-0.266	3.213	9.200	2.656
9.	Netherlands	1965	-0.607	-0.316	3.108	9.202	2.029
10.	Yugoslavia	1958	-0.533	-0.317	3.240	8.471	3.308
11.	Yugoslavia	1964	-0.537	-0.373	2.856	8.823	2.778
12.	U.K.	1935	-0.578	-0.361	2.924	9.076	3.065
13.	U.K.	1968	-0.578	-0.269	3.122	9.845	2.401
14.	Japan	1951	-0.352	-0.262	3.776	8.135	3.255
15.	Japan	1965	-0.482	-0.231	3.382	9.244	2.674

behaviour is illustrated in Figure 1, the arrow indicating the direction of change over time.

Figure 1



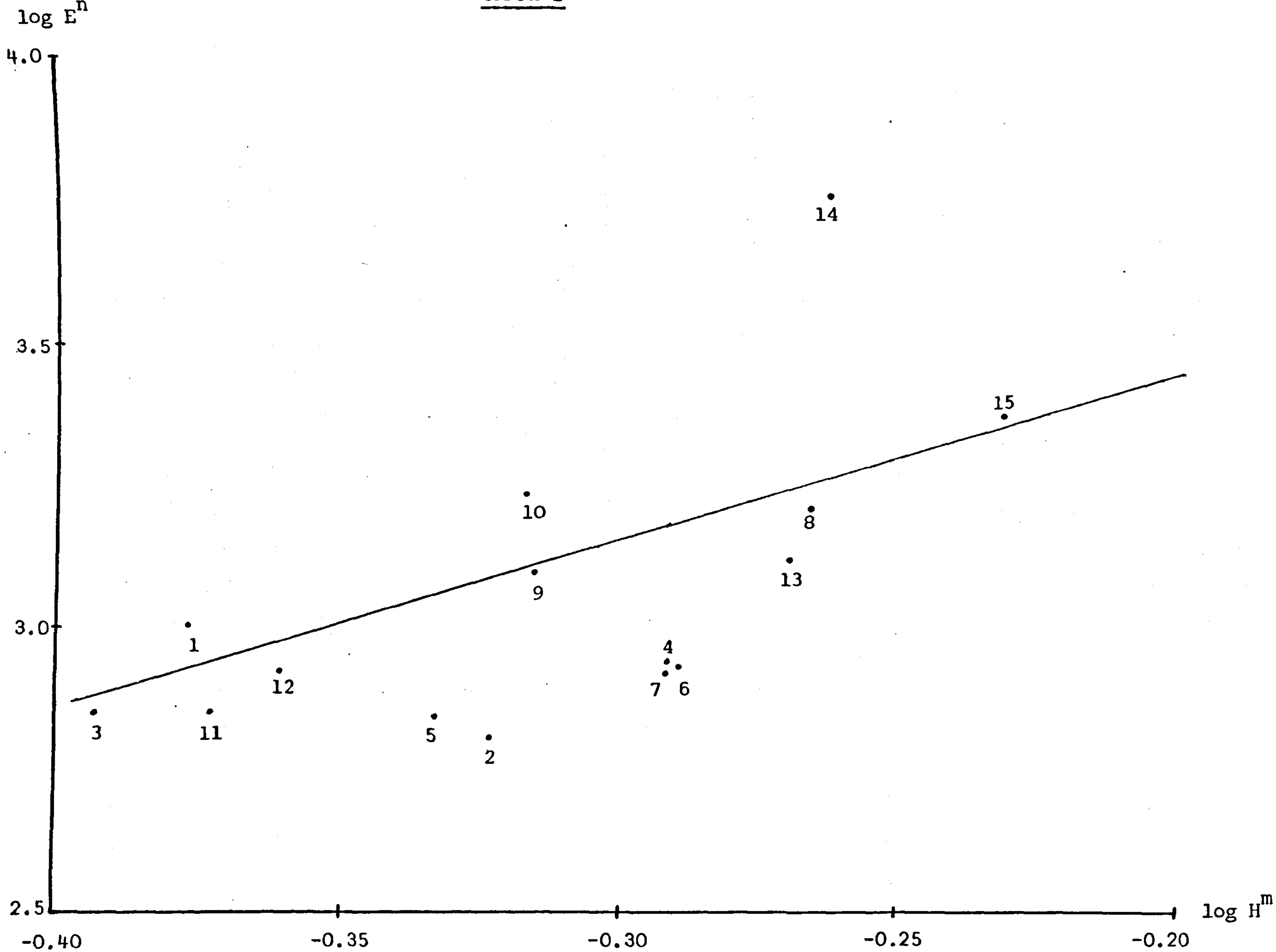
This behaviour runs counter to our hypothesis of E_n increasing with H_n. But we can recall that as well as there being the possibility of E_n increasing with complexification, it might also be reduced by increasing "efficiency". The data in Table 3 seem to support the existence of this dual effect. When log E_n is regressed against log H^m for the fifteen observations, the fitted equation is:

$$\log E_n = 4.13 + \underset{(2.912)}{3.40} \log H^m \qquad R^2 = 0.395$$

(The number in parentheses is the t-value).

The R^2 is not particularly impressive, as one might anticipate with pooled cross-section and time series data. The coefficient of log H^m is positive, though, and the t-value indicates that it is significantly different from zero at the 2% level, and significantly greater than zero at the 1% level. We are therefore reassured that $\partial \log E^n / \partial \log H^m > 0$, as hypothesised. However, when the data and the fitted line are plotted, as in Graph 2, we see that the behaviour of the individual countries is erratic. One might surmise that this arises from the cross-sectional

GRAPH 2


log E^n
4.0
3.5
3.0
2.5
log H^m
-0.40
-0.35
-0.30
-0.25
-0.20
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

behaviour of the data being at variance with the time series behaviour. To test this it was postulated that energy dissipation in a given country was related to the degree of organisation, but also changed over time. The model used was:

$$E_n = e^a (H^m)^b e^{\gamma t}$$

or:

$$\log E_n = a + b \log H^m + \gamma t$$

The fitted equation is:

$$\log E_n = 4.51 + \underset{(2.978)}{3.66} \log H^m - \underset{(0.793)}{0.0053t} \qquad R^2 = 0.425$$

The coefficient of $\log H^m$ is again significantly greater than zero at the 1% level. The coefficient of t is less than zero at only the 25% level. So we now have $\partial \log E_n / \partial \log H^m > 0$ and $\partial \log E_n / \partial t < 0$ (perhaps).

To fully understand the time series behaviour of the data we also need to know the sign of $\partial \log H^m / \partial t$. To find this we postulate the model: $H^m = e^a (E_n)^b e^{\gamma t}$. The fitted equation is:

$$\log H^m = -0.75 + \underset{(2.978)}{0.12} \log E_n + \underset{(1.235)}{0.0014t} \qquad R^2 = 0.463$$

As we would anticipate, $\partial \log H^m / \partial \log E_n > 0$, but also we see that $\partial \log H^m / \partial t > 0$, with the coefficient of t being significantly greater than zero at the 15% level. We can therefore assert with some confidence that $\partial \log E_n / \partial \log H^m > 0$, and with more temerity that $\partial \log E_n / \partial t < 0$ and $\partial \log H^m / \partial t > 0$.

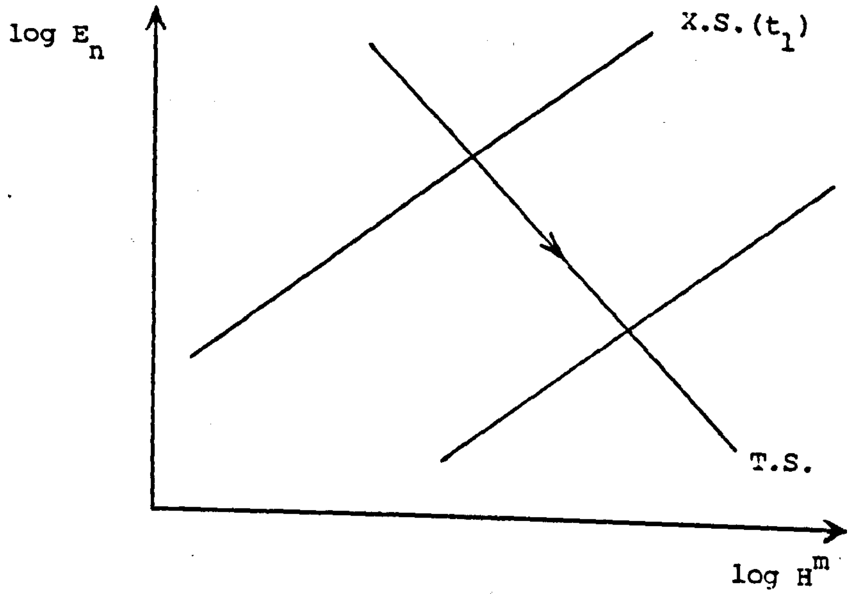
Thus over time the cross-section (X.S.) fitted line is moving downwards, in the direction of the time series (T.S.) line, as in Figure 2.

Our tentative conclusions from this analysis are now in correspondence with the downward sloping curve for the UK, 1963 - 1970.

There is a positive slope to the relationship between E_n and H^m when considered <u>between</u> countries at a given time, but a downward slope <u>within</u>

a country over time. This is in line with the speculation that dissipation may be increased by increasing organisation, but decrease over time due to increasing "efficiency".

Figure 2



We can now turn to the relationship between E_{ind}/P and H^m. We have already noted that E_{ind}/P is probably a less good measure of energy dissipation than E_n. It is, however, considerably more accessible.

The data for UK 1963-70 indicate that $\partial \log (E_{ind}/P)/\partial t > 0$ and $\partial \log H^m/\partial t > 0$; i.e. $\partial \log (E_{ind}/P)/\partial \log H^m > 0$.

Although this is in agreement with our original hypothesis, it now runs counter to the time series behaviour just noted between E_n and H^m.

When the pooled data are examined as above, the fitted equations for these variables are:

$$\log (E_{ind}/P) = 13.82 + \underset{(1.977)}{13.61} \log H^m - \underset{(0.661)}{0.025t}$$

$$R^2 = 0.247$$

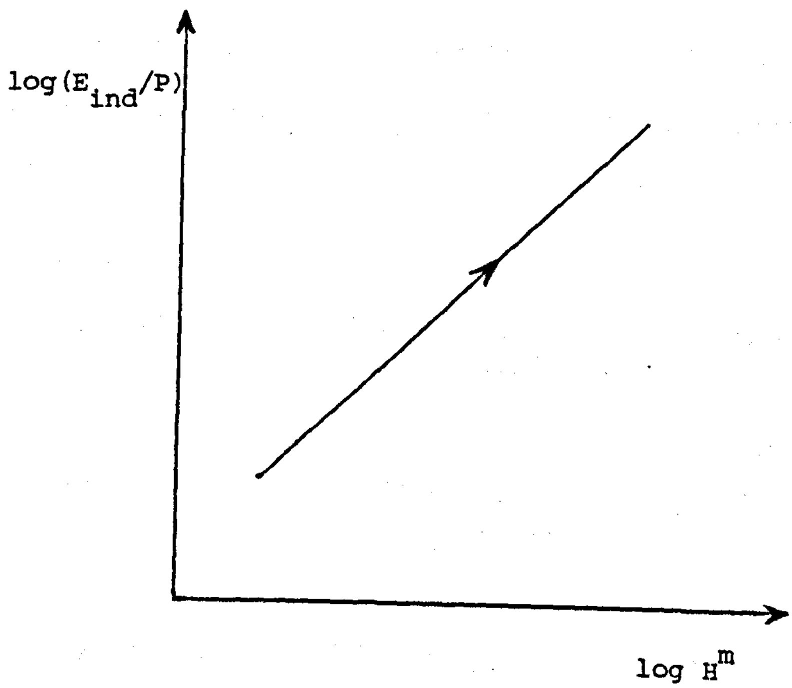
$$\log H^m = -0.55 + \underset{(1.977)}{0.018} \log (E_{ind}/P) + \underset{(1.140)}{0.0015t}$$

$$R^2 = 0.296$$

Again we see $\partial \log (E_{ind}/P)/\partial \log H^m > 0$ at the 5% significance level, and $\partial \log (E_{ind}/P)/\partial t < 0$ (30% level) and $\partial \log H^m/\partial t > 0$ (20% level).

This is in line with the earlier pooled data analysis, but it is interesting to note that we find $\partial \log(E_{ind}/P)/\partial t < 0$, as most work in this area indicates that $\partial \log (E_{ind}/P)/\partial t > 0$. If we therefore reject this finding, and assert that $\partial \log(E_{ind}/P)/\partial t > 0$, then the time series and cross-section findings give the same slope, and are in accord with the UK 1963-70 figures.

Figure 3



The contradiction between this time series behaviour, indicated in Figure 3, and that for H^m and E_n might be explained in terms of the effect of final demand increasing over time, and/or changing in structure, the resulting increased energy dissipation per capita masking the "efficiency" effect.

Next we examine the relationship between H^t and E_{ind}/P. For the UK 1963-70 we once more see that $\partial \log H^t/\partial t > 0$ and $\partial \log (E_{ind}/P)/\partial t > 0$ giving $\partial \log (E_{ind}/P)/\partial \log H^t > 0$.

The pooled data gives:

$$\log (E_{ind}/P) = 9.24 + \underset{(0.311)}{1.13} \log H^t - \underset{(0.198)}{0.0084t}$$

$$R^2 = 0.010$$

$$\log H^t = - 0.74 + \underset{(0.311)}{0.0071} \log (E_{ind}/P) + \underset{(0.735)}{0.0024t}$$

$$R^2 = 0.049$$

Here the low values of R^2 and the appalling t-values only tell us that we can assert no significant relationship exists between these variables.

We therefore pass on to examine the relationship between $\log (E_{ind}/P)$ and $\log (E_{ind}/G)$, where $G \equiv$ GDP. For the UK 1963-70 we have $\partial \log (E_{ind}/P)/\partial t > 0$ and $\partial \log(E_{ind}/G)/\partial t < 0$, giving $\partial \log(E_{ind}/P)/\partial \log (E_{ind}/G) < 0$.

The pooled data give fitted equations:

$$\log (E_{ind}/P) = 5.06 + \underset{(1.937)}{0.75} \log (E_{ind}/G) + \underset{(0.497)}{0.019t}$$

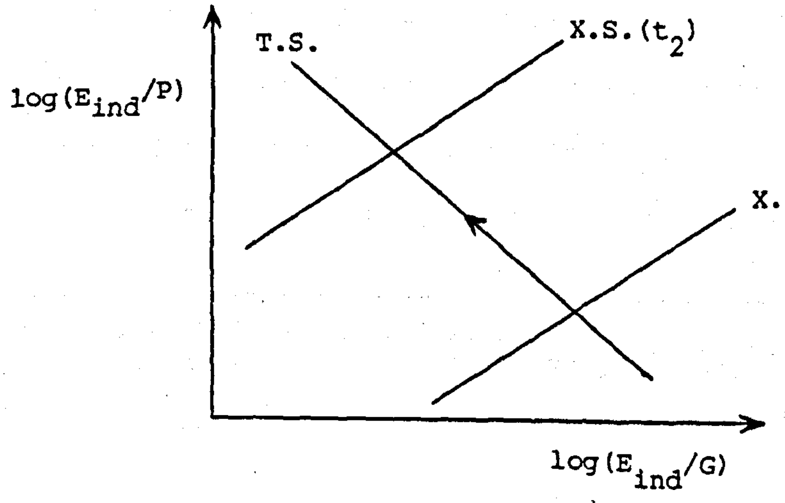
$$R^2 = 0.239$$

$$\log (E_{ind}/G) = 1.84 + \underset{(1.937)}{0.32} \log (E_{ind}/P) - \underset{(1.323)}{0.031t}$$

$$R^2 = 0.322$$

Here we have $\partial \log (E_{ind}/P)/\partial \log (E_{ind}/G) > 0$ at the 5% level, $\partial \log(E_{ind}/P)/\partial t > 0$ (50% level) and $\partial \log (E_{ind}/G)/\partial t < 0$ (15% level). This is in agreement with the UK 1963-70 evidence. The relationship between cross-section and time series effects represented by these slopes is shown in Figure 4.

Figure 4



We should first note that the time trend revealed for log (E_{ind}/P) is upwards here, in line with other estimates, but the significance level is very poor. The significance level for the time relationship for log (E_{ind}/G) is quite good, though.

We see that the time series relationship between dissipation and organisation as measured by E_{ind}/P and E_{ind}/G exactly contradicts that we noted earlier between E_n and H^m. Before analysing this contradiction we can first ensure that the time trends for E_{ind}/P and E_{ind}/G are soundly based. We can do this only at the expense of using energy statistics referring to <u>total</u> energy use, where use of energy by domestic consumers is included with industrial energy use. That is, in the above notation we must use $E = E_{ind} + E_{dom}$, instead of the preferred E_{ind}.

Data for thirty-one countries were taken from Darmstadter <u>et. al</u>.,[3] for the years 1955, 1960 and 1965. Following Slesser's[4] comment, the hydro-power contributions for these countries have been converted to the effective fossil fuel consumption that would have been necessary to produce that much electricity. This procedure was not used for the UK 1963-70 data or the pooled data, as there consideration is only taken of energy consumed by each industry, rather than of the energy consumed in the economy as a whole, including conversion losses (see Appendix D).

The data used was for thirty-one countries in all, composed of nineteen developed "Western" countries, including Japan and Israel, seven Eastern Bloc countries and five developing countries. Since domestic energy use is included, which climate dictates will mostly be higher in Eastern Bloc countries and lower in developing countries than in Western countries, and as GDPs of Eastern Bloc countries are often inflated by exchange rate controls , the underlying relationship between E_{ind}/P and E_{ind}/G may be

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3. J. Darmstadter, P. Teitelbaum and J. Polach, <u>Energy in the World Economy</u>, John Hopkins University Press, London, 1971, p. 867.

4. M. Slesser, <u>Energy in the Economy</u>, MacMillan, London, 1978.

masked. To attempt to eliminate this masking two dummy variables were introduced, their values being (0,0) for Western countries, (1,0) for Eastern Bloc countries, and (0,1) for developing countries. We might make some predictions as to the signs of the coefficients of these dummy variables by considering the effects on E and G of the climate, and of currency controls.

Let us assume that overall energy use, E, can be written as the product of a "basic" dissipation, E*, and an "effect of climate" multiplier, θ, where $\theta = 1$ for Western countries. Let us also assume that currency controls inflate GDP, and this inflation is by a factor β; i.e., $G = \beta G^*$, where G^* is the "true" GDP. So $E/P = \theta E^*/P$ and $E/G = E^*/\beta G^*$.

So: $\log (E/P) = a + b \log (E/G)$

becomes $\log (\theta E^*/P) = a + b \log (\theta E^*/\beta G^*)$

i.e. $\log (E^*/P) = a + b \log (E^*/G^*) + ((b-1) \log \theta - b \log \beta)$

For Eastern countries we expect $\theta > 1$, so if $b > 1$ we expect this climate effect to be positive. The value of β we assume to be greater than one for Eastern countries, so the "currency" effect will be negative. The overall effect may therefore be positive or negative.

For Developing countries we anticipate $\theta < 1$, so for $b > 1$ the climate effect will be negative. We would anticipate that $\beta \approx 1$ for these countries, so the overall effect should be negative, for $b > 1$.

When a regression was performed with no dummies and no assumed time trend, using pooled cross-section and time series data, the fitted equation was:

$$\log (E/P) = 6.57 + \underset{(8.927)}{1.53} \log (E/G) \qquad R^2 = 0.467 \text{ (DF=91)}$$

Including dummy variables D_1 and D_2 the equation becomes:

$$\log (E/P) = 6.59 + \underset{(13.77)}{1.88} \log (E/G) - \underset{(6.127)}{0.67} D_1 - \underset{(7.832)}{0.91} D_2$$

$$R^2 = 0.717$$

We see that b, the coefficient of log (E/G) <u>is</u> greater than one, and the coefficient of D_2, the Developing countries dummy has a negative coefficient, as predicted, while the Eastern countries dummy is also negative and smaller in size. We note that including the dummy variables has produced a marked improvement in the t-value of b, the coefficient of log (E/G). This is now significantly greater than zero at better than the 0.05% level, while the coefficients of the dummy variables are significantly less than zero also at the 0.05% level.

When an exponential time trend is included the fitted equation is:

$$\log (E/P) = 6.48 + \underset{(13.92)}{1.83} \log (E/G) - \underset{(6.260)}{0.66D_1} - \underset{(8.146)}{0.91D_2} + \underset{(3.095)}{0.030t}$$

$$R^2 = 0.745$$

The significance levels for the coefficients of log (E/G), D_1 and D_2 are maintained, while the coefficient of t is significantly greater than zero at the 0.25% level.

The time trend for log (E/G) was found by fitting the alternative equation:

$$\log (E/G) = -2.23 + \underset{(13.92)}{0.38} \log (E/P) + \underset{(7.785)}{0.34D_1} + \underset{(7.086)}{0.38D_2} - \underset{(1.851)}{0.0084t}$$

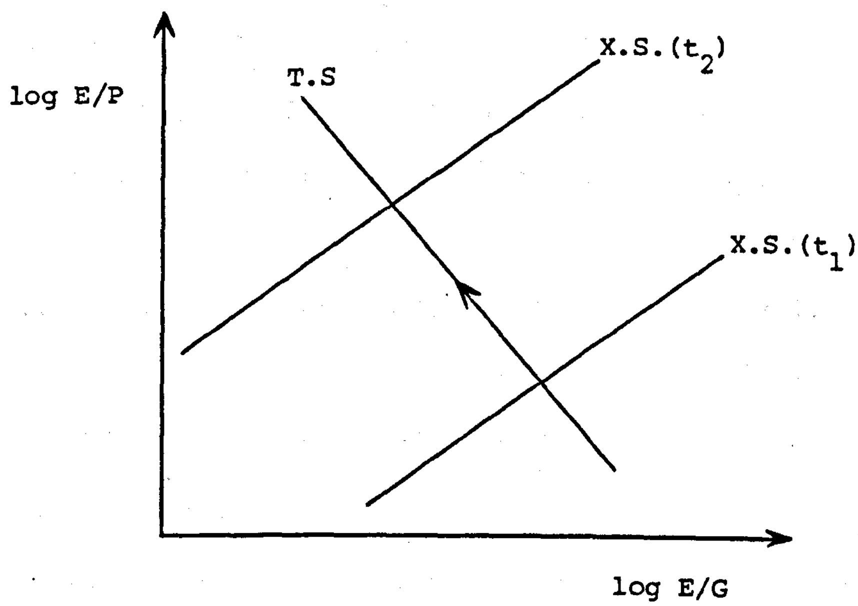
$$R^2 = 0.738$$

The coefficients of log (E/P), D_1 and D_2 are all consistent in size and sign with those in the former equation, and retain their significance levels. From these two equations we therefore have $\partial \log (E/P)/\partial \log (E/G) > 0$, $\partial \log (E/P)/\partial t > 0$ and $\partial \log (E/G)/\partial t < 0$. This behaviour is displayed in Figure 5.

This is consistent with the earlier findings for the pooled data for $\log (E_{ind}/P)$ and $\log (E_{ind}/G)$, as displayed in Figure 4. This particular result finds backing from other studies in terms of the cross-section and time series analysis of energy coefficients[5,6,7]. To see why let us recall that the energy coefficient, R, is defined by:

$$R = \frac{\Delta E/E}{\Delta G/G}$$

Figure 5



As noted in the previous chapter, for preference E should be replaced by E_{ind}. Taking limits and rearranging gives:

$$dE/E = R\ dG/G$$

Integrating gives:

$$\log E = K + R \log G$$

i.e. $$d \log E/d \log G = R$$

Now the relationship used above is of the form:

$$\log (E/P) = a + b \log (E/G)$$

i.e. $$\log E = a/(1 - b) + (b/(b - 1)) \log G - (1/(b - 1)) \log P$$

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5. J. Darmstadter, "Energy and the Economy", Energy International, August 1970, p. 31.

6. J. Darmstadter, J. Dunkerley and J. Alterman, How Industrial Societies Use Energy, John Hopkins University Press, London 1977.

7. V. Smil and T. Kuz, "European Energy Elasticities", Energy Policy, June 1976, p. 171.

So: $$\frac{d \log E}{d \log G} = \left(\frac{b}{b-1}\right) - \left(\frac{b}{b-1}\right) \frac{\partial \log P}{\partial \log G}$$

i.e. $R = (b - R_p)/(b - 1)$ where $R_p \equiv \partial \log P/ \partial \log G$

i.e. R_p is the elasticity of population with income.

Rearrangement gives:

$$b = (R - R_p)/(R - 1)$$

Now we would usually think of R_p as being small compared with R. For example, using data from Humphrey and Stanislaw[8] for the UK, $R_p \approx 0.2$ this century. Given that $R_p < 1$ we see that $R > 1$ implies $b > 0$, and $R_p < R < 1$ implies $b < 0$. Now cross-section analyses seem to mostly give $R < 1$, while time series analyses seem to give $R > 1$. These findings are perfectly consonant with our analysis giving positive cross-section slopes and negative time series slopes.

In particular, the energy coefficients calculated by Humphrey and Stanislaw[8] for the UK are greater than unity during most of the nineteenth century, and less than unity during most of the twentieth century. Fitting an exponential time trend to their data between 1835 and 1975 gives:

$$\log R = \underset{(4.670)}{0.88 - 0.0091t} \qquad R^2 = 0.627 \ (DF = 13)$$

The coefficient of t is significantly less than zero at the 0.5% level. This time trend indicates that the energy coefficient became less than unity in about 1900. As was noted in the previous chapter, the energy coefficient calculated using E will probably overestimate the preferred coefficient using E_{ind}. We can therefore feel confident that the preferred coefficient has been less than unity during this century. That is, during this century in the UK it has been the case that $\partial \log (E_{ind}/P)/\partial \log (E_{ind}/G) < 0$.

However, the discrepancy remains between the time series behaviour observed when using (E_{ind}/P) and (E_{ind}/G) as compared with using E_n and H^m.

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8. W.S. Humphrey and J. Stanislaw, "Economic Growth and Energy Consumption in the UK, 1700 - 1975", Energy Policy, March 1979, p. 29.

We might hope to understand this discrepancy by recalling that even when E_{ind} is used in preference to E, changes in E_{ind} may reflect several different effects taking place in the economy. In the previous chapter it was shown that two central difference representations of ΔE_{ind} are possible. The first, using:

$$E_{ind} = \underline{c}\,(\underline{I} - \underline{A})^{-1}\underline{Y}$$

gives:

$$\Delta E_{ind} = \Delta\,\underline{c}(\underline{I}-\underline{A})^{-1}\underline{Y} + \underline{c}\Delta(\underline{I}-\underline{A})^{-1}\underline{Y} + \underline{c}(\underline{I}-\underline{A})^{-1}\Delta\underline{Y} + \tfrac{1}{4}\Delta\underline{c}\Delta(\underline{I}-\underline{A})^{-1}\Delta\underline{Y}$$

The term $\Delta\underline{c}(\underline{I} - \underline{A})^{-1}\,\underline{Y}$ was called the "Intra-Industry" term, reflecting as it does changes in E_{ind} caused by changing direct energy intensities

The term $\underline{c}\Delta(\underline{I} - \underline{A})^{-1}\,\underline{Y}$ reflects changes in E_{ind} caused by changing interrelations between the producing sectors, and is therefore called the "Inter-Industry" term.

Finally, $\underline{c}\,(\underline{I} - \underline{A})^{-1}\,\Delta\underline{Y}$ expresses the effect on E_{ind} of changes in the size and structure of final demand. Therefore this is called the "Demand (Goods)" term.

The alternative approach is to use:

$$E_{ind} = \underline{u}\,(\underline{I} - \underline{Q})^{-1}\,\underline{e}^{Y}$$

giving:

$$\Delta E_{ind} = \underline{u}\Delta(\underline{I} - \underline{Q})^{-1}\,\underline{e}^{Y} + \underline{u}(\underline{I} - \underline{Q})^{-1}\Delta\underline{e}^{Y}$$

The term $\underline{u}\Delta(\underline{I} - \underline{Q})^{-1}\,\underline{e}^{Y}$ indicates the change in E_{ind} caused by the change in the structure of industrial energy dissipation for a constant structure of dissipation directly to meet final demand. This is called the "Indústry" term.

$\underline{u}\,(\underline{I} - \underline{Q})^{-1}\,\Delta\underline{e}^{Y}$ is the change in E_{ind} caused by the changed structure of energy dissipation to directly meet the demand for goods. This is called the "Demand (Energy)" term.

Neither of these two terms can be directly identified with the three main terms in the first approach. Obviously the Inter-Industry term is similar to the Industry term, and the Demand (Goods) term is related to the Demand (Energy) term. But both the Industry and the Demand (Energy) term will also incorporate parts of the changes associated with the Intra-Industry term.

Both Demand terms we would expect to be positive, reflecting increasing purchasing over time.

If an economy is becoming more organised over time we would expect the dissipation caused by increasing interconnections to be increasing, which might be expected to cause both the Inter-industry and the Industry terms to be positive.

Increasing "efficiency" in production by each sector would be indicated by a negative Intra-Industry term.

Both decompositions of ΔE_{ind} are given for the UK 1963-70 and the pooled data in Table 4. There are eleven observations in all. Of these seven (64%) have positive Inter-Industry terms, nine (82%) have negative Intra-Industry terms, and nine (82%) positive Demand (Goods) terms. This is in agreement with the notion of economies exhibiting increasing organisation and simultaneously increasing "efficiency".

If one assumes that all economies exhibit positive contributions to ΔE_{ind} through increased organisation, but only "mature" economies have efficiency effects large enough to outweigh this, then one might anticipate that the energy coefficient for developed countries would be lower than for developing countries. Darmstadter _et_. _al_.[9] cite the overall energy coefficients for Western developed countries between 1950 and 1975 as 0.85, and for Developing countries between the same dates as 1.67. The decreasing trend of the energy coefficient for the UK between 1835 and 1975, noted earlier, also is consistent the supposition that economies

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9. Darmstadter, Teitelbaum and Polach, _op_. _cit_., p. 37.

TABLE 4

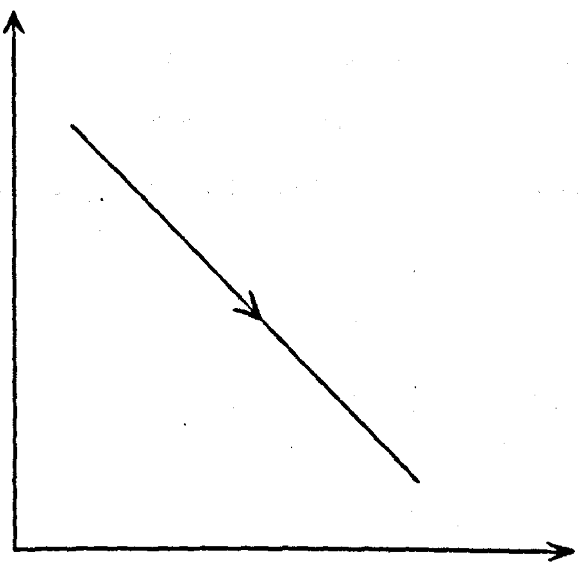
Country	Period	$\Delta c(I-A)^{-1}Y$ Intra-Industry	$c\Delta(I-A)^{-1}Y$ Inter-Industry	$c(I-A)^{-1}\Delta Y$ Demand (Goods)	$\frac{1}{2}\Delta c\Delta(I-A)^{-1}\Delta Y$ Mixed	Total	$u\Delta(I-Q)^{-1}e^{Y}$ Industry	$u(I-Q)^{-1}\Delta e^{Y}$ Demand (Energy)	Total
U.K.	63-68	-121.9	-38.7	260.7	0.1	100	106.6	6.6	100
U.K.	68-70	-295.2	56.2	339.5	-0.5	100	-103.3	203.2	100
Philippines	50-65	-206.1	145.8	163.3	-3.1	100	168.3	-68.3	100
India	51-60	79.4	319.0	-278.7	-19.7	100	-136.5	236.5	100
India	60-64	-88.6	-58.4	259.2	12.2	100	112.3	-12.3	100
Netherlands	48-53	116.9	1.7	-18.6	0.1	100	104.7	-4.7	100
Netherlands	53-57	-49.9	86.2	65.9	-2.2	100	-3.7	103.7	100
Netherlands	57-65	-972.5	-264.3	1255.3	81.6	100	266.2	-166.2	100
Yugoslavia	58-64	-144.9	-14.7	258.2	1.4	100	112.6	-12.6	100
U.K.	35-68	-135.1	35.0	208.8	-8.7	100	70.0	30.0	100
Japan	51-65	-53.0	16.8	166.2	3.6	100	137.7	-37.7	100

become more efficient as they develop.

Eight of the eleven observations (73%) have positive industry effects and seven (64%) have negative Demand (Energy) effects. These findings are consistent with the previous decomposition if one assumes that much of the effect of the Intra-Industry term appears in the Demand (Energy) term, though it should be noted that the correlations between the two sets of terms for any given country do not appear to be good.

One further point is that if the Demand (Goods) term is extracted from ΔE_{ind}, the combined Intra- and Inter-Industry terms are negative for seven (64%) of the observations. Now this suggests that the overall dissipation of economies over time is decreasing when demand effects are excluded, though organisation seems to be increasing, as instanced by the positive Inter-Industry term. That is, $\partial \text{Dissipation}/\partial t < 0$, and $\partial \text{Organisation}/\partial t > 0$.

Figure 6



This result for the time series behaviour of economies is in agreement with that noted when using E_n and H^m. It would seem, therefore, that the discrepancy between the results of the time series analysis of E_n and H_m, and the time series analysis of (E/P) and (E/G) can mainly be put down to the effects of changing demand.

We are now in a position to draw some tentative conclusions. The organisation of economies does seem to increase over time. That is, the sectors making up that economy seem to be becoming more interconnected. Also, at a given time, between countries there does seem to be positive correlation between organisation and energy dissipation. That is, economies with high levels of organisation tend to have high rates of energy dissipation, and those with lower levels of organisation tend to have lower dissipation rates.

Within a given country, though, the rate of energy dissipation seems to be decreasing when the effect of changing demand is excluded. The signs and relative sizes of the Intra-Industry and Inter-Industry effects indicate that this is because the increased dissipation generated by increased organisation is more than outweighed by the decreased dissipation consequent upon increases in the "efficiency" of the economy.

Our starting hypothesis that dissipation increases with organisation therefore seems to be borne out when static comparisons are made between economies, but does not seem to hold for dynamic analyses within any given economy.

APPENDIX A

Some Theorems on H^t

Definition 1

$$H^t(n^2, E) = \sum_{i=1}^{n} \sum_{j=1}^{n} (e_{ij}/E) \log (e_{ij}/E)$$

where $\sum_i \sum_j e_{ij} \leq E.$

i.e. $H(n^2, E)$ is defined on an n x n table.

Definition 2

$$H_c^t (n, E) = -\sum_{i=1}^{n} (e_{ij}/E) \log (e_{ij}/E)$$

where $\sum_{i=1}^{n} e_{ij} \leq E.$

i.e. H_c (n, E) is defined on a column of n elements.

Definition 3

$$H_r^t (n, E) = -\sum_{j=1}^{n} (e_{ij}/E) \log (e_{ij}/E)$$

where $\sum_{j=1}^{n} e_{ij} \leq E$

i.e. H_r(n, E) is defined on a row of n elements

Lemma 1 $H^t(n^2, E + \delta E) \approx (1 - \delta E/E)\, H^t(n^2, E)$ for $\sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij} = 1$

Proof

Now $$H^t(n^2, E + \delta E) = -\sum_{i=1}^{n} \sum_{j=1}^{n} (e_{ij}/E + \delta E) \log (e_{ij}/E + \delta E) \qquad \text{(i)}$$

And $$e_{ij}/E + \delta E = (e_{ij}/E)\,(1 + \delta E/E)^{-1}$$

$$\approx (e_{ij}/E)\,(1 - \delta E/E) \text{ for } \delta E << E \qquad \text{(ii)}$$

Also $$\log (e_{ij}/E + \delta E) = \log \{(e_{ij}/E)\,(1 + \delta E/E)^{-1}\}$$

$$= \log (e_{ij}/E) - \log (1 + \delta E/E)$$

Now for $\delta E << E$ have $\log (1 + \delta E/E) \approx \delta E/E$

So $$\log (e_{ij}/E + \delta E) \approx \log (e_{ij}/E) - \delta E/E \qquad \text{(iii)}$$

Substituting for (ii) and (iii) in (i) we get

$$H^t(n^2, E + \delta E) \approx - \sum_{i=1}^{n} \sum_{j=1}^{n} (e_{ij}/E)(1 - \delta E/E)\{\log(e_{ij}/E) - \delta E/E\}$$

$$= -(1 - \delta E/E)\{ \sum_{i=1} \sum_{j=1} (e_{ij}/E) \log (e_{ij}/E) -$$

$$\sum_{i=1}^{n} \sum_{j=1}^{n} (e_{ij}/E) \ (\delta E/E)\}$$

$$= (1 - \delta E/E) \ \{H^t(n^2,E) + \delta E/E\}$$

$$\approx (1 - \ E/E) \ H^t(n^2,E) \text{ for } \delta E/E << H^t(n^2,E)$$

Lemma 2

Consider a two compartment system, with elements a and b, where $o < a < 1$, $o < b < 1$, $o < a + b < 1$, with measure $H^t(2,1)$. If the two elements are aggregated then the new system has measure $H^{t'}(1, 1) < H^t(2,1)$, with $\max\ (H^t(2,1) - H^{t'}(1,1)) = 2a \log 2$.

Proof

Now $\quad H^t(2,1) = -a \log a - b \log b$

and $\quad H^{t'}(1,1) = -(a + b) \log (a + b)$

The change in H^t is $\Delta H^t = H^t(2,1) - H^{t'}(1,1)$

$$= -((a \log a + b \log b) - (a + b) \log (a + b))$$

Now suppose $\quad b \leqslant a$

set $\quad b = ma, \ o \leqslant m \leqslant 1$

So $\quad \Delta H^t = -((a \log a + ma \log ma) - a(1 + m) \log a(1 + m))$

$$= a((1 + m) \log (1 + m) - m \log m)$$

Now $o \leqslant m \leqslant 1$ so $\log (1 + m) > 0$ and $\log m < 0$

i.e. $\Delta H^t > 0$ so $H^{t'}(1,1) < H^t(2,1)$

Now $\dfrac{d\Delta H^t}{dm} = a \{\log (1 + m) + (\frac{1+m}{1+m}) - \log m - \frac{m}{m}\}$

$$= a \log (1 + \frac{1}{m})$$

$$\geqslant 0 \quad \text{for} \quad m \geqslant 0, \ a \geqslant 0$$

We see that ΔH^t is a monotomic increasing function of m, so ΔH^t is maximised when m is maximised, i.e. have max ΔH^t at max m = 1, i.e. at a = b.

$$\text{So max } (H^t(2,1) - H^{t'}(1,1)) = a(2 \log 2 - \log 1)$$
$$= 2a \log 2.$$

<u>Theorem 1</u>

If a sector is added to an n-sector economy, causing an increase in its dissipation by δE, then a sufficient condition that H^t increases is that the increase in energy dissipation as a proportion of the original energy dissipation is less than $1/n^2$, i.e. $\delta E/E < 1/n^2$.

<u>Proof</u>

Consider $H^t(n^2,E)$ with $\sum_{i=1}^{n} \sum_{j=1}^{n} e_{ij} = E$

Add another sector to the economy, increasing energy dissipation by δE. The new H^t is given by:-

$$H^t((n+1)^2, E + \delta E) = - \sum_{i=1}^{n} \sum_{j=1}^{n} (e_{ij}/E + \delta E) \log (e_{ij}/E + \delta E)$$
$$+ \sum_{i=1}^{n} (e_{in} + 1/E + \delta E) \log (e_{in}+1/E + \delta E)$$
$$+ \sum_{j=1}^{n} (e_{n+1j}/E + \delta E) \log (e_{n+1j}/E + \delta E)$$
$$+ (e_{n+1\ n+1}/E + \delta E) \log (e_{n+1\ n+1}/E + \delta E)$$
$$= H^t(n^2, E + \delta E) + H_c^t(n, E + \delta E)$$
$$+ H_r^t(n,E + \delta E) + H^t(1,E + \delta E)$$
$$= H^t(n^2,E + \delta E) + H^t_{add}$$

Now by Lemma 1, $H^t(n^2,E + \delta E) \simeq H(n^2, E)(1 - \delta E/E)$ i.e. H^t for the central part of the system is reduced.

But the total change in H^t is composed of this reduction plus the extra component H^t_{add}.

Now the simplest possible addition is where the new row and column contains a single non-zero element, which is δE. In this case we have:-

$$\min H^t_{add} = -(\delta E/E + \delta E) \log (\delta E/E + \delta E)$$

Now $\quad \delta E/E + \delta E \simeq \delta E/E$ for $\delta E << E$

So $\quad \min H^t_{add} \simeq -(\delta E/E) \log (\delta E/E)$

$$= (\delta E/E) \log (E/\delta E)$$

Now the reduction of H^t for the central core is $(\delta E/E)\, H(n^2,E)$ obviously this is maximised when $H(n^2,E)$ is maximised, i.e. when $e_{ij} = E/n^2\ \forall i,j$. Then $\max H(n^2,E) = \log n^2$. Now the most stringent condition on the system for H^t to increase is:-

$$\min H^t_{add} > (\delta E/E) \max H^t(n^2,E)$$

i.e. $\quad (\delta E/E) \log (E/\delta E) > (\delta E/E) \log n^2$

i.e. $\quad E/\delta E > n^2$

i.e. $\quad \delta E/E > \frac{1}{n^2}$

Note The very stringent condition in Theorem 1 can be relaxed to allow n of the 2n + 1 new cells inhabited, their value of each component being $\delta E/n$.

In this case:

$$H^t_{add} = -n(\delta E/n\ (E + \delta E)) \log (\delta E/n\ (E + \delta E))$$

$$\simeq (\delta E/E) \log (nE/\delta E)$$

The condition for H^t to increase now becomes

$$(\delta E/E) \log (nE/\delta E) > (\delta E/E) \log n^2$$

i.e. $\quad \delta E/E < \frac{1}{n}$

Theorem 2 On disaggregating one sector of an economy into two sectors, H^t for that economy can never decrease, the maximum increase depending upon the relative amount of energy dissipated by the dissaggregated sector to the total energy dissipation by the economy.

Proof

Our economy initially has:

$$H^t(n^2,E) = -\sum_{i=1}^{n}\sum_{j=1}^{n} (e_{ij}/E) \log (e_{ij}/E)$$

Let us represent this as an (n - 1) + 1 sector economy.

i.e. $$H^t(n^2, E) = -\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} (e_{ij}/E) \log (e_{ij}/E) + \sum_{i=1}^{n-1}(e_{in}/E) \log (e_{in}/E)$$

$$+ \sum_{j=1}^{n-1} (e_{nj}/E) \log (e_{nj}/E) + (e_{nn}/E) \log (e_{nn}/E)$$

$$= H^t((n-1)^2,E) + H^t_r(n-1,E) + H^t_c(n-1,E) + H^t(1,E) \qquad (i)$$

$$= H^t((n-1)^2,E) + H^t_{add}$$

Let us disaggregate sector n into two sectors, n' and (n + 1)', i.e.:-

$$H^{t*}((n+1)^2,E) = -\sum_{i=1}^{n-1}\sum_{j=1}^{n-1} (e_{ij}/E) \log (e_{ij}/E) + \sum_{i=1}^{n-1} (e_{in'}/E)$$

$$\log (e_{in'}/E) + \sum_{j=1}^{n-1} (e_{n'j}/E) \log (e_{n'j}/E)$$

$$+ \sum_{i=1}^{n-1} (e_{i(n+1)'}/E) \log (e_{i(n+1)'}/E) + \sum_{j=1}^{n-1}(e_{(n+1)'j}/E)$$

$$\log (e_{(n+1)'j}/E) + (e_{n'n'}/E) \log (e_{n'n'}/E)$$

$$+ (e_{n'(n+1)'}/E) \log (e_{n'(n+1)'}/E) + (e_{(n+1)'n'}/E)$$

$$\log (e_{(n+1)'n'}/E) + (e_{(n+1)'(n+1)'}/E)$$

$$\log (e_{(n+1)'(n+1)'}/E)$$

$$= H^t((n-1)^2,E) + H_r^{1*}(n-1,E) + H_c^{1*}(n-1,E) + H_r^{2*}(n-1,E)$$
$$+ H_c^{2*}(n-1,E) + H^{1*}(1,E) + H^{2*}(1,E) + H^{3*}(1,E)$$
$$+ H^{4*}(1,E) \qquad \text{(ii)}$$

The last four terms in (ii) are due to the disaggregation of the last term in (i).

In the simplest possible case only one of the two dissaggregated sectors contains non-zero elements, i.e.

$$e_{in'} = e_{in},\ e_{nj'} = e_{nj},\ e_{i(n+1)} = e_{(n+1)j} = 0$$
$$e_{n'n'} = e_{nn},\ e_{n'(n+1)'} = e_{(n+1)'n'} = e_{(n+1)'(n+1)'} = 0$$

In this case

$$H_r^{1*}(n-1,E) = H_r^1(n-1,E)$$
$$H_c^{1*}(n-1,E) = H_c^1(n-1,E)$$
$$H(1,E) = H^1(1,E)$$
$$H_r^{2*}(n-1,E) = H_c^{2*}(n-1,E) = H^{2*}(1,E) = H^{3*}(1,E) = H^{4*}(1,E) = 0$$

i.e. min $H^*((n+1)^2,E) = H(n,E)$. So there has been no effective dissaggregation.

The most complex case is where:

$$e_{n'j} = e_{(n+1)'j} = e_{nj}/2 \quad (j \leq n-1)$$
$$e_{in'} = e_{i(n+1)'} = e_{nj}/2 \quad (i \leq n-1)$$
$$e_{n'n'} = e_{(n+1)'n'} = e_{n'(n+1)'} = e_{(n+1)'(n+1)'} = e_{nn}/4$$

i.e.
$$H_r^{1*}(n-1,E) = H_r^{2*}(n-1,E) = -\sum_{i=1}^{n} (e_{in}/2E) \log (e_{in}/2E)$$

$$H_c^{1*}(n-1,E) = H_c^{2*}(n-1,E) = -\sum_{j=1}^{n} (e_{nj}/2E) \log (e_{nj}/2E)$$

$$H^{1*}(1,E) = H^{2*}(1,E) = H^{3*}(1,E) = H^{4*}(1,E) = (e_{nn}/4E) \log(e_{nn}/4E)$$

i.e. $$\max H^{t*}((n+1)^2,E) = H((n-1)^2,E) - (2\sum_{i=1}^{n-1}(e_{in}/2E)\log(e_{in}/2E)$$

$$+ 2\sum_{j=1}^{n-1}(e_{nj}/2E)\log(e_{nj}/2E)$$

$$+ 4(e_{nn}/4E)\log(e_{nn}/4E)$$

$$= H^t((n-1)^2,E) - (\sum_{i=1}^{n-1}(e_{in}/E)\log(e_{in}/E) + \sum_{j=1}^{n-1}(e_{nj}/E)$$

$$\log(e_{nj}/E) + (e_{nn}/E)\log(e_{nn}/E) +$$

$$+ \log 2\,(\sum_{i=1}^{n-1} e_{in}/E + \sum_{j=1}^{n-1} e_{nj}/E + 2\,e_{nn}/E)$$

$$= H^t((n-1)^2,E) + H^t_{add} + \log 2(\sum_{i=1}^{n} e_{in} + \sum_{j=1}^{n} e_{nj})/E$$

$$= H^t(n^2,E) + \log 2(\sum_{i=1}^{n} e_{in} + \sum_{j=1}^{n} e_{nj})/E$$

So the maximum increase in H^t due to disaggregation of one sector is proportional to the relative amount of energy dissipated by the disaggregated sector to the total energy dissipation by the economy.

Theorem 3 On aggregating two sectors of an economy into one sector, H^t for the economy can never increase, the maximum decrease depending upon the relative amount of energy dissipated by the aggregated sector to the total energy dissipation of the economy.

Proof

Our economy initially has:-

$$H^t(n^2,E) = -\sum_{i=1}^{n}\sum_{j=1}^{n}(e_{ij}/E)\log(e_{ij}/E)$$

Suppose sector (n-1) and n are aggregated into sector (n-1)', giving $H^{t*}((n-1)^2,E)$. We can write :

$$H^t(n^2,E) = H^t((n-2)^2,E) - \{ \sum_{i=1}^{n-2} (e_{i(n-1)}/E) \log (e_{i(n-1)}/E)$$

$$+ \sum_{j=1}^{n-2} (e_{(n-1)j}/E) \log (e_{(n-1)j}) + (e_{(n-1)(n-1)}/E)$$

$$\log(e_{(n-1)(n-1)}/E) + (e_{n(n-1)}/E) \log (e_{n(n-1)}/E) \}$$

$$- \{ \sum_{i=1}^{n-2} (e_{in}/E) \log (e_{in}/E) + \sum_{j=1}^{n-2} (e_{nj}/E) \log (e_{nj}/E)$$

$$+ (e_{nn}/E) \log (e_{nn}/E) + (e_{(n-1)n}/E) \log (e_{(n-1)n}/E) \}$$

Similarly $H^{t*}((n-1)^2,E)$ can be written:

$$H^{t*}((n-1)^2,E) = H^t((n-2)^2,E) - \sum_{i=1}^{n-2} ((e_{i(n-1)} + e_{in})/E) \log ((e_{i(n-1)} + e_{in})/\text{I}$$

$$+ \sum_{j=1}^{n-2} ((e_{(n-1)j} + e_{nj})/E) \log ((e_{(n-1)j} + e_{nj})/E)$$

$$+ ((e_{(n-1)(n-1)} + e_{n(n-1)} + e_{(n-1)n} + e_{nn})/E)$$

$$\log((e_{(n-1)(n-1)} + e_{n(n-1)} + e_{(n-1)n} + e_{nn})/E)$$

So the change in H^t is given by:-

$$\Delta H^t = H^t(n^2,E) - H^{t*}((n-1)^2,E)$$

$$= - \sum_{i=1}^{n-2} \{ (e_{i(n-1)}/E) \log (e_{i(n-1)}/E) + (e_{in}/E) \log(e_{in}/E)$$

$$- ((e_{i(n-1)} + e_{in})/E) \log ((e_{i(n-1)} + e_{in})/E) \}$$

$$- \sum_{j=1}^{n-2} \{ (e_{(n-1)j}/E) \log (e_{(n-1)j}/E) + (e_{nj}/E) \log(e_{nj}/E)$$

$$- ((e_{(n-1)j} + e_{nj})/E) \log ((e_{(n-1)j} + e_{nj})/E) \}$$

$$- \{(e_{(n-1)(n-1)}/E) \log (e_{(n-1)(n-1)}/E) + (e_{(n-1)n}/E)$$

$$\log (e_{(n-1)n}/E) + (e_{n(n-1)}/E) \log (e_{n(n-1)}/E)$$

$$+ (e_{nn}/E) \log (e_{nn}/E) - ((e_{(n-1)(n-1)} + e_{(n-1)n}$$

$$+ e_{n(n-1)} + e_{nn})/E) \log ((e_{(n-1)(n-1)} + e_{(n-1)n}$$

$$+ e_{n(n-1)} + e_{nn})/E)\}$$

Now if one of the aggregated sectors contains only zeros then all of the above vanishes, and there is no reduction in H^t, as there was effectively no sector to be aggregated away to begin with, i.e. the minimum reduction in H^t under aggregation is zero.

Now by Lemma 2 the maximum reduction occurs when $e_{i(n-1)} = e_{in} = e_{(n-1)j} = e_{nj}$ $\forall$ i, j $\leq$ n-2 and $e_{(n-1)(n-1)} = e_{(n-1)n} = e_{n(n-1)} = e_{nn}$

$$\text{So max } \Delta H = - \sum_{i=1}^{n-2} \{2(e_{in}/E) \log (e_{in}/E) - (2e_{in}/E) \log (2e_{in}/E)\}$$

$$- \sum_{j=1}^{n-2} \{2(e_{nj}/E) \log (e_{nj}/E) - (2\,e_{nj}/E) \log (2e_{nj}/E)\}$$

$$- \{4(e_{nn}/E) \log (e_{nn}/E) - (4e_{nn}/E) \log (4e_{nn}/E)\}$$

$$= \sum_{i=1}^{n-2} (2 \log 2)(e_{in}/E) + \sum_{j=1}^{n-2} (2 \log 2)(e_{nj}/E) + (4 \log 4)(e_{nn}/E)$$

$$= \log 2 \{\sum_{i=1}^{n} 2(e_{in}/E) + \sum_{j=1}^{n} 2(e_{nj}/E)\}$$

Now in this case $2e_{in} = e_{i(n-1)'}$, and $2e_{nj} = e_{(n-1)'j}$

$$\text{i.e. max } \Delta H = \log 2(\sum_{i=1}^{(n-1)} e_{in} + \sum_{j=1}^{(n-1)'} e_{nj})/E$$

So we see that the maximum decrease in H^t due to the aggregation of two sectors is proportional to the relative amount of energy dissipated by the aggregated sector to the total energy dissipation of the economy.

<u>Theorem 4</u> If one sector of an economy decreases (increases) its energy dissipation, proportionally across its functions, then H^t for that economy is increased (decreased) if the proportion of total energy dissipation due to that sector is greater (less) than its proportional contribution to H^t. Otherwise H^t is decreased (increased).

<u>Proof</u>

For the original economy we have

$$H^t(n^2,E) = -\sum_{i=1}^{n}\sum_{j=1}^{n}(e_{ij}/E)\log(e_{ij}/E)$$

Suppose the sector which changes is sector n. Then the change in the system will be to simply reduce the elements of row n by a fixed proportion.

Let us divide up $H^t(n^2,E)$:

$$H^t(n^2,E) = H^t(n^2 - n,E) + H^t_r(n,E)$$

where
$$H^t(n^2 - n,E) = -\sum_{i=1}^{n-1}\sum_{j=1}^{n}(e_{ij}/E)\log(e_{ij}/E)$$

and
$$H^t_r(n,E) = -\sum_{j=1}^{n}(e_{nj}/E)\log(e_{nj}/E)$$

We shall reduce the energy dissipation of sector n from E_n to $E_n - \delta E$. So the total energy dissipation by the economy becomes $E - \delta E$, and the elements of row n become $e_{nj}(1 - \delta E/E_n)\ \forall j$.

So we now have:-

$$H^t(n^2,E - \delta E) = H^t(n^2 - n, E - \delta E) + H^t_r(n, E - \delta E)$$

Now $H^t(n^2 - n, E - \delta E) = - \sum_{i=1}^{n-1} \sum_{j=1}^{n} (e_{ij}/(E - \delta E) \log (e_{ij}/(E - \delta E))$

$$\simeq (1 + \delta E/E) H^t(n^2 - n, E), \quad \text{by Lemma 1}$$

Similarly $H_r^{t*}(n, E - \delta E) \simeq (1 + \delta E/E) H_r^{t*}(n,E)$

Where $H_r^{t*}(n,E) = - \sum_{j=1}^{n} (e_{nj}/E)(1 - \delta E/E_n) \log (e_{nj}/E)(1 - \delta E/E_n)$

$$= - \sum_{j=1}^{n} ((e_{nj}/E) - (e_{nj}/E)(\delta E/E_n)(\log(e_{nj}/E)$$

$$+ \log (1 - \delta E/E_n))$$

Now $\log (1 - \delta E/E_n) \simeq 0$ for $\delta E << E_n$

So $H_r^{t*}(n,E) \simeq - \sum_{j=1}^{n} ((e_{nj}/E) \log (e_{nj}/E)) - (\delta E/E_n)(e_{nj}/E)\log(e_{nj}/E)$

$$= (1 - \delta E/E_n) H_r^t (n, \delta E)$$

So $H_r^{t*}(n, E - E) \simeq (1 + \delta E/E) H_r^{t*}(n,E)$

$$\simeq (1 + \delta E/E) (1 - \delta E/E_n) H_r^t (n,E)$$

$$\simeq (1 + \delta E/E - \delta E/E_n) H_r^t (n,E)$$

So $H^t(n^2, E - \delta E) = (1 + \delta E/E) H^t(n^2-n,E) + (1 + \delta E/E) H_r^t (n,E)$

$$= (1 + \delta E/E) H^t(n^2,E) - (\delta E/E_n) H_r^t (n,E)$$

The change in H^t is given by

$$\Delta H^t = H^t(n^2,E - \delta E) - H(n^2,E)$$

$$= (1 + \delta E/E) H^t(n^2,E) - H^t(n^2,E) - (\delta E/E_n) H_r^t (n,E)$$

$$= (\delta E/E) H^t(n^2,E) - (\delta E/E_n) H_r^t (n,E)$$

i.e. $\Delta H^t > 0$ iff $(\delta E/E) H^t(n^2,E) > (\delta E/E_n) H_r^t (n,E)$

i.e. iff $E_n/E > H_r^t (n,E)/H^t(n^2,E)$

i.e. iff the proportion of energy dissipation due to sector n is greater than its proportional contribution to H^t.

APPENDIX B

The Expected Value of H^t

H^t is defined on a set of n^2 random numbers $\{Z_{ij}\}$ by

$$H^t = - \sum_{i=1}^{n} \sum_{j=1}^{n} \left(\frac{Z_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij}}\right) \log \left(\frac{Z_{ij}}{\sum_{i=1}^{n} \sum_{j=1}^{n} Z_{ij}}\right)$$

For ease of exposition we redefine the set $\{Z_{ij}\}$ as the set $\{Z_i\}$ with $r = n^2$.

i.e.
$$H^t = - \sum_{i=1}^{r} \left(\frac{Z_i}{\sum_{i=1}^{n} Z_i}\right) \log \left(\frac{Z_i}{\sum_{i=1}^{n} Z_i}\right)$$

now for large r the Z_i act as independent variables, so we can write:

$\sum_{i=1}^{r} Z_i = r \langle Z \rangle$, where $\langle Z \rangle$ is the expected value of Z.

so we have:- $$\langle H^t \rangle = \langle - \sum_{i=1}^{r} \left(\frac{Z_i}{r\langle Z \rangle}\right) \log \left(\frac{Z_i}{r\langle Z \rangle}\right) \rangle$$

$$= - \frac{1}{r\langle Z \rangle} \sum_{i=1}^{r} \langle Z_i \log Z_i - Z_i \log r - Z_i \log \langle Z \rangle \rangle$$

$$= - \frac{1}{r\langle Z \rangle} \left(\sum_{i=1}^{r} \langle Z \log Z \rangle - \log r \sum_{i=1}^{r} \langle Z \rangle - \log \langle Z \rangle \sum_{i=1}^{r} \langle Z \rangle \right)$$

$$= - \frac{1}{r\langle Z \rangle} \left(r \langle Z \log Z \rangle - r\langle Z \rangle \log r - r\langle Z \rangle \log \langle Z \rangle \right)$$

$$= \log r + \log \langle Z \rangle - \frac{\langle Z \log Z \rangle}{\langle Z \rangle}$$

We recall that $r = n^2$, hence:

$$\langle H^t \rangle = 2 \log n + \log \langle Z \rangle - \frac{\langle Z \log Z \rangle}{\langle Z \rangle}$$

APPENDIX C

The Expected Value of H^m

H^m is defined on a matrix $\underline{V}$ of the form:-

$$V_{ij} = Z_{ij} \;\forall\; i \neq j$$

$$V_{ij} = 1 + Z_{ij} \;\forall\; i = j$$

Let us suppose the $\{Z_{ij}\}$ are a set of random numbers.

Now from Appendix B we know:-

$$\langle H^m \rangle = \log r + \log \langle V \rangle - \frac{\langle V \log V \rangle}{\langle V \rangle}$$

For ease of exposition we redefine V:-

$$V_i = Z_i \quad i = 1, m$$

$$V_i = 1 + Z_i \quad i = m, r$$

Here $r = n^2$ and $m = n^2 - n$.

Now $$\langle V \rangle = \frac{1}{r}\sum_{i=1}^{r} V_i = \frac{1}{r}\left(\sum_{i=1}^{m} Z_i + \sum_{i=m+1}^{r} (1 + Z_i)\right)$$

$$= \frac{1}{r}\left(\sum_{i=1}^{r} Z_i + (r - m)\right)$$

$$= \frac{1}{r}\left(r \langle Z \rangle + (r - m)\right)$$

Also $$\langle V \log V \rangle = \frac{1}{r}\sum_{i=1}^{r} V_i \log V_i$$

$$= \frac{1}{r}\left(\sum_{i=1}^{m} Zi \log Z_i + \sum_{i=m+1}^{r} (1 + Zi) \log (1 + Z_i)\right)$$

$$\approx \frac{1}{r}\left(\sum_{i=1}^{m} Zi \log Z_i + \sum_{i=m+1}^{r} (1 + Zi)(Z_i)\right),$$

for small Zi

$$= \frac{1}{r}\left(\sum_{i=1}^{m} Zi \log Z_i + \sum_{i=m+1}^{r} Zi + \sum_{i=m+1}^{r} Z_i^2\right)$$

$$= \frac{1}{r}\left(m \langle Z \log Z \rangle + (r - m)\langle Z \rangle + (r - m)\langle Z^2 \rangle\right)$$

Substituting for $\langle V \rangle$ and $\langle V \log V \rangle$ we get:

$$\langle H^m \rangle = \log r + \log \left(\frac{1}{r}\left(r\langle Z \rangle + (r-m)\right)\right) - \frac{m\langle Z \log Z \rangle + (r-m)\langle Z \rangle + (r-m)\langle Z^2 \rangle}{r\langle Z \rangle + (r-m)}$$

$$= \log \left(r\langle Z \rangle + (r-m)\right) - \frac{m\langle Z \log Z \rangle + (r-m)\left(\langle Z \rangle + \langle Z^2 \rangle\right)}{r\langle Z \rangle + (r-m)}$$

Substituting $r = n^2$ and $m = n^2 - n$ we get:-

$$\langle H^m \rangle = \log \left(n^2\langle Z \rangle + n\right) - \frac{(n-1)\langle Z \log Z \rangle + \langle Z \rangle + \langle Z^2 \rangle}{n\langle Z \rangle + 1}$$

APPENDIX D

Data Sources and Methods of Computation

UK 1963, 1968, 1970

The Input-Output flow tables used were the "Industry x Industry Flow Matrices (Table D)" of the corresponding UK Input-Output Tables[1]. The tables for 1968 and 1970 were published in 90 x 90 form, while that for 1963 was in 70 x 70 form. Using the detailed sectoral descriptions by Minimum List Heading of the Standard Industrial Classification, the 1968 and 1970 tables were aggregated down to 70 x 70 form.

Total direct energy consumption (fuel delivered) for each sector was taken directly from the Digest of UK Energy Statistics[2], Table 17, for the corresponding years. As the sectors used there were not the same as those used in the 1963 I-O table, a certain amount of disaggregation was required. This disaggregation was done by assuming that the aggregated sectors all had the same ratio of energy dissipation to value of output.

It was recognised that the UK economy is very dependent upon imports, and that much of the UK energy dissipation is, in a manner of speaking, carried out abroad. It was felt that if this aspect of energy dissipation were ignored important details of the structural changes in the UK economy would be missed.

To account for this extra dissipation it was assumed that imports required for their production the same direct energy dissipation per unit value as the corresponding domestically produced commodity. Thus using the vector of imports, $\underline{q}$, the vector of total demands, $\underline{x}$, and the vector of domestic energy dissipation, $\underline{e}$, the modified vector of energy

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1. Input-Output Tables for the United Kingdom, HMSO, London, 1968; 1973; 1975.
2. Digest of United Kingdom Energy Statistics, HMSO, London (monthly).

dissipation was defined by :

$$e_i^* = e_i\ (x_i + q_i)/x_i$$

Using the table of flows, x_{ij}, the vectors of final demand, $\underline{y}$, total demand, $\underline{x}$, and the modified energy dissipation vector, $\underline{e}^*$, the energy dissipation table was constructed by the transformations:

$$e_{ij} = (x_{ij} \cdot e_i^*)/x_i$$

$$e_i^y = (y_i \cdot e_i^*)/x_i$$

The table was expressed in units of kWh. From this table of energy dissipations, the dissipation matrix, $(I-Q)^{-1}$ was constructed, as described in Chapter 7.

The UK GDP for the three years was taken from Economic Trends[3]. There GDP is given at 1970 factor cost.

E_{ind} was taken to be the sum of energy dissipated by all industries, excluding conversion losses. That is, E_{ind} was taken to be the sum of all the elements in the above energy dissipation table.

The population, P, for the three years was taken from the Monthly Digest of Statistics[4].

Using the dissipation table, the dissipation matrix, GDP, E_{ind} and P the values for the variables H^t, H^m, E_n, E_{ind}/P and E_{ind}/GDP were constructed, as described in Chapter 8.

Pooled Data

The Input-Output flow tables were provided by the Economics Department of Bradford University. They had already been processed and aggregated down to a consistent 10 x 10 basis.

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3. Economic Trends, HMSO, London (monthly).
4. Monthly Digest of Statistics, HMSO, London (monthly).

Total energy consumption was taken from Darmstadter et al.,[5] and for the Netherlands some sectoral detail of energy consumption was available from the OECD Statistics of Energy[6]. Other countries had the vector of energy use, e, estimated by assuming that energy was homogeneous and at a fixed price. Thus, using the notation of Chapter 7, it was assumed that:

$$e_i = E.(x_{gi}/x_g)$$

and

$$E_{dom} = E.(y_g/x_g)$$

The energy dissipation table was constructed as for the UK 1963-70, expressed in units of kWh.

The dissipation by the energy sector due to its own demand, e_{gg}, obviously mainly reflects conversion losses. No details were available as to the size of these losses, but as it was clear that most of e_{gg} could be accounted for in this way, e_{gg} was set at one-tenth of its initial calculated value.

Import details were unavailable for many of the countries, so to maintain consistency no effort was made to augment the energy dissipation vector.

GDP and population figures were taken from the relevant UN Statistical Yearbook[7]. GDP was converted to US $ and deflated to 1960 values, using the price index in Fite and Rees[8].

The values of the variables were then obtained as for the UK 1963-70.

As the provenance of the 10 x 10 tables was rather dubious, their

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5. J. Darmstadter, P. Teitelbaum and J. Polach, Energy in the World Economy, John Hopkins University Press, London, 1971.
6. Statistics on Energy, 1955-1969, OECD, Paris, 1971.
7. Statistical Yearbook, UN, New York (annual).
8. G.C. Fite and J.E. Reese, An Economic History of the United States, Houghton Mifflin, Boston, 1965.

internal consistency was examined using perturbation analysis. Each of the 100 entries in the original table (x_{ij}) was altered in turn, by a fixed proportion, and from this altered table the matrix $(\underline{I} - \underline{Q})^{-1}$ was obtained, and hence H^m. The deviation of H^m from its original value was examined in each case. For most of the elements of most of the tables examined, the proportional induced deviation of H^m was approximately -10% of the proportional applied perturbation to the element. For example if x_{ij} were perturbed by 15%, the observed deviation in H^m was approximately -1.5%.

The only exception to this general stability was for the table for India 1955-6. As a result, this table was excluded from the data used.

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