Mathematics for Non-Specialists:

A Study of Two Undergraduate Courses

R.A. Pickering

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Abstract

The results of studies of two courses in mathematics for non-specialists are presented. The first of these was designed as a subsidiary option for arts-based students. Initially, the investigation was concerned with establishing the aims and objectives which applied to a course of cultural rather than vocational interest and the implications of these for syllabus content. Consideration was given to appropriate modes of data collection, bearing in mind the particular demands of the research aims. Information gathered from both the staff and students involved over a three-year period established the nature of the course, how it operated, what were its strengths and weaknesses and how people reacted to it. The effectiveness of the use of study booklets as a supplement to the lectures was investigated and some actual and possible developments are described.

The second part of the research was concerned with a service course in mathematics for first-year undergraduates reading for a degree in economics. One part of the investigation attempted to discover the attitudes of the students to the subject and how relevant it was thought to be to their main subject. Of major interest was the design and formative evaluation of an instructional scheme containing a significant component of self-paced study. The effectiveness of the instruction was considered with regard to the development of basic techniques, the ability to solve problems and the confidence of students in applying mathematical methods. It was possible to assess the difficulties that students experienced in applying mathematical

techniques to problems in economics. Discussions with specialist economics lecturers established the aims of the course with greater clarity than the formal syllabus was able to convey and also provided data to supplement the views of the students about their own experiences.

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CHAPTER 1

INTRODUCTION AND GENERAL DESCRIPTION

1.1 Introduction

This work contains the results of studies carried out on two undergraduate courses which were currently available at institutions in the United Kingdom. The investigation grew from an interest in the teaching problems associated with courses in mathematics for non-specialists at undergraduate level. It was fortunate that it was possible to devise a programme of research based on two courses which, although both in this same category, nevertheless had quite distinct characteristics. Both were undergraduate courses in mathematics lasting one academic year for which the normal entry requirement was an O-level pass in the subject; in each case it was necessary that the individual passed the subsidiary course in order to progress on the respective degrees. However, despite these similarities, the two courses had markedly different aims and had developed in different ways.

The first course was an optional subsidiary mathematics study, available as one of a group of science options at a particular University. The University in question required students to have a broad-based undergraduate curriculum and the regulations constrained students reading for non-science-based degrees to include the study of a science subject as one subsidiary to their main or principal disciplines. As a result, some one hundred and forty or so enrolled for the course of subsidiary mathematics which was the subject of the investigation.

The second course was the mathematics section of the Quantitative Methods (mathematics and statistics) studied as a compulsory subject in the first year of degree course for a B.A. in Economics and offered at a Polytechnic in the United Kingdom. During the period of the investigation, roughly thirty students were enrolled on the degree each year.

In the former case, little justification was possible for the inclusion of mathematical studies in the curriculum on the basis of relevance or support for other subjects. Thus, the course was designed to give a view of the cultural significance of the subject and this gave it an unconventional character. The second case provided a contrast to this. The relevance of mathematics to the proper study of economics was apparent to the students in many ways. This did not give rise to misgivings about the usefulness of the subject but was more likely to produce anxiety about making sufficient progress to cope with later studies.

Thus the two courses were conceived in different institutions, with their different traditions and histories and were designed to satisfy different aims. In the following section, the major features of each course are presented and may be compared.

1.2 Major Features of the Courses under Investigation

At the University where the subsidiary mathematics course was offered, students normally undertook to study two principal and two subsidiary subjects for their first degree. A subsidiary subject was studied for one year. The regulations required students who had chosen arts principal subjects to study science as a subsidiary and mathematics was one of the options available. It was recognized by the Mathematics Department concerned that a course which satisfied the demands of non-vocational or cultural aims would be unconventional and not be of the more familiar kind of so-called service courses. The course designers were thus faced with an initial problem of what were worthwhile and sensible cultural aims and how could these be reflected in syllabus content. In addition, the teaching environment was significantly different from that which a university lecturer normally encountered. The students had a much weaker background than usual and their grasp of technique was limited. If they had been influenced by a popular view of mathematics which saw it as barely comprehensible to the layman, it would be likely that they approached the subject with some reserve.

One problem therefore, was to establish from the staff and students involved what were the attitudes, opinions and objectives which combined to produce the overall climate in which the course operated. Another major task was seen to be an investigation of the content and nature of cultural mathematical studies and what objectives were appropriate to govern the choice and treatment of the syllabus.

The objectives of the research were formulated from such initial considerations. These are discussed in more detail later but briefly they were noted as (i) an analysis of the nature and content of a culturally-based course in mathematics, (ii) a descriptive survey of the factors, including the attitudes, opinions and aspirations of the staff and students, which together made up the climate in which the course operated and influenced its development, (iii) an identification of those characteristics which specified the teaching task and how this differed from that normally encountered and (iv) to seek pointers to any fruitful lines of development.

The second study involved a quite different course in a different institution. In their popular image, polytechnics are seen as providing courses which are more overtly vocational than those found in universities. Their courses are closely monitored by an external agency* to ensure that the aims and intellectual demand are of a required standard. Hence the mathematics syllabus was contained in a prescribed document which represented the consensus of opinion on what mathematics was needed for the proper study of economics at the tertiary level. The course represented therefore, a typical example of what have become known as service courses. In this instance, the Department of Economics and Social Studies which administered the degree delegated the teaching of mathematics and statistics to the Department of Computing and Mathematics.

The instructional method had to be considered in the light of the demands of a fixed syllabus and the need for students to acquire sufficient mastery of the subject to apply it satisfactorily to problems in economics. Other factors to be taken into account were the wide range of student ability (ranging from no formal qualification to A-level pass) and the influence of specialist staff who used mathematical treatment in their teaching of economics. It was clear that the objectives of this study were going to be somewhat different from

* Council for National Academic Awards

those considered above for the subsidiary mathematics course. For example, in this case the syllabus content was fixed, was the result of many years of trial and adjustment and was known to satisfy the demands of the parent Department. What was of interest however, was information about staff and student attitudes to mathematics, the development of independent enquiry in students and their use of mathematical skills in problem-solving. One major difference in the two studies was that in this second case, the investigator was also the lecturer involved in teaching the mathematics.

Again, as previously, the objectives of the research are considered in detail in later chapters, but they may be stated briefly here as (i) the survey of the attitudes, opinions and expectations of those people involved that formed the background in which the course operated, (ii) the design and formative evaluation of a teaching method appropriate to the identified course aims, (iii) an investigation into the factors affecting and ways of developing the application of mathematics in problem-solving. The two investigations were carried out concurrently and the majority of the data gathering occurred during the period from October 1977 to June 1980.

1.3 An Outline of the Contents of the Following Chapters

In chapter 2 a survey of relevant literature is presented. Chapters 3 to 12 are concerned with the subsidiary mathematics course for arts undergraduates. First the philosophy of a broad-based cultural approach to a subject is presented (chapter 3) and this is followed (chapter 4) by a discussion of appropriate aims and content for this kind of course with mathematics specifically in mind. The next chapter (chapter 5) discusses the background and considerations behind the particular choices of research procedure. Chapters six to eight set out the information gathered from students concerning their attitudes, expectations and reactions to the course. The description is completed by the presentation of data obtained from discussions and conversations with the members of staff involved (chapter 9), which contains details of their attitudes to the course and its aims and their reactions to the teaching experience. In chapter 10, as well as considering more general aspects of the objectives of the teaching approach, there is a discussion of the effectiveness of two short study booklets which were designed as learning aids to supplement the lectures in two topics of the syllabus. This is followed by a summary of the contents of the previous five chapters and the conclusions that were drawn from the investigation. There is also a discussion of possible lines of development (chapter 11). The first section is concluded by an interesting postscript which describes some changes in the course which took place during the 1980/81 academic year, whilst this document was in preparation (chapter 12).

Chapters 13 to 19 are concerned with the investigation into the teaching of mathematics to undergraduate economists. Initially, the background is provided by a description of the degree course as a whole and the identification of the aims of the mathematical studies. The rationale for the choice of instructional scheme is given and the research objectives and procedures are specified (chapter 13). The next chapter is concerned with the details of the instructional scheme design and its administration (chapter 14) and this is followed by the section containing the presentation and analysis of data obtained from the students over the three years of study. (Chapters 15, 16 and 17.) Next, a different viewpoint is presented by evidence collected from specialist economics lecturers who were closely associated with the degree and also from some second year students who were able to give their opinions after further experience on the course (chapter 18). A summary and conclusions completes the account of this study (chapter 19) and the whole work is brought to a conclusion with a final review (chapter 20).

The research programme and procedures are to be found in volume 2, appendices 1 to 10, bound separately. This separate volume also contains the bibliography. It should be consulted as an integral part of the study presented in volume 1.

CHAPTER 2

A REVIEW OF RELEVANT LITERATURE

This chapter is a brief review of background literature which provided a basis for the two studies. The general works are described first, later publications refer to more specific aspects. The final paragraphs are concerned with those sources which were used in the formulation of the research procedure.

The growing concern with changes in education had its origin in a combination of the effects of learning theory and changes in the aims of education. Bruner (1960) noted the growth of interest in curriculum planning, looked at the process of learning in terms of the psychological evidence and remarked on the need to emphasise structure not content. A list of five major categories of learned capabilities were defined by Gagné (1965) in a work that goes on to categorize the basic forms of learning and the conditions needed for these to occur. Of interest to the teacher of mathematics to non-specialists was a discussion of students' behaviour in coping and defending; this suggested that established negative attitudes may have a profound effect on the ability to learn a particular subject (Bruner, 1966, chapter 7). Ausubel (1968) produced a comprehensive book on the psychology of education. For anyone studying instructional method, the chapter which gave a critical review of the aims of discovery method was of particular interest (chapter 14). Sceptical of some of the more extravagant claims, Ausubel acknowledged the utility of "guided discovery", a theme which was also referred to by Gagné and Briggs (1974) and Gagné (1975). These texts were mainly concerned with the influence of the psychology of learning on instructional design and the first of these develops a detailed scheme for the design, implementation and evaluation of instruction based on behavioural objectives. A humanistic approach to learning was advocated by Rogers and Maslow (Sahakian ed, 1976, ch.12) who contrasted some of the traditional assumptions about the aims of education and principles that can be abstracted from humanistic philosophy.

Starting points in the consideration of learning objectives were detailed by the identification of taxonomies of educational objectives in the cognitive (Bloom, 1956) and affective domains (Krathwohl, Bloom and Masia, 1964). A critical look at the assumptions of Bloom and his associates was taken by Ormell (1974) who suggested some modification in the approach to formulating the purposes of instruction. A paper specifically concerned with learning and teaching in higher education suggested that learning was too "technified" that is, one looked for specific answers to specific questions and it was suggested this was not the true purpose of learning (Dahlgren and Marton, 1978).

Some indication of the personality factors that may be significant in teaching students who are more at home in other disciplines were found in Hudson (1968) and Entwhistle (1977). These authors pointed to the classifications of divergent-convergent, holist-serialist, introvert-extrovert and suggested that there may be unexpected attitudes and responses generated in the non-specialist. The problem of teaching mathematics from a mainly cultural standpoint was put in the context of a liberal view of education by Phenix (1964), Hirst P.H., (in Archambault ed, 1965) and Whitfield (1971). The objectives of mathematical education itself received comment from Avital and Shettleworth (1968), Servais and Varga (1971), Courant and Robbins (1941) and McNelis and Dunn (1977).

More specific references to courses having a distinctly cultural role were made by Kline (1962), Smithies (1970), Man-Keung Siu (1977) and two publications describing experiences in teaching non-specialists at sixth-form level (Mathematicl Association, 1965 and 1967). Course content and teaching method received comment from Brueckner, Grossnickle and Reckzeh (1957), Hirst K. and Biggs (1969), Beard (1972), Thwaites (1972), MacDonald (1976) and Allen (1977) whilst Gibson (1970) described the barriers to progress created by students' misconceptions of the nature of the subject (in that case, social science).

A wide variety of text-books were available which contained material claimed to be suitable for the non-specialist approach. The wide range of topics covered by different authors indicated that cultural aims were not defined in terms of content alone. Typical examples were Douglis (1970), Dowdy (1971), Graham (1973), Bower (1973), Jacobs (1970), Hunkins and Pernot (1977) and more recently Averbach and Chein (1980). Most authors were evidently concerned with the process as well as the content of mathematics and attention was given to adopting a student-centred approach. Other publications looked at particular aspects; an example was a paper on the role of mathematical proof (Searle, S.R., 1977). An interesting comparison with the research described here was a case study of a course in physics designed for Arts undergraduates at the University of Lancaster (Heywood and Montagu -Pollock, 1977).

An overall view of the features of conventional service courses and their design problems was given by MacDonald-Ross and Rees (1972). There were some discussions of the aspects associated with teaching mixed ability groups and the relative merits of different instructional method. Berril and Sampson (1974) reported on mixed-ability teaching in schools, Costin (1972) gave a useful comparison of the effectiveness of different teaching techniques. Many authors reflected a concern with the individualisation of study methods. The fundamental paper on the method of self-paced study was that of Keller (1968) and implementations of student-centred learning schemes were described, for example, by Stroud (1971), van der Klauw and Plomp (1974), Milson (1975) and Romiszowski, Bajpai and Lewis (1976). The methods were sufficiently well-developed to prompt surveys. Personalized systems of instruction were reviewed by Boud, Bridge and Willoughby (1975) and the types of instruction then in use were classified by Goldschmid B. and Goldschmid M.L. (1973) and Goldschmid M.L. (1976).

The use of technology in individualisation of instruction was widely documented. General reviews of instructional media were given by Hawkridge (1973), Romiszowski (1974), Brown (1975) and Hooper (1975) and (1976). Individual implementations involved a whole variety of combinations, for example, a computer administered questionnaire in conjunction with tape-slide instruction was described by Denis and Lombaerde (1975). That individualised instruction need not be dependent on sophisticated technology was demonstrated by the description of a civil engineering course where instruction involved the use of a programmed booklet (Croxton and Martin, 1965).

A discussion of teaching methods was found in Allanson (1968) who advocated, with particular reference to scientists, that mathematics be presented as relevant to the real world and also in Davis and Hills (1972) who described a systematic approach to teaching electrical engineering to a mixed-ability group. An application involving discussion groups, lectures and study notes was detailed by Woodall (1976). Attempts to deal with the specific mathematical difficulties of undergraduates in Economics and Biology were described by Tagg (1970), whilst the problems associated with teaching mathematics to craft technicians were the subject of Rees (1974) and (1975). The aims and objectives of teaching mathematics to non-specialists came under the scrutiny of Elton (1971).

The subject of self-paced study in particular was the concern of review papers that looked at Keller-plan implementations in the United Kingdom (Elton, Boud, Nuttall and Stace, 1973) and also undergraduate science courses (Elton, 1975). A book by Hills (1976) was a detailed account of the development over a five year period of some self-teaching courses at the Universities of Surrey, England and Concordia, Montreal. The factors affecting the programming and control of individualised systems were outlined by Romiszowski (1975). Modifications to the basic Keller plan were proposed by Davies (1976) and (1977), where the variety of educational aims were matched by a similar variation in instructional method and an attempt was made to assess the relative merits of "hybrid" schemes.

With reference to the aims and procedures of evaluation, a comprehensive review was given by Taba (1962) and more recent trends and implications were detailed in Tawney (ed. 1976) and a review of evaluation methods by Stake (1976). A hand book of research methods was the work of Burroughs (1971) and a comparison of the relative merits of different research methods was given by Mouly (1970). A thought-provoking paper on the approach to evaluation as illumination was that of Parlett and Hamilton (1972). A valuable discussion of the problems of inference and proof associated with non-quantitative forms of evidence was undertaken by Becker (1958). General principles of evaluating the degree of success of teaching was the subject of Beard (1967) and problems associated with evaluation based on student opinion were discussed by Bryant (1967). A comparison was given of two approaches to educational research by a description of two case studies of innovations at two different universities in the United Kingdom (Sheldrake and Berry, 1975). An account of an attempt to assess the creative aspects of mathematics in schools was given by Dunn (1975). A review of methods of measurement of affective variables in learning mathematics was published by Aiken (1976).

More recent publications which are concerned with evaluation include discussions of the methodological and epistemological issues (Galton ed., 1980 and Smetherham ed., 1981) and practical guidance in the design of evaluative programmes from Harlen (ed., 1978) and Morris and Fitz-Gibbon. (1978) Hamilton (ed., 1977) has presented a collection of papers illustrating the development of evaluative methods whilst Stenhouse (ed., 1980) has assembled an interesting set of case studies.

The details of all references and associated literature are contained in the Bibliography which follows the Appendices.

CHAPTER 3

A CULTURAL MATHEMATICS COURSE: THE RATIONALE AND SYLLABUS DESIGN

3.1 Introduction

In looking at a course in mathematics which has no direct bearing on the students' main areas of study, it was natural to be faced with the question "Why study mathematics at all?". Students may be prompted to ask the question if they felt their time was wasted or presented little reward for their efforts. Staff may be prompted to ask the same question if they felt the course was time-consuming and detracted from more important (more advanced?) courses. Students, at least, would not necessarily take on trust that what the University felt was educationally good for them must therefore be so. In this chapter, an argument is presented which calls for a liberal educational experience as the necessary ingredient for the cultured person. Some views are put forward concerning the nature of a mathematics course of cultural rather than vocational value and the implications in terms of syllabus content.

3.2 Views of Human Knowledge

Considering first the nature of human knowledge, the views of three different authors were compared. Hirst (Archambault ed., 1965) in discussing the nature of a liberal education saw it as essential in the understanding of how man ought to live. It was seen not in terms of collections of information but as ways of understanding experience. It was a necessary feature of knowledge that there were public criteria whereby truth was distinguishable from falsehood. Each form of knowledge had its own distinct logical structure. Hirst denoted seven distinct primary disciplines, of which mathematics was one. He concluded with a plan for a liberal education which was not constructed in terms of the accumulation of information. The vast growth of knowledge made the choice of topics a difficult operation so he suggested that what was important was sufficient exposure to the concepts for someone to know "how it works". It was also noted that critical appreciation may require some understanding of the technicalities and that consideration must be given to the abilities and interests of the students.

Phenix (1964) categorized human knowledge in terms of what he referred to as realms of meaning. He saw the problem as that of human beings engaged in discovering, creating or expressing meaning. His analysis of the realms of meaning led him to propose six categories, one of which, symbolics, contained mathematics. The distinct realms corresponded to distinctive human functions of expression, communication and perception and each had its own logic and ways of comprehending.

After referring to what he saw in the modern world as a growing threat of meaninglessness, Phenix went on to make a plea for a determined search for meaning in education. This he described as the fullest possible realisation of the distinctly human capacities and required the implementation of a liberal education. Basically he advocated taking material from the scholarly disciplines, since they represented the accumulation of human endeavour and possessed a framework in which fact was distinguished from opinion. The representative ideas and distinctive logic of each discipline were important and material should be chosen so that the pupil was brought to see the wood by way of the trees. He also advocated the use of "guided discovery" methods and the choosing of material that appealed to the imagination.

The third view of human knowledge examined was that taken by Whitfield (1971). He suggested that modern social and economic developments have produced something of an educational crisis. He concluded that school organisation, at least, should be primarily concerned with the disciplines of knowledge. He went on to define a discipline as a flexible conceptual structure which organises knowledge in a way suitable for learning. Hence it was necessary in order to plan a liberal education to be able to differentiate between the forms of knowledge.

Taking Phenix's realms of meaning as a frame of reference, Whitfield went on to construct a scheme for the education of the 9 - 16 year old and suggested that later age groups demanded a more specialised education. He divided mathematics into five-twelfths symbolics, five-twelfths empirics and two-twelfths aesthetics and considered that the curriculum should emerge from a synthesis of the views of the philosopher and the psychologist.

It can be seen that all three approaches recognized the existence of distinct forms of knowledge (or realms of meaning), each having its own criteria and tests of truth and falsity. However the categorizing is done, it is advocated that some attempt is made to expose students to <u>all</u> forms of knowledge, that is, to impart a liberal education. There was agreement that education should be based on traditional disciplines, these not being based on arbitrary divisions but on the historical development of man's scholarship and the unique logic in each form. Anxieties were created due to the proliferation of human knowledge and it was suggested that material was selected to reduce confusion, the representative ideas in each area should be exemplified, the distinctive logic should be exploited. There were important concepts which could serve as organising principles. Phenix advocated guided discovery and seeing the wood by means of the trees, Hirst noted that critical appreciation may require some understanding of the specialist and all agreed with the need to exploit the interest and imagination of the student. This view of education which emphasises the distinctive values of the various disciplines and requires the cultured person to be exposed to all the different forms is in contrast to a philosophy which presupposes the inherent logical unity of all knowledge. Although subject divisions are imposed, it is thought there is an essential relatedness and one purpose of education is to develop an appreciation of how all knowledge holds together. (Brubacher, 1962.) Thus the multiplicity of studies which has resulted from the expansion of human knowledge, leading to problems of confusion, superficiality or congestion, may be counterbalanced by adopting an "integrated curriculum".

There is little question that in certain circumstances, this approach has proved fruitful in the promotion of learning by problem-solving, project work and the exploration of broad topics. However, there are problems to be faced. For example, what is it that is being integrated? Should the subject frontiers be swept away or should relationships be developed across them? How does one guard against reducing knowledge to a formless mass? The operation of such a curriculum depends on great flexibility of organization and a highly experienced band of tutors.

The two philosophies, exemplifying the differences or integrating the various forms of knowledge, both respond in different ways to the problem created by the demands of society for more and more specialisation. The individual feels alienated by his inability to comprehend what the expert is doing. (Two, or more, cultures?) An example of this is the impact of computer science; the non-expert sees the new technology as a threat rather than a stimulus to his spirit of enquiry. There is no reason to see computer science in the narrow context of its usefulness in the modern world. There are features which can contribute to a liberal education, for example, the use of symbolism, the logical structure of the algorithm and the precise requirements of programming are all worthy of study by the non-specialist.

3.3 Views on Significant Aspects of Mathematics

If the case for the study of mathematics for its cultural interest is accepted, one is faced with a problem of constructing a suitable syllabus. This section looks at ideas about what are the essential concerns of mathematics, particularly in relation to courses for the non-specialist. The following section then considers the implications in terms of syllabus design.

To begin with, it did not appear helpful to think of mathematics in terms of what mathematicians do; they indulged in a variety of activities, most of them highly specialised and incomprehensible to the non-expert. Mathematics, as we saw earlier, is significant as a way of thought. In the introduction to the book "Ideas in Mathematics" it says:

> "... we exemplify and try to cultivate a mathematician's habits of thought, which are as distinctive as the dancer's habits of posture and motion or a musician's habits of playing his instrument."
>
> (Douglis, 1970, p ix)

An emphasis can be noted on the processes involved rather than the material content. Other authors were eager that the cultural significance of the subject was appreciated. Courant and Robbins (1941) claimed that two thousand years of history provided evidence that mathematics was an essential part of the equipment of every cultured person. This theme was supported by Brueckner, Grossnickle and Reckzeh (1957) who said that mathematics had social and cultural significance, whilst Man Keung Siu (1977) gave this as one reason for teaching mathematics to "maths haters", together with its connection with the real world and to create a means of communication with others. This last point was raised in a report concerned with the teaching of mathematics to non-specialists beyond O-level.

> "... every undergraduate should know enough of what mathematics is about to be able to discuss it intelligently with his fellows."
>
> (Mathematical Association Report, 1965, p 3)

This assertion was based not only on the demands of intellectual development but also on future career prospects. It was considered important that those destined for high administrative office should have some knowledge of mathematics.

Kline (1977) made this same point when he said that liberal arts courses were perhaps the last opportunity of the mathematician to influence future policy-makers.

A book which was at one time used as a text-book on the subsidiary mathematics course was "Mathematics - Art and Science" (Dowdy, 1971). In the introduction, the author stated his aim in writing as:

> "My purpose is to develop an appreciation of mathematics and an awareness of its contribution to our culture."

He went on to explain that he saw the best way of doing this was to guide the non-mathematician in a few in-depth experiences. Dowdy held the study of mathematics to involve:

> "... experimentation, intuition, conjecture and, finally, proof or disproof."

A similar aim was contained in the preface to the book "Mathematics - A Human Endeavor" (Jacobs, 1970). Although it acknowledged that mathematics was indispensable to most activities of the modern world, its true significance did not depend on its practical use. The author hoped that the book would develop an interest in students who previously had little or none. The exercises in each chapter emphasised inductive thinking and discovery, and it was hoped to show that:

> "Mathematics has its own kind of beauty and appeal to the person who is willing to look."
> (Jacobs, 1970, p xii)

Another approach was presented in a paper published in the "Bicentennial Tribute to American Mathematics"

(Tarwater ed., 1977). In "The Freshman Liberal Arts Course" Kline stated that the value of such courses did not lie in their content; judgment, weighing of evidence, inductive reasoning and reasoning by analogy were more vital. His main point was that these were best developed by attempting to understand the physical world. Students were unlikely, in his opinion, to respond to an abstract approach and he gave a warning to the specialist in his comment:

> "Mathematicians offer topics they value, but mathematics proper has very little to contribute to a liberal arts education."

It will be evident in the following section (3.4) that different authors had rather different views about how best to achieve the aims of a non-specialist course, but it may be noted that there was some agreement on what the general aims should be. These may be summarized as follows:

(i) the appreciation of the distinctive forms of mathematical thinking,

(ii) the appreciation of the role of mathematics in our culture, with a knowledge of the development of some of its significant ideas,

(iii) the development of a mathematical way of thinking, involving inductive thinking, conjecture, reasoning by analogy etc.,

(iv) the awareness of the unique role of mathematics in understanding the real world.

Having noted these aims it is possible to consider the choice of appropriate syllabus material.

3.4 Published Material On Course Content

To provide some basis for comparison, several books were examined which were concerned with a "non-technical" approach. In particular, the contents of four books were compared. The titles reflected the authors intentions, being "Ideas in Mathematics" (Douglis, 1970), "Mathematics for Liberal Arts Students" (Richman, Walker and Wisner, 1967), "Mathematics a Creative Art" (Bower, 1973) and "Elementary Concepts in Mathematics" (Jones, 1970). The result demonstrated that there was no prescribed syllabus for a course of this nature, there being few topics which were common to all four texts.

Douglis, for example, provided fifteen chapters, each one devoted to what he considered was a significant aspect of the discipline. It was interesting to observe that some chapters were about history (e.g. Babylonian Mathematics, ch.4), some about mathematical structure (e.g. Polynomials, ch.5; Theory of numbers, ch.11) and some about processes (e.g. Mathematical Induction, ch.10). In this particular book, there was a danger of being overwhelmed by the volume of material and losing sight of the author's intention of trying to demonstrate habits of thought. To be fair, the introduction did state that the user was expected to be selective, but there was an impression given that so much had been included as a means of ensuring that something, at least, would turn out to be useful.

Dowdy developed his aims by devoting each of his nine chapters to one significant topic. These included, for example, number theory (ch.1), projective geometry (ch.3) and mathematical logic (ch.5), with a minor exception, the topics could be studied in any order. It was also stated in the introduction that only brief glimpses were included of the history and applications of mathematics.

A different approach was taken by Jacobs in his book where he stated a deliberate omission of any discussion of sets and other topics of the "new maths", in the belief that such ideas had been overemphasised. Despite this intention, there were chapters on probability, statistics and topology, all of which, it could be argued, were non-traditional at this level.

In developing his theme for a liberal arts course, Kline said that in his opinion, the commonest course of this nature, based on set theory, symbolic logic, theory of numbers and similar mathematical systems was too sophisticated. Students were unlikely to be intrigued and may be confused. He also dismissed a presentation of "technical" mathematics as "mere colour mixing" and did not favour courses that presented "puzzles, trivialities and curiosities". Appreciation not skill was a justifiable goal.

In the paper, Kline referred to the implementation of his philosophy in his book " Mathematics - A Cultural Approach" (Kline, 1962). The difficulty was that by choosing his stand firmly in the applications in the real world, Kline was sometimes confronted with topics which technically were beyond the ability or experience of prospective students. For example, in a fairly lengthy discussion of projective geometry in Art, the whole presentation was a description of the use of perspective. This may be considered valuable on other counts, but it may be questioned how much weighing of evidence, inductive reasoning or reasoning by analogy were generated.

Some authors made the point that initially, an interest in mathematics came from concrete experience of the world around us. Avital and Shettleworth (1968) emphasised the search for insight through the choice of good models and the development of mathematical modelling. They claimed that mathematical knowledge was gained by attempts to solve problems, the nature of mathematics was a study of relationships and comprehension was enhanced by contact with a large variety of models taken from the environment. Although it was acknowledged that relationships ultimately became abstracted, Courant and Robbins argued that mathematics must be concerned with the real world:

> "A serious threat to the very life of science is implied in the assertion that mathematics is nothing but a system of conclusions drawn from definitions and postulates that must be consistent but otherwise may be created by the free will of the mathematician. If this description were accurate, mathematics could not attract any intelligent person."
> (Courant and Robbins, 1941, p xvii)

Drawing the various points of view together, it was apparent that to search for a unique syllabus was unfruitful and that it was more appropriate to judge material in terms of how it was to be used. The emphasis was not on what was in the syllabus but why it was thought necessary to include it. Thus, it was important to bear in mind what the course was trying to achieve. A second consideration mentioned as important in this context was to attempt to make the subject appeal to the student and stir his interest. One interesting approach to this problem has been a fairly recent attempt to provide motivation by presenting a series of intriguing problems and use these as the basis to develop the necessary manipulative mathematics for deriving the solutions. (See, for example, Hunkins and Pernot, 1977 and Averbach and Chein, 1980.) In discussing mathematics as an art, Servais and Varga (1971, pp.16-17) emphasised the search for patterns and went on to describe the unifying concepts that were present in modern mathematics which could be exploited to economise on learning effort. They felt that students should be helped to enjoy mathematics, for without interest, learning required overwhelming effort.

3.5 Themes of Cultural Relevance

In the first part of this chapter, it was argued that the study of mathematics should be part of a liberal education because of its significance in our culture and the distinctive ways of thought that it embodied. Some overall aims were identified (p. 24) and they were seen to be based on an appreciation of the significant ideas of the subject and an involvement in some genuine mathematical activity. The manner in which the aims were to be achieved was viewed differently by different authors. Several themes were used in different publications which were thought suitable for a cultural approach, namely history, mathematical significance, practical application, recreation and everyday use.

The history of mathematics was considered to be valuable for several reasons. Many students with an arts background would be stimulated by the historical aspect and the subject itself gave an indication of how mathematics had contributed to our cultural development. The development of some topics involved not too serious manipulative difficulties and students might be encouraged to discover that various topics had, in the past, stretched the minds of even great mathematicians. However an important consideration may be the integration of what is essentially a descriptive treatment with the generation of some stimulating mathematical activity.

With mathematical significance a distinction would need to be drawn between what was accepted as significant by the mathematician and the non-mathematician. This was seen as important; if Kline's point made earlier was true, then courses based on significant mathematical systems did not stimulate student interest. A problem lay in the fact that significant mathematical systems are often abstract, yet students were limited to thinking in concrete terms by virtue of their experience of the subject. The moral was that in any development, it was important to consider the viewpoint of the student and to identify what was the appropriate starting point.

The significance of practical applications raised the greatest divergence of views amongst the various authors. In some cases, the use of mathematics in the real world was held to be a strong source of student interest and it was possible to choose topics that were within the concrete experiences of the students. It was in this context that authors often chose the "intriguing problems" which could be used to stimulate student motivation for coping with the underlying mathematics. However one difficulty here was that some applications which were thought to be of significance would present students with too much technical difficulty and the presentation would have to be on a rather descriptive level.

Recreational mathematics was a sphere in which there was much to arouse the interest of the student. An advantage was that problems could be formulated from simple material and the assumptions were clear. A great deal of worthwhile activity could be generated, recreational was not synonymous with trivial. This approach had become well known through the publications of Martin Gardner in Scientific American and were exemplified by such recent books as that of Averbach and Chein (1980). An important point concerning this topic was seen to lie in the care required to avoid giving students the impression that they were merely playing games. It was also the topic most likely to cause a mathematician some disquiet if incorporated in an "academic" course.

If, as various people had suggested, the students found a source of interest in their experiences of the real world, then the everyday uses of mathematics could be a profitable field of study. The main difficulty with this was thought to lie in the differences between the opinions of lecturers and students on what were the most worthwhile aspects of study. Student expectations were unlikely to be profound. It is worth noting the point made earlier, that student experience may contain useful starting points from which to develop topics that the mathematician will accept as significant. Two examples may be given of this. The layman is confronted almost daily with some form of statistics, a topic which he is often well motivated to study. Also, his concern with home economics may, for example, be used to enlarge his interest into a study of linear programming. Both of these topics give considerable scope for the formulation of mathematical models and the activity of problem-solving. These are both worthwhile objectives which are compatible with the overall aims noted earlier.

This chapter has created the background for a discussion of the subsidiary mathematics course under investigation. The following chapter sets out the details of the course as it existed at the beginning of the investigation.

CHAPTER 4

THE UNIVERSITY COURSE IN SUBSIDIARY MATHEMATICS FOR ARTS UNDERGRADUATES

4.1 Introduction

This chapter describes the course which was the subject of the first of the two studies. The details of the course are given and its objectives are identified. A discussion of the syllabus content is presented, a comparison is made with the syllabus adopted by the course tutors and the aspects identified in published material in chapter 3. Conclusions are drawn about the choice of course material.

4.2 Description of the Subsidiary Mathematics Course

Examination of the University Undergraduate Prospectus indicated the background and rationale of this course. The usual pattern of undergraduate study was to take two principal subjects for three years and two subsidiary subjects for one year. At least one of these was to be a science subject and at least one was to be chosen from the humanities and social sciences. Thus, students studying two principal arts subjects would choose a subsidiary from the list of sciences. The subsidiary mathematics course considered here was described as intended for students whose interest in mathematics was cultural rather than technical, and who had not taken mathematics beyond G.C.E. O-level. The emphasis was on the development of the subject and on its ideas and achievements.

The information given in the prospectus was augmented by information obtained in discussion in 1979 with the then course supervisor. It was explained that historically, quite a lot of students choosing to do a subsidiary course in mathematics were following principal courses in humanities or social sciences. Among existing courses was one centred largely on statistics, which was appropriate for students following such subjects as economics, psychology, sociology or natural science but not necessarily suited to those in other social science or arts subjects. The aim was to produce a more general course that would be of some greater interest to this category of student and one which took a broad view of mathematics. This aim gave the course an unusual feature in that there was coverage of a very wide field of topics. It was hoped that it would have more to offer the student than simply the passing of the final examination and so satisfying a University requirement. From the beginning, a "linear sequence" approach was rejected and advantage taken of the width of coverage by making the topics as independent as possible; this helped to avoid the situation in which students who became lost at the beginning were lost from then on.

From its inception in 1972, the course attracted increasing numbers so that each year throughout the period of this investigation, something like one hundred and forty students were enrolled. Formal tuition consisted of two one-hour lectures and a one-hour tutorial each week for each student. The lectures were usually given to the entire group. An exception to this was the class of 1978/79 which was split into two groups for one of the weekly lectures. For tutorials, the class was split into tutorial groups led by a member of staff. The lectures were given by six or seven different members of staff, each of whom would arrange a short course of lectures on a single or possibly two topics. Generally these same lecturers formed the body of tutors, although over the years, individual commitments might dictate that a person did one job but not the other. Thus in 1979/80, six staff were involved with lecturing and four of these acted as tutors, whereas one other person tutored without any lecturing.

The major means of assessment was a written, unseen, examination paper at the end of the session. In order to give substance to the desire not to make passing the examination the over-riding concern, significant weight was given to the effort made by the student in his year's study as manifest by attendance and the performance in the week-by-week assignments set and marked by the tutors.

The syllabus showed minor variations from one year to another. The syllabuses in use in 1977/78 and 1979/80 are shown in Appendix 2. Variations resulted from the wish to retain only topics which were thought to be suitable (as suggested by students' answers to assignments and to examination questions) and from the availability of lecturing staff. For example, the section on probability in the 1978/79 programme was omitted in the following year, since the lecturer concerned was not available. The time released by this was given over to further study of calculus because of a general feeling that this topic was not well understood by the students and it was considered worthwhile to devote more time to it. Similarly, musical scales had been discarded because it did not fit easily into the overall pattern of topics and did not generate any particular interest in students who were without a musical background.

4.3 The Course Aims

Some mention of the aims of the course has already been made in the previous section. Information additional to that in the prospectus was obtained from discussion with staff members (see chapter 9, sections 4 and 5).

It was discovered that the aims uppermost in the minds of the staff were the generation of some genuine mathematical activity in the students and the appreciation of the precise way of reasoning in mathematics. Although it was considered reasonable in some topics to introduce descriptive material, there was a real desire to present the course so that activity rather than reception was a key feature. Within these overall aims, the lecturers also identified objectives related to specific aspects of the syllabus. Thus, it was maintained that the students should encounter and come to appreciate significant examples of mathematical systems, that the historical development possible with some topics was a source of student interest, that conjecture and proof were worthy of study if a precise way of reasoning was to be appreciated and the use of mathematics in the real world was an important feature of the subject. It was appreciated however, that a student's limited experience was a factor in how far some of these could profitably be pursued. This was the case, for example, in trying to adopt a rigorous approach but there was a definite concern to avoid a jumble of formulae or a "cook-book" of methods.

Although, quite naturally, individual members of staff gave different emphasis to the various aims and objectives, it could be seen that the general purposes were very similar in intention to those aims which were identified in the literature and noted in 3.3.

4.4 The Syllabus

During this investigation, the syllabus for Subsidiary Mathematics was grounded in the study of number systems in the first term, geometric systems at the beginning of the second term and algebraic systems in the later part of term two and in term three. Interleaved with these broad themes were further topics, for example flow-charts, calculus, probability and theories of motion. Topics were moved around slightly from one year to the next and sometimes suffered deletion (see Appendix 2). Although the topics were thought of as being as independent as possible, it was interesting to note that the course supervisor remarked that topics were not as separate as all that and the course developed some structure quite naturally. There was enough remaining of the original concept to help those students who found the subject technically difficult. He went on to say that a good student had the possibility of seeing the course as more than a collection of topics and of appreciating how the subject did fit together.

The topics were looked at to see how they embodied those aspects of a cultural approach which were identified in 3.5. Firstly, the history of the subject was evident in the treatment of such topics as geometry, number systems and theories of motion. It was interesting to note that the historical background was often used to arouse the interest of the student and to lead on to related problems which required some active response. Thus, for example, the development of number systems went on to ideas of conjecture and proof in simple number theory.

It would appear that an over-riding consideration in the choice of syllabus topics on this course was one of mathematical significance. This was concluded from the time allocated to careful developments of number systems, geometric systems and algebraic systems. In the latter case, the programme introduced sets, groups, propositional calculus and Boolean algebra and the opportunity was taken to illustrate the concept of isomorphism.

Applications of mathematics included maxima and minima (in calculus), switching circuits (in Boolean algebra) and relative velocities (theories of motion). The teaching methods in calculus and theories of motion included the use of simple illustrative materials which were effective in focussing student attention (see 8.2).

The themes that were least evident in the programme were recreational mathematics and the everyday uses of mathematics. Although it has been mentioned earlier that a study of statistics might be included under this last heading, it was noticed that when this topic had been part of the syllabus, the emphasis had been rather more on the historical development of probability and not so much on the everyday uses. It will be seen in chapter 9 that the lecturing staff were least enthusiastic about topics concerned with everyday uses.

It was concluded in the previous chapter that the manner of interpretation of the syllabus content was of major importance, that is, the purpose of studying a particular topic might be as important as the actual content. Two examples of this have been mentioned, the exercises on conjecture and proof in number systems and the isomorphism inherent in algebraic systems. Further examples were the idea of proof by induction in measuring and counting and the development of axiomatic systems in the various geometries. This last topic was also an example of how difficulties may arise when dealing with the subject at class level. Having decided that geometry was a medium to illustrate axiomatic systems, was the technical detail involved in studying Euclidean, non-Euclidean, finite and Cartesian systems within a few weeks too much for weaker students?

The investigator saw the implication of this as requiring lecturers to give careful thought to the translation of the syllabus into course material. Consideration was needed of such aspects as the proper balance between descriptive and problem-based content, how the treatment of a topic fitted in with the overall aims of the course and how the presentation could arouse the interest of the students and yet lie within the compass of their ability.

4.5 Conclusions

Implementation of the subsidiary mathematics course was based on the aims of providing a broad-based view of the subject, developing an appreciation of its significant ideas and their development and involving students in some genuine mathematical activity. The choice of more or less independent topics taught by different lecturers allowed the students opportunities to make fresh starts and so reduce the risks of becoming lost and frustrated. This relative independence also had the effect of giving the syllabus a degree of flexibility, which enabled topics to be introduced or modified with relative ease.

The idea of providing a broad view was thought to be a reasonable basis on which to develop the generally agreed aims of a cultural course. It has been argued that there is no "prescribed core" of topics necessarily contained in such a course and those chosen seemed appropriate enough in the circumstances which existed. What was also important, was the need to give careful attention to the objectives that were to be achieved in teaching each part of the syllabus.

The syllabus had a structure in which topics were grouped around some major themes. This was thought to have some advantage in that the themes would counterbalance the tendency for individual topics to appear disjointed and also, for better students to gain some understanding of the power of generalisation in mathematics. Later chapters describe how the students (chapter 8) and the staff (chapter 9) viewed the syllabus and what was achieved in practice. In chapter 10, some further discussion is given of course material in connection with the development of study booklets.

CHAPTER 5

THE RESEARCH PROCEDURE: IDENTIFICATION OF OBJECTIVES AND METHODS

5.1 Introduction

(A detailed description of the research programme eventually adopted is in Appendix 1, including a calendar of events.) This Chapter contains a brief survey of evaluative techniques, the identification of the objectives of this study, an examination of possible research procedures, a description of the methods actually chosen and a discussion of factors which bear on the value of the data. (See Ch. 13 for the second case study.) A retrospective examination of the research and the various contributory factors is in Chapter 20. (p.335 et seq.)

5.2 Evaluative Methods: the Background

Different methods of evaluation have evolved to respond to the changes that have occurred in the aims of education. In chapter 3 (section 2) it was seen that several authors (e.g. Hirst, 1965, Phenix, 1964 and Whitfield, 1971) were conscious of the influences of a growth in human knowledge and changes in the needs of society. There has been a recognition that education is primarily concerned with not only imparting a fixed body of knowledge but also with the structure of knowledge. Bruner (1966, ch.2), noted that lack of real evidence concerning transfer of training posed problems in dealing with growing needs in education and suggested a solution - the conjecture that the key to transferability was a study of structure and not content.

The change in emphasis in educational aims was referred to by Rogers in a discussion of the "humanistic" view of learning. (Sahakian ed., 1976, ch.12.) He wrote that the traditional concept of learning was based on suppositions which included, for example, that students could not be trusted to pursue their own learning, that presentation equalled learning and that truth was known, knowledge was a closed book. It was claimed that this led students to think of education in terms of the accumulation of course grades. A similar construction was given by Brueckner, Grossnickle and Reckzeh (1957) who identified the assumptions in a traditional theory of learning of a mechanistic neurological process, an authoritative prescription through deductive procedures and the application of repetitive drill. In contrast, both of these works proposed alternative ideals. These, for example, gave weight to purposeful experiences and activities which the individual felt were relevant, the perception of relationships, the search for insight and the process of learning through doing. Phenix (1964) claims that students will cope better with a changing world if they are familiar with basic modes of thought and investigation.

In "Change and Development in Evaluation Strategy", Harlen (Tawney ed., 1976, ch.3) named four influences producing changes in evaluation. The first was the changes in the aims of education which required new kinds of data for the decision-makers. The chosen objectives may require evaluative information which is provided by methods other than criterion-referenced testing. The second was the move away from the passing on of knowledge and towards a preparation for dealing with new experiences. Greater value was given to encouraging pupils to challenge rather than accept and to derive confidence in their own powers of reasoning. The third was the development of new teaching methods based on the findings of educational psychology which stressed the need to consider the individual and his way of thinking. Finally, there was a concern with the shortcomings of the traditional approaches to evaluation.

5.3 The Development of Evaluative Models

The conditions in education which were outlined above were instrumental in producing changes in evaluative models. Harlen (ibid, 1976) identified two traditional strategies for evaluation. These were derived from the experiences of the experimental psychologist and the work of the school Inspectors. The "psychometric" model was based on the formulation of specific objectives, controlled experiments and the interpretation of the achievement in terms of test scores. The second model relied on "informed judgment" by experienced observers. The key feature here was that the product of education was too complex and broadly spread to be expressed in terms of prescribed objectives. Its evaluation was best represented by the collated views of highly experienced observers. Deficiencies were held to exist in both models; in the first it was necessary to restrict outcomes to those which could be objectively measured whereas in the second, the conclusions tended to be subjective and unreliable and dependent on individual value judgments. In both models, Harlen concluded that it was difficult to formulate practical guidelines from the results that were obtained. The following refers to the experimental strategy:

> "when put into practice, this classical experimental model, which seems so logically attractive, fails to give the information which is most useful for either formative or summative evaluation. What makes it so attractive and reasonable is precisely its weakness; it is part of an over-simplified view of curriculum development."
>
> (Tawney ed., 1976, p.35)

Developments have led to the variety of models which were mentioned earlier. One description of currently existing approaches begins by identifying eight common dimensions in evaluation classification. (Stake, 1976.) In his explanation, Stake points out that no one method is suitable for all situations, the information needs of each investigation will be different. Any particular study is unlikely to lie at the extreme points of any of the dimensions, there will be variation from one study to another. The dimensions may be briefly summarized as follows:

Formative-Summative - The distinction may be simply on the uses made of the results. Summative evaluation of a completed component of a course may differ little from formative evaluation of part of the course. The best distinction is not when but why.

Formal-Informal - Informal evaluation is an abiding human act. Formal evaluation demands standards of accuracy, validity, credibility etc.

Case Particular-Generalisation - The distinction between the study of a programme as fixed and the study of a programme as representative of others.

Product-Process - The primary attention is directed either towards the outcomes of the programme under investigation or to its transactions.

Descriptive-Judgmental - On one hand, studies may be highly descriptive of students and settings but with little direct reference to criteria of worth. On the other hand, others are especially concerned with the multiplicity of values that may exist in the educational setting.

Preordinate-Responsive - Preordinate studies lay more stress on formulating objectives, hypotheses and prior expectations; responsive studies are organized around phenomena encountered as the programme unfolds.

Wholistic-Analytic - A "case-study" approach attempts to treat the programme as a totality, whereas an alternative approach chooses to concentrate on a small number of key characteristics.

Internal-External - Evaluations may be conducted by personnel either from within the institution or from elsewhere.

Stake pointed out that these dimensions were not entirely independent, for example, an internal evaluation is more likely to be formative rather than summative and to be descriptive rather than judgmental. He then gave details of nine typical approaches covering the whole range of evaluation. Some, like the "Prestige panel" model which involves evaluation by an outside panel of eminent, but possibly inexpert people, are not closely related to the investigations described here. There were three which were thought to be worthy of consideration.

The first of these was described as "student gain by testing". It requires tests to be developed based on pre-specified objectives. The approach is that of the educational psychologist and the key elements are the statement of goals and the analysis of test scores to compare goals and achievement. The advantages are the precise measures of student progress that are possible; the drawbacks are the possibility of over-simplifying the educational aims and the lack of concern for the processes involved in the teaching. It may be possible to judge precisely a point of under-achievement without this providing any clue as to why this is so.

A second model was named "Transaction-observation". This is in contrast to the student-testing approach and may be described as a disciplined study of the educational processes, with attention given to the "learning milieu". Issues are often drawn from the proceedings rather than from theory or goal statements. It produces a broad picture of the programme and highlights possible conflicts about values. A source of weakness is the tendency to rely on subjective perception of events and to ignore causes.

The purpose of the third approach was described as the generation of explanations and tactics of instruction. This "instructional research" method lays emphasis on the use of controlled conditions, multivariate analysis and the formation of conditions which allow the results to have some general value. The outcome is identified as the development of new teaching material or new principles of teaching. Difficulties arise from the rather artificial conditions that need to be created for the necessary level of control. There is also little emphasis given to the "humanistic" goals of education.

The mode of evaluation must be one which is compatible with the chosen approach and the objectives that need to be achieved. This implies not only an overall model of evaluation but also a careful consideration of available techniques to decide which methods are best suited to producing the appropriate data. Before going on to explain in section 5.5 which approach and data gathering methods were thought best for this particular investigation, a brief survey is given of available procedures.

A basis for judging the appropriateness of an evaluation technique has been stated as the provision of evidence which is valid, reliable, germane to the argument and, where necessary, is in time to be used in any decision making. (Steadman, in Tawney ed., 1976, ch.4). Steadman divided the functions into four groups; formative feedback, attainment testing, attitudinal change and motivation assessment and the description of the curriculum in terms of the context and processes involved.

The techniques which were of direct relevance to the first of these categories were given as questionnaires, checklists, teacher diaries, group discussions and interviews. If the objective was seen as attainment testing, the procedures that were stated as appropriate were the use of control groups and pre- and post-testing.

The discussion of the methods of attitude measurement were of particular interest by virtue of the fact that in both parts of this research, there was some interest in the students' attitudes to mathematics (see 6.4 and 15.2). Attitude measurements have traditionally involved the use of attitude scales which are in the form of questionnaires; one method requires respondents to indicate their degree of agreement with various statements. Analysis of their replies reveals a profile which is a measure of the professed attitude. Another approach is the semantic differential scale where objects or concepts are judged by placement at a suitable point along a bi-polar scale, such as good-bad, active-passive etc. Typically, the scale contains seven intervals which are assumed to be of equal magnitude. The reliability of scales requires the initial provision of a large pool of items which is refined down in a lengthy process of trial and judgment. The validity of scales is also problematical; there is a source of difficulty in the fact that a test devised for one particular circumstance does not fit the precise demands of another situation. With semantic differential scales, the words used to define each dimension may not have the same meaning for different people and their personal judgment may not be adequately represented by an equally-spaced interval scale. One of these objections may be overcome by using a repertory grid test, which is similar to the semantic differential test but allows the individual some freedom to define his own dimensions. An overall reservation on the value of test material is the poor correlation between professed behaviour in pen and paper tests and decisions made in real life; attitudes measured in tests have poor predictive value (ibid, p.67).

In discussing the aim of describing the context and processes of the curriculum, the main components were given as the identification of the external features (e.g. school type and setting), the employment of the "case-study", the use of observation schedules and finally, participant observation.

Observations were said to be of value because they provide evidence of actual changes in the classroom; it is possible that the adoption of new methods does not have the desired effect on the actual experience of the student or pupil. The difficulty with the compilation of valid data by this process is a considerable demand on the time of the investigator(s). Unless observations are restricted to reasonably straightforward objective events, observers may need some training if reliable and valid results are to accrue. This is so if observers are required to make judgments about the intentions of certain events when they are recorded. The case study is widely used in evaluation and is very flexible. At its best, it will provide information about the interrelationships involved as well as the context. The drawbacks to this technique are its lack of controls and apparent subjectivity and that the review may be too involved with the individual problem and fail to give information of more general nature.

It was noted that the roles of participant and observer are a matter of balance. If the observer attempts to be too objective, he may not perceive some of the more hidden aspects which may be of real interest. On the other hand, the teacher may suffer influence from unconscious motivations, self-justification and post-hoc rationalizations. If the various factors are combined in the right proportions, the observer who is also a participant offers the greatest chance of achieving the insight required for effective educational change.

All these considerations were influential in the early stages when the research programme was formulated. Several later publications had important influence during the implementation. For example, the widening interest in evaluative methods was apparent in works which outlined differences in approach. Alternatives to the controlled experiment could be found in "Beyond the Numbers Game", (Hamilton ed., 1977) whilst a book which demonstrated how the emphasis on evaluation can vary in different innovatory programmes was a collection of fifteen case studies of curriculum projects. (Stenhouse ed., 1980)

Further evidence of growing interest was the appearance of practical guides for the would-be evaluator. Harlen (ed., 1978) was concerned that the practising teacher should understand the detail involved in the process of evaluation. Elsewhere, there was advice, for example, on methods of data collection, sampling strategies, setting up observation systems and record-keeping. (Morris and Fitz-Gibbon, 1978)

There was evidence of the continuing discussions concerning aims and methods. The way in which problems arose because of differing perceptions of the role of evaluation was described (Smetherham ed., 1981, p.17) and the effects this could produce on the practice of the art. (p.89) It was also emphasised that important lessons could be learned from the social sciences about procedures relevant to educational evaluation. (ibid, p.111)

The views of several contributors were also assembled by Galton (ed., 1980). The "traditional" and "newer" paradigms of evaluation were brought into perspective (Ch.4) and it was suggested that further developments may result in a synthesis of the different philosophies, stimulated by the development of new measuring instruments.

In the following sections, the tasks of this particular study are identified and appropriate procedures are selected.

5.4 The Aims and Objectives of the Research

The overall aim was to undertake a descriptive survey of the currently existing course and to identify the major issues which shaped its character. This implied the need to secure a reliable picture of the situation, to identify the general nature and philosophy of a non-vocational course with which to compare this situation, to identify the goals of the course and to ascertain how far they were met and finally to draw conclusions about what developments may be considered worthwhile. Within this framework, the following objectives were formulated:

(i) to examine the purposes and content of a non-vocational or cultural course,

(ii) to create a profile of the typical student on this particular course,

(iii) to identify the salient features as seen by the student,

(iv) to identify the salient features as seen by the staff involved,

(v) to obtain some knowledge of the way students responded to the experience of the course,

(vi) to consider fruitful lines of future development,

(vii) to prepare study booklets and investigate their effectiveness as part of the overall learning experience.

5.5 Research Procedures

Once the objectives of the research were stated, it was possible to consider the appropriate research procedures. The philosophy adopted was to seek what has been described as evaluation as illumination (Parlett and Hamilton, 1972). The aim of this type of study was stated as being to look at a particular course, the way it operated and to discover what the people involved with it regarded as its strengths and weaknesses.* In the case of this study, one feature of significance was the attitudes adopted to the philosophy of a broad undergraduate education, another was what the staff and students viewed as important aspects to study, a third was the reactions of the staff to the course, and a fourth what the students' interests and motivations were. All these could be identified as contributing to the environment in which the course operated. Initially it was necessary to discover what issues were regarded as of most concern. As the investigation went on over three years, it was possible to focus progressively on the important aspects. Thus, for example, it was possible to construct an idea of what the student saw as important aims of such a course and to contrast these with what the student perceived as the aims that actually applied to the existing course.

It was reasonable to consider the possibility of an extension of this approach to a more objective assessment. Two possible aims were formulated. The first might have been to consider some measure of comparison between the course under consideration and an accepted norm. This was considered to be an unfruitful approach basically because no such norm could be identified. The difficulty lay in the fact that such courses were uncommon in the particular context so it was impossible to construct, a priori, any generally accepted

* This is a Transaction-Observation approach as defined by Stake (see p.45).

views about the nature and purposes of the course itself. Indeed, it was this fact that led initially to the formulation of the objectives listed in 5.4.

Another objective approach might have been to assume the premise that all courses are capable of some improvement and so to attempt some measure of the effect of new teaching method or material. The over-riding problem here was control of the variables. Evaluation would have been based on before and after comparisons; the only real possibility here was to compare the performance of different years. One had only to consider the problems of matching different cohorts of students in background, ability and attitudes and the difficulties generated by having different lecturers and tutors, together with the considerable constraints that this type of approach could impose on the Department concerned, to realise why it was concluded that any objective data generated would have been so constrained in nature as to add little to the broad picture that it was hoped to portray.

The development of student attitudes could be revealed by carrying out a longitudinal study, that is, obtaining data from a representative sample of students over a number of years. This was felt to be a difficult task; it would be necessary to select a reliable sample which needed to be kept intact over a considerable period of time. The students involved on this course were drawn from a wide area within the University and their common feature might only be the few hours spent together for Subsidiary Mathematics. Once the course was completed a major problem for the investigator was foreseen in trying to arrange fruitful interviews.

Also, to be of real value, the study would need to follow up the students after their graduation and this proved to be impractical. Consequently, other approaches were considered more profitable.

The inclusion in the investigation of the evaluation of a body of new course material was not thought to be appropriate. It has already been argued that the syllabus content cannot be expressed as a fixed body of knowledge (chapter 3) and the aims of the course designers play a major function in determining the choice of material. Without a detailed study of these, it would have been difficult to ascertain how new topics would fit into the existing structure. This implied that any introduction of material would have to follow an initial study of the course itself and take place in the last stage of the research. It was felt that this would demand a diversion of effort which would adversely affect the achievement of other objectives of the investigation. There were also practical constraints arising from the fact that the investigator was not a member of the course team and therefore was unable to offer any contribution to the teaching of possible new topics. The introduction of study booklets did involve a small measure of formative evaluation, but the purpose here was not to replace part of the course but rather to complement the existing methods of instruction.

A case-particular approach was chosen rather than an investigation of more generalised aspects of learning because it was thought to be more relevant to the unusual nature of this course; information concerning similar courses referred almost exclusively to the conditions in the United States.

Thus it appeared worthwhile to undertake a case-study which would reveal the exact nature of the course and how it operated.

5.6 Data Gathering Methods

Following the considerations detailed in 5.3 and 5.4 it was possible to conclude that the two methods of data collection likely to be most appropriate were the questionnaire and the interview. The advantages and disadvantages of both were weighed.

Technically more complex methods of collecting attitudinal data, for example, the semantic differential and repertory grid techniques referred to above were not pursued, partially because of a lack of conviction as to their validity and reliability but chiefly because the main method chosen (informal interview, partially backed up by questionnaire) seemed likely to yield data on a wide variety of aspects of the course. Thus, though alternative methods would have provided some cross-reference (triangulation, see, for example, Webb et al. (1966) p.3), it was considered that from a practical point of view it would be more productive to concentrate on making the programme of interviews as intensive as possible rather than dissipate energy in other directions. The questionnaires to students (Appendix 4) which were designed on the basis of initial interview responses, did provide some element of "triangulation".

The questionnaire looked attractive because it was the method by which very wide coverage of the population was feasible, thus minimising the effect of bias. However, certain problems have been noted by different authors in interpreting the results from questionnaires. The most obvious was the possibility of misinterpretation of the questions, also the style of required answer may not be adequate to express the respondent's true feeling. (Typically, responses are required in the form of a rating scale.) Another factor which might affect the interpretation was if the respondents felt pressured in some way by the questions, resulting in replies being evasive. Moreover there was the problem of what to do about non-respondents. It was not uncommon to find the results of surveys to be based on quite moderate response rates. Mouly (1970, p.243) suggested that non-responders were significantly different from responders and that the shortfall could not be taken care of by a simple weighting of the actual returns. It was usually impractical to design the system either to allow for large-scale follow-up to check on the reliability of the data or to adopt methods to bring the responses to near total coverage (ibid, p.256). It was felt these disadvantages would operate in the present investigation.

On the other hand, interviews were potentially the source of more detailed and richer information. There was also the opportunity to seek clarification of incomplete or obscure replies and interesting leads could be followed up by further questions or prompts. Indeed, it was envisaged that some of the more interesting points were likely to emerge as interviewees went into more detail about their first responses or found themselves prompted by a remark made by a colleague.
On the debit side, it was noticed that one significant drawback of the interview was the need to use much smaller samples because of the time element. In this particular investigation,

the issue was complicated by the fact that groups of interviewees were necessarily to be drawn up from volunteers, with the resultant possibility of bias. A further factor which could produce bias was the unconscious weighting of questions by the interviewer; the degree of objectivity in a question was difficult to assess. It also became clear that if the information obtained in this way was to be worthwhile, some technical proficiency would be needed by the interviewer.

With the stated objectives in mind, it was decided to use the interview as the major source of data. It was felt that the greater detail in the responses would be worth the possibility of any loss in precision due to the small sample. Nevertheless, because of this last point, it was decided that the limited use of questionnaires would be the source of useful corroborative data with which to compare the results from the interviews.

Another source of information that was considered was the scrutiny of examination scripts. This was a possible source of objective data on how students coped with individual topics. However, evidence that students found a topic difficult was not necessarily an indication that they had failed to appreciate its significance or that there was no worthwhile change in attitude. Formal permission to carry out the scrutiny would have required taking the investigation outside the host Department and since it was felt that it was more appropriately treated internally, this line of enquiry was not pursued.

Extensive observation of lectures and tutorials would have provided valuable data on the "classroom transactions"

(Stake, 1976, Ch.3) involved in the course. Unfortunately, practical constraints, chiefly related to the large time commitment this would have required of the investigator, meant that only a limited amount of such observation could be undertaken. This provided useful corroborative evidence of staff and student views on the way in which lectures and tutorials operated. (See p. 112 and Appendix 6.)

Interviews were planned with small groups rather than individuals. This was to promote an atmosphere in which the "threat" of the interviewer would be reduced. A further advantage was some degree of interaction, remarks made by one member of the group could spark a response in someone else.

Several difficulties were present in this approach. One was that a student of strong over-riding personality would impose views on the group that might not be expressed individually. A second might be that students in a group may feel pressured to express a conventional rather than personal opinion. These problems were overcome to some extent because most students were interviewed more than once (up to a maximum of four times); as the groups were different on different occasions, each individual did not experience the same pressure throughout. (Only one example was noticed of a student who tended to reply to questions with, "I agree with the previous speaker."). A practical problem was that since the information was in fairly diffuse form, a considerable amount of time was required to extract specific points from the bulk of data.

The interviews were recorded on tape and raw transcripts were made later. It was thought that people found it easier to talk than to write, a further reason for using an interview rather than an open-ended questionnaire. As stated earlier, the interviewees were volunteers to a request, broadcast via the tutors to individual tutorial groups. Since the same individuals were asked to turn up for an interview on four occasions throughout the year, it was felt that the response to a random sampling might be insufficient to provide a useful result. Another factor, due to the remoteness of the interviewer from the University site, was the organisational problem thrown on the staff of the Department concerned. It was felt that the difficulties encountered in rounding up any unwilling students would have been too onerous. In the light of events, this seemed justified. Even with volunteers, it was often necessary to pay several visits before a reasonable number of interviews were on the tape. There was therefore, a not inconsiderable possibility that the views expressed would be biased; a motive for volunteering being a strongly held opinion (for or against). However, consideration of several points suggested that the sample was a fairly unbiased one. These points were as follows:

(i) On the evidence of their performance in the final examination, the interviewees showed a wide range of competence in the subject, which reflected the performance of the whole student group.

(ii) The students were drawn from a wide variety of principal subject areas and were not confined to a narrow field of disciplines. (See Appendix 2)

(iii) A wide range of opinions was expressed about their experiences of mathematics at school.

(iv) The evidence from questionnaires (returned from a wider sample, see Appendix 4) provided data of comparable nature to that from the interviews.

(v) Statements about the general learning situation that could be checked either by observation of lectures and tutorials or by reference to the statements of members of staff were found to be accurate.

(vi) There was a consistency of attitude and opinion expressed about the major issues over the different years covered by the investigation.

The interviews were designed to be semi-structured in the sense that the interviewer had a list of prepared questions which were put to each student in the group in turn. These were carefully phrased so as not to lead to or encourage single-word replies. As a simple example of this, it seemed reasonable to ask, "Why did you choose to study this particular option?" but rather than ask "Do you find this subject challenging?" it was felt that the question was better as, "Some people find mathematics a particular kind of challenge, what do you think?" It was found that students were stimulated into giving quite extensive answers and often, as they sought to justify their replies, some useful information came to light. On the other hand, where necessary, clarification would be asked for and interesting points would be pursued by asking supplementary questions. In Appendix 3 a transcript is given of a complete interview with a group of students. The amount and nature of the data obtained is clearly seen in this example.

It was mentioned earlier that certain corroborative evidence was to be obtained by the use of questionnaires. These were partly concerned with the collection of information about attitudes to the course but they were also used to enquire about topics which had been found particularly difficult or rewarding. Additionally this technique was used to gain extra information about student response to the booklet on Boolean Algebra used in the 1979/80 session (see 10.5) and also to collect the views of recently graduated students on their memories of the subsidiary mathematics course (see p. 122). Generally, responses were required on a rating scale but open-ended replies were asked for in two questions and also the respondent wrote down the topics he thought particularly difficult or satisfying. The results collected during the different years of the study have been combined to provide one comprehensive picture of the course. This was in contrast to the second study in the thesis, where the results were presented year by year. The reason for this was that in the latter case, a major interest was the formative evaluation of the teaching method; changes were made throughout the course of the study which made one year different from another. In the former case, each year the philosophy and overall structure were the same, the changes being relatively minor ones of content; moreover there were no major variations in student responses from year to year. (A later development involving fewer lectures and more tutorials is described in Chapter 12.)

5.7 Quality of the Evidence

We conclude with a discussion on the nature of the evidence that is compiled by the techniques that have been described. The basic question is "Does it become so subjective as to yield little of real value?" A similar problem was considered in detail by H.S. Becker (1958) who explored the subject of inference and proof in participant observation.

Several of the points made in the paper were worth noting. The major issue was the credibility of the informants. If statements turned out to be incorrect, what did this indicate about the individual? There was the question of whether information was freely volunteered or directed by the observer. Would the reply have been the same in the absence of the observer? It was concluded by Becker that more weight could be given to the volunteered information as being more likely to represent the students' true opinions. A second factor was denoted as the observer-informant-group equation. This pointed to the problem of assessing the influence of the relationships involved between the student, his colleagues in the group and the observer.

With respect to the particular study under discussion here, some points have already been made about the evidence with reference to the student sample. One further relevant factor was that the interviewer was known <u>not</u> to be a member of the University concerned and from their responses it was possible to believe that the respondents were not inhibited by seeking to give conventional answers. Evidence of this was their freedom of expression, in the way they made little attempt to confine themselves to complimentary statements, also their frankness in putting themselves into potentially embarrassing situations. An example of this arose when some stated that they had done virtually no reading in connection with this course. In terms of the interview itself, in order to encourage the volunteering of information, the questions were phrased in as un-directed a way as possible.

In a discussion on the quality of evidence, Becker pointed out that the observer, in assessing the evidence, could not argue that a conclusion was totally true or false, but taking a quasi-statistical stance would decide how likely it was that a conclusion about the distribution of an opinion was accurate. Factors that were mentioned as pertinent were whether the information was the result of direct questions or volunteered and above all, the frequency of occurrence. Were the observed sample unanimous, a majority in favour or what proportion? Also, it was important to seek as many kinds of evidence as possible to support a given conclusion. For this reason, the information obtained from student interviews was supported by that obtained from discussion with individual members of staff,questionnaires administered to samples of students and a limited amount of observation of lectures and tutorials. Throughout the following chapters it will be seen that the evidence has been weighed with these factors in mind. The conclusion of the analysis, the presentation of a profile of the course and its participants, is given in chapter 11.

5.8 The Preparation of Study Booklets

A further feature of the research was the preparation of booklets covering the topics of Flow-Charts and Boolean Algebra and the subsequent evaluation of their effectiveness. A major aim was to investigate how individual study might play a complementary role to the lecture course. This came in response to an early indication that in certain fields, some students found the lectures difficult to follow. Two supplementary aims were formulated. One was to produce booklets that provided a comprehensive set of notes which would relieve the student of the task of writing detailed lecture notes. The second was the clarification, by discussions with the lecturers concerned, of the purposes of teaching that particular topic. This was necessary before the booklets could be written. The discussion of this part of the research, with details of the objectives and the conclusions are found in chapter 10.

CHAPTER 6

ANALYSIS OF STUDENT RESPONSES

Part 1: The Student Background

6.1 Introduction

As explained in the previous chapter, the majority of the information was collected by interviews. A small amount was also obtained by questionnaire. The interviews were carried out over a period of three years, comprising the academic sessions of 1977/78, 78/79 and 79/80. During the first of these, two different groups were interviewed once only and the approach was very much to gain experience in interview procedure and to discover what were the students' general reactions. In subsequent years, the same group of students was seen on four occasions throughout the year and more specific information was collected on the issues that had emerged in previous interviews. It was also possible to look for changes in attitude that might have occurred during the year.

The questionnaires were administered to tutorial groups towards the end of February in both 1979 and 1980 (approximately half-way through the course). In this way, information was made available from twenty-eight students, all different from the interviewees. The questionnaire is shown in Appendix 4, where a summary of the results is also given.

Some background information about the interviewees is given in Appendix 2. Where quotations appear in the text, they are accompanied by a roman numeral which allows the source to be referenced to the list in the Appendix. In presenting the results of these investigations, several points have been identified and are discussed with reference to the evidence obtained. The forms of the interviews and the questions were discussed in section 5.6. It was found that the responses were fairly open-ended and that interesting comments did emerge as supplementary replies or were stimulated by an answer given by a colleague. (A transcript of some typical interviews is given in Appendix 3.)

The results are presented in the following three chapters. In this chapter, the students' backgrounds are set out, including their views about mathematics and their reasons for choosing this particular option. In the following chapter, the student view of the aims of the course is discussed and the third chapter presents their responses to experience of the course.

6.2 Mathematical Experience at School

The first point considered was the students' mathematical experiences at school. It was expected that the majority, being arts students, would have found the subject unattractive. However, the evidence showed a wide spectrum of opinion. Of the twenty students who were asked the question, six said they found it an enjoyable subject, seven had no particular feelings either way and three owned to disliking it completely. The remaining four either expressed mild dislike or said they enjoyed some parts but not others. Questioned about their dislikes they made some interesting comments. One student said that he did not enjoy the abstract things and when asked to give an example, gave the solution of equations. Another quoted calculus and trigonometry as being abstract. Several commented on liking the arithmetic. Their remarks indicated a relationship between coping with a topic and expressing a liking; at times the two appeared synonymous. Typical comments were:

> "I could do the trigonometry and so on, could not do Venn diagrams. I could not see the practicality of it. It did not seem useful, I could not see the point of it all." (xvii)
>
> "I enjoyed some bits of it, liked the bits you could apply, like statistics." (viii)
>
> "I enjoyed maths at school when I could do it ... disliked probability and things to do with bearings and velocities and that sort of thing. I did not enjoy graphs, I liked matrices and solving equations." (iv)
>
> (November interviews)

Three people made specific reference to the strong influence of the teacher on their attitude to the subject, both for increasing and diminishing their liking. One student remarked on an extreme dislike of the subject until given a teacher who, uncompromisingly, coached them to pass the examination.

These views may be compared with others reported elsewhere. Thouless (1969) in a general discussion of mathematics teaching noted a special problem in the fact that some pupils at school acquire a distaste for the subject and become convinced they cannot do it. He said that a large section of the community, who were clearly intelligent, displayed an almost total lack of understanding of mathematical reasoning.

In an article about mathematics for student teachers, some evidence was given about the attitudes of eighteen-year-old entrants (Kerslake, 1974). Many were said to be apprehensive about further study, having given up the subject rather thankfully after O-level. This was far more common than a positive attitude. Remarks made by students were quoted. These revealed that some had found the subject "boring", there was "no fun in it" and it was "too abstract". Other comments showed that it was not seen as "relevant to everyday life" and that it should have been "more practical". There was an interesting remark that it was "satisfying when you do get it right" and one person thought that one missed something in not studying mathematics. Kerslake commented that these attitudes were not noticeably different from those she had recorded ten years earlier.

6.3 Attitude to a Science Subsidiary

Overall, fourteen students made direct comments about their opinions of studying a science-based subsidiary. Of these, six students were definite that it was a suitable thing to do, five were not in favour of it and three replies were evenly balanced between the two. One commented that studying a science was good for some but not for others. Another reply contained the suggestion that although it would be a pity to drop science, it was felt more could be got out of something they were more interested in. One suggestion, for example, was that a language subsidiary would go better with some principal subjects.

"No, I do not think so (give up the science subsidiary). It is meant to be a broad education here and I think most people should do something like that really because they would never do it on their own." (iv)
(May)

"I do not see the idea as necessarily bad, it would be good if you could gain something from it ... I would not say that science is a totally hopeless idea. I like science, if I could do it, it is useful." (xix)
(May)

"It depends on what level you are talking about. I do not really want to do a science subject and feel it a bit of an imposition, but mathematics does seem a worthwhile thing for a human being to study." (i)
(May)

"I think it is all wrong. You come to university to specialise." (xiv)
(February)

It might be thought, a priori, that students who had chosen to study at a University, with a published philosophy of broad education, would be in sympathy with the concept of including a science subsidiary in an arts-based course. It was worthy of note therefore, that on the evidence of these comments, there were some instances where this was not the case. It became evident in discussions that this situation was known to the staff who were involved with the course. The feeling was that with increasing student numbers, there were those who were not entirely in sympathy with the broad philosophy. This was supported by a student's remarks about people arriving via the UCCA clearing system; though a small group, this might well contain some who would have preferred an alternative course structure.

Other comments suggested that objections were based more on finding the subject too intractable.

> "yes, in a way (it is) trying to do too much. A lot of people can appreciate both, I would not deny that, but others are very arts-based and I am one of them." (x) (May)
>
> "... too much work. In theory it is quite attractive but in practice it works out very hard." (xx) (February)

It is as well to remember the pressures that bear on the undergraduate. A somewhat negative attitude may be expected at some stage when trying to cope with a heavy work load. It has been noted in other circumstances (see 13.3) that this response is quite typical in students who are faced with courses which they see of only peripheral relevance to their main studies.

6.4 The Students' General Attitude to Mathematics

A significant point about the general attitude to mathematics was that students were overwhelmingly of the opinion that mathematics was an important subject. Where they differed was in the degree of importance and which aspects were considered important; no-one felt mathematics to be unimportant. Their remarks showed that most saw the significance as one of personal usefulness, a previous quotation did acknowledge it as a "worthwhile subject for a human being to study".

"I think the practical aspects of maths are vital ... A lot of it has been theoretical but I think the practical side is very important." (xiv)
(November)

"I suppose it is a subject where you do something. The idea of lectures in history or philosophy is a good one to get away from the actual numbers but I always see maths as to do with numbers." (vi)
(February)

"I can appreciate this is the sort of thing that mathematicians do ... I think it should have a far more practical application." (iii)
(May)

Other comments mentioned the practical necessity of some knowledge of the subject and that the ideas of mathematics seemed important in this day and age. One remarked that you used mathematics all the time in different sorts of ways. Evidence from the questionnaire supported the feelings expressed by these comments. In answer to the question "Do you think an understanding of mathematics to be important?", five thought "very important", thirteen thought "important", seven of "some importance", one of "little importance" and none of "no importance".

A second point of interest was concerned with the idea that mathematics was a subject only for those with a special aptitude. Opinions were fairly evenly divided. Two comments were to the effect that a special aptitude was necessary. One said that it was a subject you could either do or not do, the other that you were born to think in a mathematical way. This second student went on to add that he did not possess this natural ability. Most students did not adopt quite such an extreme stance:

> "I do not think people have a special aptitude, rather an interest. People who are interested will do much better." (xvi)
>
> "Certain mathematics does require a certain state of mind. Provided you get a basic understanding, it is possible to achieve something through hard work." (xiv) (March interviews)

The majority adopted the position of accepting some level of mathematical achievement as generally possible. One remarked that the idea of a special aptitude was true to some degree; "only <u>some</u> people could do mathematics well but it depended what was meant by mathematics". Generally, students felt they could cope with some level of mathematical understanding. At this point it is interesting to look at evidence about which aspects of mathematics were deemed to be important.

6.5 Views on Significant Aspects of Mathematics

It was clear that the majority of those interviewed saw the subject in terms of its applications and quite often, reference was made to "practical mathematics". However, when asked to describe "practical mathematics", few clear ideas were put forward. Most examples concerned arithmetic and involved things like working out income tax, mortgage repayments and percentages. Some other aspects of mathematics were described as "abstract" mathematics and here again, the ideas about what constituted abstract mathematics were unusual. It has already been mentioned that, at school, one student found trigonometry too abstract and vectors and differential calculus were also put in this category. The following remark displays this feeling:

"I think studying arithmetic is important to everybody. Unless someone is interested, not any further ... Mathematics is not worth studying for its own sake." (xiv) (February)

The idea that the subject was seen as of practical importance may be illustrated by the following:

"I would prefer just to learn things that I am going to use." (xiii) (March)

"The uses are important, when you apply it to physics and things like that you really need it." (xviii) (February)

"I think mathematics is important but not the stuff we are doing at the moment ... There is no reason to study maths otherwise than because it is useful." (xvi) (February)

When students made comments on the existing syllabus, they were likely to make some judgment concerning its perceived usefulness, as in the above quotation. Similarly:

"I think it is very difficult to apply it to what we do every day." (xx) (February)

On the other hand the subject was sometimes viewed in less utilitarian terms, as illustrated by three replies:

"I think it is a challenge to try and grasp these concepts." (viii) (March)

"I do not think you have to relate the course all the time to practical applications, not necessarily." (xii) (March)

The third reply suggested that mathematics was important in the present day and although the course itself was not very useful for practical purposes it made you think mathematically. This last point was mentioned rarely; the idea that the modes of thought in mathematics were worthy of study was not a point which occurred to many students.

> "People who think logically will do better at mathematics ... you have got to ask how much people like us need to know about mathematical structures." (xv) (February)

It was noteworthy that few remarks were made suggesting that mathematics was worthy of study because of its significance in the history of our culture. This was evident also when students were given the opportunity in the questionnaire of February 1979 to write down what they considered to be a major objective of studying a mathematics option. Most replies were a list of usefulness for daily life, business, tax returns ... or simply passing the examination! It was refreshing to record the following three objectives:

> To reintroduce mathematics as a worthwhile subject for anyone to study, as opposed to the approach taken at school.
>
> To put across something of the historical development of the subject (and) to try and show that it can be enjoyed and appreciated by people who lack skill in the "mechanics".
>
> To give some idea of the application of mathematics in the realm of ideas (artistic, philosophical etc.).

An interesting comment was made by one student that was worth noting (xii). He felt that it was a good thing that mathematicians should be expected to teach everybody something. Perhaps it should not be overlooked that if mathematics is to be seen as an important part of our culture, then mathematicians have some responsibility to present it as such. A similar point was made earlier when it was pointed out that future policy makers will be amongst the non-specialists (see p.22). Comment on the possibility of discerning changes in attitude with increased maturity is made on page 122.

One feature of mathematical study often referred to was the experience of some degree of frustration. However, the contrary view was also expressed, that it could be very satisfying. Often, the opinion was held that both had been experienced at different times. Of those interviewed, eight students had been satisfied and frustrated at different times, two claimed no feeling of satisfaction and one was neutral, his feelings were "not particularly either way". Typical remarks were:

> "It (mathematics) is very satisfying if you can do it. I enjoy the challenge." (iv)
>
> "Yes (it is satisfying) but very frustrating if you cannot see the result." (xi)
>
> "If I find I have got some mathematics that I can get to grips with and do, then it is okay. It is when I am faced with something I have not much idea about, then it ceases to be a challenge." (xiv)
>
> (November interviews)

Further evidence concerning the experience of frustration was provided from the questionnaire. In answer to the question "Was the subject ever frustrating?" five replied "very often", nine "often", nine "sometimes" and only three replied "rarely" (26 replies). On the other hand, in reply to the question "Was mathematics ever satisfying?" twenty-one thought "sometimes", five "often" and none "rarely" or "never". Thus students, in general, experienced some measure of frustration when studying this subject, more so than in their arts-based subjects, but acknowledged that some measure of satisfaction was also possible.

Very often, the feeling of frustration was linked with the idea that mathematics is so precise, that they were looking for a unique solution to a given problem. There was some evidence that experience of the course modified this rather limited view.

In March, fourteen interviewees replied to a question about this feature and about half of the comments suggested that a less definite view was taken than previously. This was held by some to make the subject more interesting but others felt it was a source of confusion. One response made a point that different forms of proof, where you seemed to be able to twist things round as you wanted, made the subject appear less precise. The following were typical responses:

> "One specific instance suggests it has changed a little bit. Last term we dealt with the concept of infinity which was something I always considered as "black and white", but now I understand it is not ... It has made the subject more interesting, certainly not frustrating." (xii)

"This is why maths appeals to me, the fact that you can have different points of view. The maths we did at school was so boring because it was black and white." (ix)

"I used to see it as black and white but not any more. That is why I am disillusioned. I just wanted the black and white and we have done more of the philosophy." (xiii)

"Almost totally (right or wrong). All the maths I have come across, there has only been one answer." (xiv) (March interviews)

There were two comments about the contrast between doing mathematics and writing essays, where there was no "right" answer.

Finally, two groups were asked if they saw mathematics as a cold impersonal subject. Two students said that this was not their general feeling, four agreed that it appeared cold but also added that they did not find this annoying or off-putting. Only one student felt that this aspect put him off to some extent although he did not find it a major problem.

"Certainly, yes, it is impersonal, but it does not worry me that much." (xiv) (February)

"Not when I can do it, I enjoy it when I can do it." (xx) (February)

6.6 The reasons for Choosing the Mathematics Option

The students were asked to give their reason for choosing this subject as their science option. Of the twenty-six students who were asked their reasons, four gave an explicit positive reason for their choice. For three students, the subject was declared to be enjoyable, based on their school experiences and avowed current interest. Of these, one thought it most suited his ability, one enjoyed it at school and the third said that she liked mathematics, found it interesting and of the choice of subsidiary subjects, saw it as the obvious one. The fourth student declared an interest in science generally, and chose mathematics as likely to be the most helpful.

> "Although I was interested in other choices, I think mathematics is probably more helpful for me. It is basic but it is quite interesting as well. A positive choice, yes." (xix) (February)

There were five students who felt that the subject might be useful, bear some relevance to their other studies or that some aspects might prove to be interesting. One, for example, mentioned it as relevant to her principal subjects of Economics and Social Science. One said that he understood the use of mathematics to be important in Psychology, (one of his principal subjects) but although he realised this referred more to statistics, he felt that was not a possibility for him so he chose this particular course (xvi). Another student mentioned mathematics being relevant to economics but also mentioned that a further reason for his choice was the avoidance of practical work on sporting afternoons! One girl who wanted to teach felt that the course was an opportunity to refresh her memory of O-level mathematics and this might help her career. Many comments showed an aversion for the practical sciences, the laboratory work involved appeared as a serious obstacle to some students.

> "The only science I was likely to cope with. It could have some relevance to everyday life." (iii) (May)
>
> "Mathematics sounded easier. I thought about Physics and Astronomy but mathematics had no practicals." (vi) (November)
>
> "I had not got another science. I failed Physics at O-level, only capable of doing mathematics." (xvii) (November)
>
> "Because I am not very good at science. No liking for mathematics, I did not do very well at school." (xxi) (March)

In all, sixteen students included reasons suggesting it was partly a question of eliminating unsuitable choices. Three people mentioned avoiding practical work when sporting activities might clash, six spoke about avoiding practical work for various reasons and five referred to an inability to cope with a science subject.

That these attitudes were not untypical was suggested by corroborative evidence from a brief discussion with a tutorial group in November 1978. In the course of this, the nine students were invited to write down their reasons for choosing this option. The replies showed two people with positive reasons for their choice, two felt it had some relevance to other subjects (economics and computer science respectively), two mentioned that no practical work was involved and the remaining three thought it was likely to prove the easiest of the science options.

Apparently, a certain number of the students started with no particular enthusiasm for this course. That this was the case was supported by some comments of the staff involved (see 9.3). The effects of the level of student interest will be referred to when discussing the effectiveness of the course (chapter 8, sections 4 and 5).

6.7 What Did Students Expect to Gain from the Course?

At the beginning of their year, students were asked to state what they expected to gain from their experience on the course. It is of interest to look at their replies before going on to consider their perceptions of the aims and objectives and a comparison of what they felt actually happened with what they felt should happen.

It was interesting to note that a fair number of comments revealed a view that it was an opportunity to broaden the intellectual experience, an aim which one student remarked was in line with the philosophy of the University as a whole. One or two comments showed that students saw it as an opportunity to take a fresh look at the subject, particularly if school experience had been less than enjoyable.

> "I did reasonably well at school, had no urge to prove my ability. I thought that this course might act as a refresher, having done no maths for two years, I would leave the University reasonably competent."
> (ii) (May)*

* This student was interviewed only once, thus it was the only opportunity to ask this question.

"I will think differently, be able to see things more from a mathematician's point of view." (iv)

"Maybe it should help us to think more logically. (Do you need to understand the language of mathematics?) We do not need that." (v)

"You should have an all-round view of things, it is the whole idea of the University." (viii)

"Not sure, more knowledge I suppose, but it is difficult to know where it will come in useful." (xv)

"The knowledge of mathematics I did not have before, the ability to do arithmetic." (xvi)

"I might have a more favourable attitude towards mathematics, I hope to get rid of a bad taste left from school." (xi)

"The personal satisfaction of having done something that I have enjoyed." (xviii)

(November interviews)

In summary, of the eighteen replies to this question, four were concerned with the opportunity to broaden one's outlook or achieve some satisfaction, two mentioned an improvement in thinking logically, six were about an increase in knowledge or competence, two with producing a different attitude to mathematics and four were not sure what to expect. This may be compared with information obtained from the questionnaire. Twenty-six students replied to the invitation to write down what they thought should be a major objective of the course. Eleven said that they hoped for an increase in mathematical knowledge, six wanted something that was useful in real life, one wished for a revision of school mathematics and one to have sharper reasoning powers. Of the remainder, five wanted it to be an interesting or enjoyable contrast to their other studies, one stating that it was to stimulate their interest whilst another wished to learn of the place of mathematics in the realm of ideas e.g. arts, philosophy etc. One saw the objective as the passing of the examination.

Two points need to be taken into account in comparing these comments. Firstly, the questions posed in the interview and questionnaire were not quite the same and secondly, the interviews took place in November, whereas the questionnaire was administered at the end of February, when the students were half-way through their studies. Even so, it is interesting to record the similarity in the results and the fact that the majority view in each case was the same, i.e. an increased knowledge of the subject.

The evidence presented in this chapter showed that the students had mixed views about their school experiences of mathematics. There was also some division of opinion about the suitability of including the study of a science in the arts undergraduate course. It was generally accepted that mathematics was a subject of some importance and that they, as non-specialists, could usefully undertake some measure of mathematical study, although at a fairly modest standard. There was a lack of interest in what were deemed to be the "abstract" parts of mathematics, with more concern for the practical or useful aspects. Most students felt that there were often occasions when the subject could generate a fair amount of frustration, although a feeling of satisfaction was also produced. A number of students chose this option because they wished to avoid less-palatable science options, many remarked that an element in their choice was its usefulness. Students' expectations from the course displayed some variety of views. It was interesting to record that despite an absence of a high degree of commitment in students' initial choice of the course, their expectations, once enrolled, often involved "broadening the field of study" and "seeing from the mathematician's point of view". (See 7.4 for student reactions to possible course aims.)

CHAPTER 7

ANALYSIS OF STUDENT RESPONSES

Part 2: Views of Aims and Objectives

7.1 Introduction

This chapter is concerned with setting out the students' views of the aims and objectives of a subsidiary mathematics course. The discussion is presented in three sections. In the first section the evidence is considered on the students' perceptions of the existing aims of the course. In the second section, information is examined which displayed their ideas about possible aims, while in the final section, reactions to suggestions about possible aims are presented.

7.2 Views Concerning the Aims of the Course

Students could gain information about the course in a variety of ways. In addition to the Prospectus, there was a leaflet which gave a description of the general aims and specified the kind of student for whom the course was intended. A member of staff in the Department was responsible for enrolling students on the various options that were available and could advise individuals on their choice. Once enrolled, the first lecture was devoted to a more detailed explanation of the course and what it would achieve. There was a wide range of opinions about how effective this process had proved. During an interview in March, one girl said that she found the aims reasonably clear and that her experience of the course had not been a surprise. She thought enough information was available for a reasonable judgment to be made beforehand. She did not know why some students found the aims rather vague. The following quotations suggested different reactions to the events at the beginning of the course:

> "The first lecture we had told us that the course would give a different insight into how maths works, looking at it from the historical and other viewpoints." (xix) (March)
>
> "We are never actually told at the beginning that this course is aimed at such and such. At the beginning of the year we had a talk about subsidiary subjects. There were leaflets; they mainly said what level they expected students to reach before they joined the course ... It did not tell you much about what the course aimed to do." (xiv) (March)

There was mention that the opinions of former students figured in some people's views of subsidiary options. These rather subjective sources had not always conveyed the right information:

> "It is not easy, more difficult than I thought it would be. Some people said it would be easy, others that it would be hard. I tended to believe the former." (iv) (November)
>
> "The idea about what the course involves can be arrived at by talking to other people." (xiv) (March)

During the first interviews in November, one group of four students was evenly divided about understanding the lecturer's objectives. The following two replies indicated few problems:

"Yes, I suppose so; it is pretty basic, you can understand the mathematics. There is a logical progression." (xvii)

"Yes, I know what is going on." (xviii)

(November interviews)

Other groups gave replies suggesting that in the early stages some students had found their initial ideas were not matched by their experiences:

"I thought there would be more history. I get confused by all the numbers. There is more mathematics than I thought." (vi)

"I thought it would be more historically biased. They seem to use the historical ideas as an excuse to teach what they are teaching. I would like more history ... More to do with basic concepts than at school. In a way it is better, but very frustrating if you cannot grasp it. Some things might be more relevant, like economic models." (iv) (November interviews)

Comments made during interviews later in the year suggested that students' perceptions of the aims remained similar to those revealed earlier. In May, one student, when asked to say if she could say what the course was all about, replied:

"Not any clearer really. Sections of the course still seem a bit unrelated. They do not seem to be moving towards any particular end." (iv) (May)

However, there were responses that implied some worthwhile aims were being achieved but the students did not give the impression that they were conscious of how valuable these were.

"Obviously I think it is to teach you more about basic concepts behind mathematics and there is quite a few historical things put into it, but no, I really cannot see that it is heading in a certain direction." (vii)
"The most it has done is to shatter the odd illusion from school, like geometry, there is only one kind of geometry and things like that." (xi)
(May interviews)

One student, quoted earlier, who had said in November that the aims were clear had discovered by February that there was rather more to it than she thought. Asked if she understood what it was about, she replied:

"Not really. It said (in the Prospectus) for people with a more cultural interest in mathematics and it does not really seem to have done that. But then, they tell you it is for people who are not scientifically minded, so you think maybe there is not much mathematics involved, maybe it is more theory. You get in there and find a great deal of mathematics and it is really quite difficult." (xvii) (February)

A group of interviewees in March were asked to say what aims they could see emerging as the course unfolded. One student referred to the overall aim of presenting a broad view of mathematics. Another began by remarking that sometimes little effort was made to relate to practical applications. When prompted to enlarge on this, his opinion was that this was not a necessary objective on every occasion. He cited the example of binary arithmetic, which they had all coped with and it was interesting <u>in itself</u>. (It was noticed that this comment juxtaposed the aspects of being able to cope and a topic that is thought to have intrinsic interest.) The remaining five replies generally expressed views that the aims had not become clearer with the passage of time. In May, one student suggested that by this stage of his studies, he had no firm ideas of the aims:

> "It is a lucky bag ... I expect some theme is behind it all, but I am not aware of what it is. Lectures come as a shock each time." (ii) (May)

Some comments revealed that in certain cases, students appreciated a difference between the school and university approach to the subject. Two comments particularly referred to a contrast. In one, the student said that at O-level it was geared to passing examinations, here you were expected to understand more. His colleague was moved to say that he preferred this method to being spoon-fed at school; it was good to develop one's own ideas. The following response illustrates the value of a different approach and also, possibly the opportunity to make use of the greater maturity of the undergraduate:

> "I think it is a challenge to try and grasp abstract concepts. For instance at school I thought Venn diagrams were just a teaching technique; it never occurred to me there might be a whole philosophy behind them." (viii) (March)

A sizeable number of those interviewed appeared to have initial ideas about the aims of the course which were not realised in practice. There were two possible consequences of this mismatch. The first was that a few students might find themselves pursuing an option for which they were not particularly suited. Also, experience of the course itself may be such a surprise that the interest of the individual concerned may be diminished. If it was a fairly common practice for students choosing options to rely on advice from past students, there was a difficult problem for staff in how best to communicate reliable information so that it was more effective.

7.3 Views on What Ought to be the Aims of the Course

This section looks at what the students felt ought to be the aims of this course. Before looking at their comments, it may be noted that students did not make much distinction between "aims" and "objectives" in the technical sense. It is generally accepted that the former are the broader purposes, the latter concern more specific details. No attempt was made to impose this distinction during the discussions and at different times, comments and replies would involve both the broader and more specific aspects.

Students were not always precise in expressing opinions about what they felt were reasonable aims. Naturally, there was a wide variety of proposals and again, there was evidence that, in many cases, the subject was viewed from a utilitarian angle. The first of the comments below is of interest because of the immediate response it prompted from another student:

> "Perhaps the introduction of some other departments, say Philosophy." (ix)
>
> "No, not something that is not concrete. (Can you define concrete?) I am not quite sure." (vii)
>
> "The history is interesting if not overdone. It gives a brief interlude between the practical aspects." (x)

"To some extent (include history) but not as a main feature. I like the idea of some practical features like working out mortgages." (xi)
(November interviews)

This last remark illustrates the interest that many students expressed in what might be termed commercial arithmetic.

"I think broadened in the sense of a little more down to earth. Get people to do, say, construction. I think it needs to remain a mathematics course. I would like it linked with what we can do, say measuring. Perhaps how mathematics is used in physics might be more helpful." (vii) (February)

"I would like to see lectures on logic and things. I would like to see the approach more philosophical and historical. I would find it personally more rewarding, but maybe this would defeat the object of doing a science." (iv) (February)

This last reply, made as it was in February, illustrated that the student was unaware of a later section of the syllabus involving propositional calculus and Boolean Algebra. Later in the same interview, this student remarked:

"Let people who are not scientifically minded see that there is more to mathematics than what they did at school." (iv)

Few people would wish to argue with that. Some lack of precise ideas was illustrated by the following group of replies. The first two were made by the same student at different times.

"I like the philosophical side, it is more what I want to learn about ... (ix) (March)

"If it has to be mathematical (as opposed to historical) it might as well bear some relation to business ... but I enjoyed Boolean Algebra because that was fun." (ix) (May)

"I would prefer it to be applied to everyday life. It has more connection, at the moment it is very set apart, it is not relevant at all." (xi) (May)

"I think I would like that ... I don't know though, I would like some of the history as well." (vi) (May)

"Yes, I think it should be based more with real life than anything. Things like calculations and mortgages, how it is all done." (x) (May)

Not all students were content with the idea that the aim should be to study "useful" topics.

"I think it could be pointed out that it could be useful, but personally, I am not too bothered about things like how to work out mortgages. I suppose it could be an objective but it would not interest me." (iv)

"I think it might be more useful but I suspect I would find it deadly boring." (viii) (May interviews)

One group was asked if there should be more arithmetic? In his reply, one student rejected the useful criterion as unimportant.

"No, certainly not, I would not question what we have done so far. I think it perfectly right. The only problem is giving people a better idea of what they are up to before you actually start doing it. If they were guided more, I think they might take more interest." (xix) (February)

Commenting in a later interview about what he thought should be a major objective, this same student said:

"You cannot afford to go to too high a level anyway. Showing it in a different light is important, they might find this more interesting (than school mathematics)." (March)

One student made some interesting comments that revealed an unusual interpretation of what was "useful" mathematics. This student had a very limited mathematical background and felt that any mathematics that he could understand was useful. In his first interview he remarked:

"As far as I am concerned, I am finding it useful at the moment. (Some people would say it is more theoretical than useful.) It is useful because I can actually understand things, not in the practical sense. I am not really bothered about the practical." (xvi) (November)

It must be recorded that this view was modified somewhat as the course progressed. (See p. 106 and p. 121) This student was one who found the second term extremely heavy going. In a later interview he commented on the importance of a knowledge of mathematics:

"To me personally, no. It is important to have a basic knowledge. I do not really see the benefit of geometry, that is probably why I am not interested." (February)

Speaking of objectives, he made the following remark:

"Yes, I think more practical arithmetic (would be an advantage). I think it would make it more useful, whether I would find it more interesting I do not really know. The only use I can see for it myself is when I have to apply it to something." (February)

Asked to suggest what he would teach on the course, he gave this response:

"I found most of the stuff we did last (autumn) term quite interesting. Really, most of what we are doing is okay, but it is when they get on to things which I think are a bit useless, like geometry. Things like number bases are really worthwhile because they are a good basis. I never used to understand them at all but now they are useful simply because I understand a bit more about mathematics." (xvi) (February)

7.4 Student Responses to a List of Suggested Aims

This section reports on the students' responses when a list of possible aims was put to them. The list of aims was assembled after looking at comments from earlier interviews and was used in the March interviews of the second and third years of the investigation. The exercise was productive in that it gave students the chance to think about issues that may not have occurred to them otherwise. Each of the suggested aims is given as a paragraph heading, followed by a summary of the students' replies.

(i) To be shown how mathematicians do things

Of the thirteen students posed this question, five thought it was not a suitable aim, five that it might be of some value. One said that it was an interesting idea, one that it was quite important and one was not sure about it. Comments showed the general feeling was that it might be technically too involved and not really of interest to them, although one remarked that it had some relevance if interpreted as understanding how their contemporaries worked with their subjects. No-one commented on the passive role of the student implied by the wording of the aim.

(ii) <u>To experience some small measure of mathematical activity</u>

This was generally accepted. One student thought it was essential, another very important, one remarked that experimentation in other fields was quite worthwhile. The other replies showed general agreement that this was a reasonable aim, although one took the opportunity to qualify his acceptance by saying:

> "If I can do it and understand it, yes, but if you cannot, well ... " (xiv) (March)

(iii) <u>To get the right answer to a limited set of examples in a variety of topics</u>

The students divided more or less equally on this. Those that felt it reasonable mentioned a need to experience some success in order to maintain their motivation. It was interesting to note that most of those who disagreed, qualified their replies by adding a comment about how important it was in mathematics to get some kind of answer.

> "You feel tremendous if you get the right answer." (ix)
> "I want to understand but I want to get some credit for it. I like the idea of getting some right answers."
> (xv) (March interviews)

Another student in rejecting this as a worthwhile aim added that experiencing mathematics should include it on some occasions.

(iv) To obtain enough credit to gain a pass grade

The responses showed that students felt the pressure that they must pass examinations whatever else might be achieved. "Sadly, an inescapable factor" was how one described it. To their credit, all of those questioned felt that this aim left something to be desired, but nevertheless, it could not be ignored. Another comment was that it was very important, although by itself not a reasonable motive for studying mathematics. Despite their reservations, no-one felt that this aim could be viewed as unimportant.

(v) To be told a variety of formulae which can be applied to given problems

This proposed aim produced the sharpest difference of opinion. Comments ranged from thinking it a very worthwhile aim to feeling that it would be very boring. Five students were in favour, four would not accept this as a reasonable aim. The remaining four were either neutral or qualified an acceptance by saying that they would have liked to know where the formulae came from. One remarked that it was a bit too much like the thing they did at school. The favourable comments, on the whole, interpreted this aim as implying something of more practical value than what they had experienced. One student commented that people might take more interest if they felt it would benefit them.

"Not the aim of this course, but I wish it was. I think it would be very worthwhile. If I felt it was maths I could go out and use, I would take an interest." (xiv)

"(long pause) I don't know. Being more practical? I suppose they could have a section on it. They do not seem to be aiming for that at all. It seems so mundane. At the beginning of the year I would have said yes, but not now." (xi) (March)

(vi) <u>To develop one's own ideas and avoid the spoon-feeding methods of school</u>

All the students who were asked except two were of the opinion that this was a worthwhile aim. One of the two dissenters put it that she relied solely on being spoon-fed, she had not the interest to sit and work things out for herself. Of the nine who commented in favour, no fewer than seven had reservations about the extent to which it could be achieved in a course of this nature. The reasons they gave for this were in three categories; their lack of grasp of the subject was a handicap, the lack of sufficient time to really get inside a topic and insufficient student interest. Two other opinions were that it was worthwhile as an aim but could not be achieved because of the way this course was taught.

"Yes, I think we are spoon-fed. I do not really grasp the subject well enough to develop my own ideas. I do not think this course aims to spoon-feed but that is what it ends up as." (viii)

"Worthwhile but the course does not achieve this. Because of the way it is taught, they never give you anything to read which might help you." (xxi)

"That would be interesting. Sounds as if you may need quite a reasonable understanding of the concepts." (xii) (March)

The final comments are the replies given by one girl during an interview in May. When asked if she thought it reasonable that the course did not concern itself with "useful" mathematics, she replied:

> "Well, if that is what they are intending to do, that is the aim of it. I do not know if the aim should be different really, I do not think so. They have decided to try and put over this aspect, I suppose it does not necessarily have to be relevant to anything." (xx) (May)

Asked if she could define useful mathematics, her reply was:

> "I do not know because I do not know if any mathematics is useful, apart from arithmetic." (xx) (May)

Later in the same interview, she was asked if she was happy with the aims of the course:

> "I do not disagree with their aims. It is fair enough if that is the kind of course they want to give. I do not really know enough about it to say what else they (the aims) could be."

This last sentence was significant. When the relatively narrow mathematical experience of these students was taken into account, it was not so surprising that they had difficulty in trying to formulate aims and judge what was worthwhile. What they felt was good for them was not necessarily so when viewed from a different angle. It was worth noting that if some students were unhappy with their experiences, there would be a tendency to think that other, more worthwhile aims might exist even if they were not fully aware of what they might be.

The students' perceptions of the aims of the course were influenced not only by information made available by the

University but also by the opinions of people who had previously studied this option. In some cases, the results were to produce a mismatch between the perceptions of the students and the aims adopted by the course organizers. Two possible effects of this were noted.

When asked about the aims that ought to be paramount on this course some students again returned to the "practical" or "useful" theme without being able to give precise details about what this implied, apart from mention of the arithmetic of accountancy. This was not a universal view of a mathematics course; a broader outlook was most evident when students responded to a list of proposed aims. This showed, for example, that when they gave some thought to it, students were aware of the desirability of developing independent thinking. One aspect which was less encouraging was that not many students saw great importance in the study of the ideas and concepts of mathematics as such but tended to think in terms of useful techniques.

In considering these findings it is as well to note that holding imprecise ideas about aims of study is not a phenomenon unique to subsidiary courses. However, with a principal subject, the students have greater experience and expertise and are sustained by a greater interest and commitment. In addition, mathematics is not an easy subject to study for a non-specialist. Is it too surprising if they sometimes respond by wishing that the course were something different? In chapter 9 it will be seen that staff were aware of an element of challenge in teaching students who initially approached the course with some trepidation (p. 130) and in chapter 8 there is evidence of students finding more of interest than they first envisaged.

CHAPTER 8

ANALYSIS OF STUDENT RESPONSES

Part 3: Reactions to Experience of the Course

8.1 Introduction

This chapter completes the analysis of the student responses. The information is discussed which illustrates students' opinions concerning the topics covered in the course, on the feelings about the effectiveness of lectures and tutorials and finally concerning the effects on students' attitudes to mathematics and the possibility of any changes as a result of their experiences.

8.2 Opinions Concerning the Various Topics on the Course

During the course of interviews, students often referred to parts of the course which they found satisfying or particularly difficult. Also, when discussing their most recent experiences, the questions were posed, "Have there been any particularly good (bad) parts?" In this way, the topics which were mentioned in the two categories could be listed. To add to this, the questionnaire administered at the end of February in 1979 and 1980 asked students to write down, at most, two topics which they found difficult and two which they found satisfying (see Appendix 4). It must be remembered that at that stage students had only experienced half the lecture programme, and were not commenting on all possible topics (see Appendix 2 for lecture programme).

The most noticeable fact was that some of the same topics appeared in both lists. Some students would report a particular topic as being intractable, others that the same thing was understood well. (Understanding and satisfying were virtually interchangeable terms. There were no occasions reported when the students answered that a topic was both difficult to understand but also satisfying.) On three occasions students volunteered that some previous experience of the topic had helped and there may have been more cases like this. This was not always the case however, one girl reported little difficulty with differentiation although the subject was new to her. That individual opinions were often opposed was obvious in some interviews. As an example, one student commented as follows on Theories of Motion:

> "The way that was done, with tennis balls, that was amusing, people were looking. He was using objects to show how things were done. If they used objects ... they would get more attention." (xix) (May)

Later in the same interview, the following discussion occurred:

> "The ones (lectures) on Motion I just could not see what they were trying to get at ... (This term) I think I can see what they are trying to get at, especially the Boolean algebra. For the first time I can actually see the point, see what they are trying to arrive at. Electrical circuits, I find that quite interesting." (xvi)
>
> "Last term, the Theories of Motion reminded me too much of physics, physics haunts me!" (xv)
>
> "Quite a few cases (lectures) when I was completely lost. Certainly in Cartesian geometry. That took a

long time to sink in, because they were using different formulae. I followed it basically but not deeply." (xix)

"Funny you should talk about Cartesian geometry because I found that was all right, you could, at least, draw it on a piece of paper. It's Euclidean geometry with planes and points and all that." (xv)

"I find that easier than Cartesian geometry ... " (xix) (May interviews)

The comments on Motion and Boolean algebra can be compared with replies made by students in the previous year:

"Well, the ones on Motion have been good, much more interesting." (vi)

"The big problem I do find with Boolean algebra is when it goes on to switches, that is when I get a bit confused. It's the plans of the switches." (ix)

"I was completely foxed by Boolean algebra!" (x)

(May interviews)

Another significant point was that calculus was mentioned most often in 1978/79 as being a difficult topic. This confirmed a feeling amongst the academic staff that this topic would profit from some revision. A consequence was that in the following year, as mentioned earlier on page 34 , the calculus was given extra time. It was noticeable that in written and verbal comments that year, the topic was referred to much less frequently as being difficult. This left geometry as the topic most often said to give difficulty. Without exception, all students in the interviews remarked that at least one aspect of geometry had generated some difficulty (Euclidean, non-Euclidean, finite and Cartesian geometries were studied in the second term).

With such a wide variety of responses to various topics, it seems that the aim of retaining as much independence of topics as possible was well founded. The provision for students to make fresh starts after having become confused in previous topics was clearly a worthwhile feature of a course of this nature. The one drawback, that this philosophy can lead to a fragmented and disconnected course, is looked at in the next section.

8.3 The Possibility of the Course being too Disjointed

Those concerned with designing the course were clearly aware that there were two opposing interests (chapter 4). There was the desire to treat the topics as independently as possible, whereas the subject as a whole would benefit by exploiting the relationships that exist between the various branches of mathematics. However, attempting to develop these relationships too precisely was felt by at least one member of staff to hold the danger of making the picture too complicated for the average student. The approach that was adopted was a compromise; whilst retaining the independence as far as practicable, obvious connections were exploited where they were useful in developing the subject. An obvious example was the way that part of the syllabus involving sets, groups and Boolean algebra was dealt with. How did the students react? Of the thirteen students who made comments, only one held the opinion that topics were not clearly separable. This prompted the opposite response from another member of the group.

"I do not understand where one ends and another begins. I do not think it has been too fragmentary , I do not think it has been planned well." (xvi)

"I do see the topics as separate, that is interesting. I see each of the lectures as very finite and there is no sense of continuity at all. I suppose with sets and groups, where you see the "intersection" become the "and" of Boolean algebra, that sort of thing ... " (xv) (May interviews)

Reactions to the possible disjointedness varied considerably. Four students found it obvious and disconcerting, two students made different comments at different times, obviously finding it more frustrating on some occasions than on others, seven thought it was a reasonable way of planning the course. It was also worthy of note that some students, as in the quotation above, displayed an awareness of connections between topics.

"I am glad that the topics are independent. Obviously some are connected, like the geometries. I am not worried by the independence, it does not appear to produce serious disjointedness ... I think it is a bit fragmentary but this is a good thing, because if you got lost on one thing you are not lost forever." (xviii)

"Yes, this seems reasonable (keeping topics independent). It does not become too disjointed ... I feel what they have done is try to give us insights into a lot of little bits of all sorts of things. It does come out fragmentary but it is probably a more sensible way of trying to do it than taking a subject and trying to go into too much detail." (xx)

"I would prefer something more continuous. It might make you work harder if the work was more progressive." (viii)

"It seems to be arbitrary which subjects they pick, it is probably not but they never make it clear ... Perhaps there could be one or two loose themes so you do not sink all in not understanding the first bit - not that bad, but something to link in." (xi)

(March interviews)

The conclusion from this was that students were generally in favour of a programme with more or less independent topics but there was a delicate balance required if disconnectedness was not to lead to some frustration. It was apparent that students were capable of appreciating general concepts. If applied with care, the introduction of these concepts had the effect of making the programme more coherent, helping the students to understand and demonstrating the generalising property of mathematics.

8.4 Views on the Lectures and Tutorials

The students' views of the aims of the course have already been discussed, but it is of interest to look at their awareness of the objectives of the lectures themselves. A particular point will be considered, the students' ideas about mathematical proof. Finally evidence is considered of opinions concerning the lectures and the tutorials.

Overall conclusions about student awareness were difficult to formulate because of the number of factors involved and the degree of individual variation. Responses were influenced by the ease or difficulty inherent in the different topics, the changes in lecturers, the limited mathematical backgrounds of the students, differences in their former experiences and the degree of interest shown by different individuals. Taking the last point first, there were comments that suggested that because in some cases student interest was low, they were more willing to criticize. With their limited experience, providing lists of lecture titles was not too helpful because the words did not convey much meaning. There were comments which were a reminder that lecturers always had to bear in mind the difficulties that such students may have in perceiving the objectives of the lecture. The following are typical comments on this subject.

> "Sometimes I sit in a lecture and wonder what is the point of it all. I enjoy it on other occasions because it is an interesting academic exercise. I understand what they do but cannot see why they are trying to do it." (viii) (November)
>
> "We were told last term the schedule (of lectures) but often the titles do not mean anything anyway. Lecturers vary how clear they make things." (xx)
>
> "The actual details are sometimes confused but I know roughly what the lecturer is talking about." (xiv)
>
> "Maybe because they (students) have not got much interest they do not pay as much attention. More willing to criticize as well I think." (xviii) (February interviews)
>
> "I think this is half the problem of the course that there is no brief introduction in words about what

the topic is going to be and why it is going to be that - that sort of background." (iv) (May)
"... The only problem is giving people a better idea of what they are up to before you actually start doing it. If they are guided more I think they might take more interest." (xix) (February)

The last two comments may be considered in conjunction with the discussion of the dissemination of information about the course which appears on page 82, where it was noted that amongst other things, the first lecture was devoted to explaining the course and its aims. A comment on this issue is also made in the conclusions (p. 183).

Turning to the student reaction to the process of mathematical proof, it was not surprising that, without exception, this was viewed as a difficult task. It was easy to underestimate the scale of the problem to the students. As one member of staff remarked, these students lacked the background of convention and expertise which gave the experienced mathematician his starting points. Several of the students who were interviewed reported no experience of proof at school. Many comments reflected the difficulties experienced in understanding what were the assumptions in particular problems. (It has been found that even first year undergraduates reading Mathematics have displayed rather hazy ideas about "proof".) (Bell, 1976)

Before presenting some representative quotations, it is helpful to note that the ideas of conjecture and proof were developed during the first term, particularly in the topic of number theory. Reductio ad absurdum and proof by induction were introduced in connection with number systems and counting and measuring. The concept of logical deduction was central to the discussion in the second term of geometric systems and their development from different sets of axioms.

> "I find proof difficult, not something I can cope with easily. If the question were to present an example which you could see proved, then I could understand it, but I also see that that is not sufficient proof. It is the generalisation step where I get lost."
> (xii) (March)
>
> "I have more difficulty than my friends, but yes, I can cope. I can see the idea of proof by contradiction, it is not easy though." (xvi)
>
> "I do not like proofs. It is not real. I cannot find the way of writing it." (xviii)
>
> "This approach is new, still having difficulty. How to prove it defeats me. You can see what to prove, it's obvious, but my mind is not trained ... we did not know that mathematicians found this difficult (irrational numbers). You felt much better when you found the Greeks had problems too." (xi)
> (November interviews)
>
> "It often does seem that you can twist things round to get a proof but I am sure there are guidelines but I do not see them." (viii)
>
> "What I do not really understand is where they produce these constants.* What they have got is letters and I think if they substituted values in and showed you how it actually worked with real numbers that would

* The student here was referring to the use of algebraic symbolism.

be much more help ... No, I do not really follow different methods of proof." (xxi)

"I would agree it is very difficult to know what I can and cannot do to prove something. I do not understand why you can prove in one way on some occasions and not on others. Nobody attempts to show you why you can use it in that situation. I tend to get lost." (xvi)

"I can see why we start from axioms in geometry, but we seem to spend so much time on that. I would take his word for it and get on with the problem. I could not follow induction." (xv) (March interviews)

It was evident that of those interviewed, a number were not at all clear as to why the mathematician was so concerned with proofs and proving. They were not all fully aware that different methods of proof existed and they were certainly lacking in the background and experience necessary which would have enabled them easily to comprehend the framework within which a proof was constructed. It was interesting to contrast two of the quotations above. One had not understood the necessity to employ symbols in mathematical statements, whilst an earlier comment (xii) illustrated that the distinction between proof and demonstration had been appreciated. Generally, the remarks illustrated the difficulty experienced by the non-specialist, particularly in the case of the geometries.

In looking at student reactions to the lectures, it was expected that all students would report some difficulties at various times in following the arguments. The precise nature of mathematical reasoning would make this inevitable.

"Lectures - yes, I always go to them, they are a good guide-line. They do help, but the homework would be much more difficult without the tutorials." (xviii) (November)

"The information is there if you want to listen to it." (xv) (February)

"I think they (the lecturers) are putting quite a lot of effort into it." (xvii) (February)

"They (lectures) are essential and on the whole they are successful, except for minor things like the over-head projector." (xiii) (February interview)

(This last comment referred to a recent experience with the overhead projector when some students felt the lecturer rolled on the material before they had time to copy it down.)

Other comments demonstrated that student reactions were as mixed as one might find on any course.

"I follow the lectures until about the middle. I feel more in tune with my tutor. Perhaps it is just that it is a lecture and there are a lot of people ... I cannot pinpoint it but there always seems to be some point in a lecture when having gone through the basics there is a sudden jump. It is different for everyone." (xi) (November)

"What sometimes annoys me is I do not realise what the whole lecture is about until the end." (iv) (March)

"Still a problem (getting lost in lectures). While you try and work something out they have gone on. I do not know why the element of challenge is disturbing. Perhaps I have not enough interest to really find out." (vii) (February)

"I think I and my friends have decided which lectures are worth going to. Perhaps an unfair judgment because of different material presented in different ways ... Overall I do not think they have been bad, some have been very interesting." (xii) (March)

There were reminders that for these students, mathematical reasoning was not easy.

"They seem to start off very slowly and then suddenly take off and I get lost." (i)

"They do assume you have understood it and they go a stage further, a lot of people have not understood initially. I often felt that it would have been all very interesting had I understood it." (ii)
(May interviews)

"Everything follows from what comes before so if you get lost, that is it." (iv) (November)

A feature that was mentioned most as affecting people's responses to the lectures was the size of the group. There were six comments recorded which directly stated that the impersonal nature of the large lecture group was disheartening.

"It is the large lecture theatre. You feel you are sitting at the end of the earth." (xi)

"It is so big, so much space. The lecturers are not even looking at you." (ix) (May interviews)

This was given some support by a comment from a student in the 78/79 class for which the group was split into two for one of the weekly lectures.

"The lectures on Wednesday morning seem much better. The two split groups are smaller." (vi) (May)

It is interesting to note that the large size of the lecture group was mentioned by several of the staff as being a handicap. (Chapter 9.) Only one student was of the opinion that a large group presented no problem.

Three major factors that influenced the students' reactions to the lectures were the large lecture group, individual reactions to the different topics in the programme and the responses of the students to the study of, for them, a difficult subject. It must be remembered that the student, in addition to the lecture programme, attended a weekly tutorial which allowed for an informal discussion within a small group. (For student comment, see below.) Although getting lost in lectures is not an uncommon experience, students on this course seemed less inclined to go away and puzzle things out for themselves than one would expect of a specialist group. This was a factor which made the task of lecturing that much more difficult. In addition, the students' limited experience was more likely to result in a feeling that lectures moved too swiftly in some instances.

When discussion turned to the operation of the tutorial system, it was pleasing to find that, apart from minor points, students were unanimous in their satisfaction with the way things were done. There were only isolated remarks concerning problems of detail; there was only one recorded remark during any of the interviews of a student having a problem with a particular tutor. On this evidence the policy of moving tutors around at termly intervals was obviously satisfactory. Students were sufficiently enthusiastic to suggest that there might be more tutorial periods and fewer lectures. They found the smaller groups and the relative ease of access to the tutor far more congenial. They commented that there was the opportunity to raise problems without embarrassment. Though there were one or two replies that suggested that students were reluctant to raise their problems the overwhelming opinion was contrary to this, the tutorials seemed successful from this point of view. Some support for this was forthcoming in the answers that seven students gave to the direct question asking if they felt exposed when studying their mathematics. Five replied in the negative, the main reason being given that they felt everyone was more or less in the same position. One replied that he personally felt inhibited about raising problems, the seventh student thought that it might be a more general problem since she had noticed some people always sat at the back and never asked questions. Here is a sample of the students' comments about the tutorials.

> "The tutorials do clarify a little bit, but the whole subject is beyond me. The tutorials probably function reasonably well." (i)
>
> "People have asked in the past and answers do not seem to help. Some people do not seem to want to let on that they find it difficult." (ii)
>
> "Sometimes I raise my problems, not all the time. I do not like making a fool of myself, if everyone else understands and I do not." (xiv)
>
> "Things are often explained to us in the tutorials, for example the relation between set theory, Boolean algebra and switching." (iii) (May interviews)
>
> "Tutors are very willing to do what we want really; for example, the last tutorial went over irrational numbers and convergence and did not get round to what we should have done ... It is more the attitude really that you are doing it together, it is much better." (iv)

"I would rather have more tutorials and fewer lectures. I do not take much in from lectures ... I quite enjoy the tutorials, the tutor is friendly and takes the view that we do not all understand mathematics." (viii) (November interviews)

"Yes, at least you can go back and look at material and explain. It is easier in groups, people do raise their problems." (vii) (February)

"Yes, I think people feel you can address problems to the tutor and talk about them. Tutors have always been very helpful and approachable." (xii) (March)

"It is because it is a much smaller group. You can say exactly what your problem is, you cannot do that in a lecture ... I enjoy the tutorials. He goes much slower and they are very helpful in going through the handouts." (ix) (May)

"The tutorials, you seem to get down to the actual maths and really get to work it out; whereas in the lectures I do not think you are learning that much about how maths works. I find that after a lecture I have got a lot of material but I do not know really how it fits in. The lecture leaves everything a little bit blurred. The tutorials complement the lectures." (xiv) (November)

These quotations suggest that the opinions of the students were reasonably consistent throughout the year and that they concluded their studies favourably inclined to the tutorial system.

In summary, factors were identified which were thought to influence the students' reactions to the lectures and it was found that although some people found difficulties with these, there was overwhelming agreement that the tutorials were successful. The major feature which seemed instrumental in forming this opinion was the ease of communication in the smaller, less formal, groups.

Because of the problem with communication, it was possible that as the class grew larger over the years, the general atmosphere became more influenced by some students who found that they could not sustain their interest at a sufficiently high level. It was also a feature that with their backgrounds, the students found it difficult to appreciate the objectives of a particular line of study. This emphasised the importance of lecturers trying to make their objectives as clear as possible to the students. "Tell them what you are going to say, then say it, then tell them what you have said" is a good maxim, but there is also good reason to explain why you think it worth saying in the first place. It appeared that little needed to be done to improve the effectiveness of tutorials. The only point that emerged was that certain sections of the syllabus did not generate any tutorial work. This was the case in some parts of the geometry. Bearing in mind the students' comments about discussing problems in the tutorial, this might be a handicap.

Some further evidence which also related to the lectures and tutorials came from the observation of a limited number of them. In all, the investigator sat in on three lectures and four tutorials, involving four different members of staff. The notes taken during a lecture and a tutorial can be seen in Appendix 6. The main features observed are summarised below.

With regard to the lectures, it was possible to appreciate that some students did not find the atmosphere of the lecture theatre too congenial. Although some one hundred or so students were present, this was small compared to the capacity of the room and they displayed their usual tendency to fill space starting from the back of the room. Communication required one to raise one's voice significantly compared with the normal classroom and this may have influenced the lecturers in their use of questions. The problem of making oneself heard ("Shout out what you think is the value") seemed to affect the students' willingness to reply and respondents appeared to come from the front half of the room.

Students had to work hard at note-taking. In two of the three lectures the pace was quite brisk and students had to be fairly attentive to follow the argument and keep track of the display on the overhead projector. There were no occasions recorded of questions or queries arising from students during the lecture.

Each lecturer made an effort to convey the objectives of the lecture to the students. When the lectures were a continuation of earlier work, several minutes were spent on recapitulation. In the example given in the appendix, a new aspect was introduced by linking it with previous study and quite extensive use was made of illustrative examples. It was seen that, where appropriate, the lecturer made a summary of the material.

There was the obvious difference in physical size when tutorial groups were observed. Communication was much less formal, more conversational and most noticeably a two-way process. Those tutorials which were observed showed the tutors making extensive use of questions which demanded responses, but a major difference from the lectures were the number of questions asked by the students. It was evident that the tutor was willing to change either his pace or direction in response to students' queries. However, it was noted that questions posed by the tutors often anticipated the problems which students were likely to have.

It was recorded in the four tutorials that nearly every student present made a comment at some time. However, it was not assumed that those who said little were gaining little from their experience. All students were seen to take notes at various points during tutorials. There were no obvious signs to an observer of inattention on the part of the students. Tutors did not give the impression of having a prescribed amount of material to cover, probably their experience indicated the most likely rate of progress and they chose the examples accordingly. There appeared to be a satisfactory balance between a recapitulation of lecture material and the co-operative study of exercises by tutor and students.

8.5 Effect of the Course on Student Attitude to the Subject

It could reasonably be argued that one of the most significant aims of a course such as this was to give the students the experience of mathematical ideas which displayed a mode of thought different from that of their principal subjects yet which is an important part of our culture (Phenix, 1964). This would normally be different from their experiences associated with the subject at O-level. Given the opportunity to study the subject in this context, it was envisaged that the experience would be judged to be a worthwhile part of undergraduate study and result in some modification of an attitude developed in a narrower school environment. Evidence is considered of this aspect in this section.

The students' initial attitudes may be judged by considering the feelings they retained about school mathematics, their reasons for choosing this course and their expectations of it together with other comments about the subject made during the first interviews held in November. In chapter 6, evidence was presented of the students' attitude to school mathematics and their reasons for choosing this option (p. 76). It was noted that previously, there was a wide variation in their liking of the subject, a reasonable number had enjoyed all, or part of their school experience. There was a minority who made a positive choice of this option and some were influenced by such factors as a feeling that they would not cope with an experimental science. Expectations were based on seeing mathematics as an important subject worthy of study but, in some cases, being unsure how far this should go. The views of the outcomes were based mainly on an increase in their knowledge of the subject and an appreciation of its usefulness. (See 7.3)

One question posed in the first interview was if the subject presented any particular challenge. Five of the eleven replies indicated that, at that stage the challenge was a source of interest, two said that is provided some interest and the remainder did not view it as particularly interesting.

> "Actually, I am quite enjoying it now. I like it when I can understand something." (vii)
> "I find it quite interesting to see how the Greeks thought." (viii) (November interviews)

The first quotation carries the suggestion that at the time of the interview, some students were pleasantly surprised by their experience. There were other remarks of this nature. For example:

> "Yes, it broadens your horizons. I am glad I am doing maths. I find I can relate it to economics." (vi) (November)
>
> "I chose maths because I thought it was an easier option. Having done it, I have been pleasantly surprised, I did find it interesting. Whether it will be practical or not I do not know." (xv) (February)

Another occasion when students' comments gave some indication of their attitude to the subject was when they were asked to say what they hoped to gain from the course, the details of which were reported in section 6.7. It was noted that the majority of replies involved an increased knowledge of the subject, others were concerned with discovering a different outlook, doing something of interest and finding it useful in real life.

Briefly, the various strands of evidence suggested that, in the beginning, the average student adopted an attitude which acknowledged the importance of the subject, showed an interest in acquiring more knowledge than might have proved the case at school but was not quite sure about the degree of commitment this involved. The view of the subject was influenced by notions of its useful or practical aspects.

Some information indicative of a different view of the subject being adopted came from six students, who in March, were asked to say in what way they thought their views might have changed. Two reported no appreciable change, one that his feelings were much influenced by the work-load and three were definite about a change of view. One remarked that the subject was no longer so definite (by this she implied different from being told just what to do at school), another said that it was a case of seeing the subject in a different light. The third replied:

> "Yes I think so (a change). I always thought maths was something that could only be understood by mathematicians. To some extent I still hold that view but it is not quite so true now ... it makes maths more 'do-able'." (xiv) (March)

The attitudes revealed in the final interviews in May were of interest because these were likely to display any changes produced by the students' experiences over the previous year. It was possible that the imminence of the examination may have contributed to a more negative response than otherwise. However, reference back to earlier interviews suggested that the comments were not unduly influenced.

At this stage, of the twelve people who were questioned, seven could be said to have a more favourable attitude or to have retained a favourable attitude, two had retained a mildly negative attitude and three felt that their attitude had declined by virtue of their experience. It was noted that many remarks pointed to a retention of a "functional" view of mathematics and that since it was likely that the technical details would be forgotten, the biggest effect was "more likely" a change in attitude. The impression was given that students did not attach much importance to attitude change.

> "I can understand what my mathematician friends are talking about sometimes, even if it is only the terms

I can understand. I do not think it has really changed my attitude. I suppose I have done things which have proved to be quite interesting. That might make me more happy about mathematics. I did not really get on with mathematics at school, I was not really any good at it ... I think it is saying that mathematics is not as horrific as you thought it was at school and it can be different and interesting and not all numbers. It had to be hard, I see mathematics as a hard subject. I think we were introduced to things I had not heard of before so I suppose it achieved that sort of objective." (iv) (May)

This comment was interesting in that it mentioned three features; being able to communicate with mathematicians, experiencing some interesting ideas and changing the view engendered by school, and showing that the subject was not all about numbers; this without feeling any great change in attitude! These three outcomes would be considered well worth the effort by the course organizers. This was not an isolated comment. The following was in similar vein:

"I think I will just forget about it. I did it because I had to do a science. I do not think it has given me an awful lot ... I did (dislike mathematics) but not any more. I think the course has produced a change because I have mixed with the lecturers and other people here who are involved and I can see how some things would really interest them. I just wish they could drop down to my level a bit ... " (ix) (May)

Again, the communication feature was mentioned and although the response implied not much had been gained, there was a change in attitude and an appreciation of aspects of the subject which was not present earlier. An opinion that was based on seeing the subject in terms of techniques is illustrated by the following:

> "More likely a change in attitude because I do not think it is the sort of thing I am going to come across really, the type of maths that we have done. But I think it helps to understand a lot of other things ... (Are the ways of thought in mathematics significant?) I do not know. Arts students tend to have a funny way of looking at things anyway ... I cannot say I developed a new scientific way of looking at things." (vii) (May)

The following comment was from a student who felt his attitude had not improved:

> "Yes I do (dislike mathematics). I do not know if I have changed. At the beginning of the year I began thinking maybe it was useful, now I am not so sure ... Not really much effect, made one worried about passing the examination. (At the beginning of the year you saw understanding anything as being useful.) I think I have probably changed now, got a bit fed up with it. It started off as a challenge, now it has just become a worry ... To start with I found it interesting. It is difficult, I think if I had enough time to actually get into it I could probably get the interest back, but I have not the time." (xvi) (May)

> "... I do not think it (my attitude) has changed but I do see mathematics in another way, I have not increased my love for mathematics ... I do not mind doing maths,

but as I said, it is an obstacle to me. I do not regard it as a challenge in respect that it is interesting to learn about." (xix) (May)

Earlier in the year, this last student had said that he did not question what they had done so far. This quotation suggests that the functional view had won the day. The final comment illustrates the view of a student who was not particularly enthusiastic but had come to appreciate the precise way of reasoning in mathematics:

"My attitude from beginning to end is that this is a subsidiary, pass it and you will be alright ... The advantage is not the specific topics we have done; I do not think I will be able to go home and put calculus into practice, but it teaches you to think logically, step-by-step and that is always good." (xv) (May)

Remarks made by some students at different times were an indication that although they thought they had experienced little or no change in attitude, nevertheless <u>some</u> change was evident. For example, one student commented:

"No I do not hate mathematics. I would not go out of my way to involve myself in it. My attitude has not changed ... "

But earlier in the same interview this student said:

"... I think I feel better inclined towards mathematics, been a bit better than O-level anyway." (viii) (May)

During the interviews in May, opinions were expressed about the course being worthwhile or successful. Although the number of comments recorded was relatively small, they did suggest that students did not necessarily link the two attributes. There were five references to it being successful and only one of these was a negative reply. On the other hand, the ten references to it being worthwhile were almost equally divided. That it was seen as successful was taken as a reflection on the sound overall quality of the lectures and tutorials, students felt the course was achieving what it set out to do. The factors which influenced it being thought worthwhile included the difficulty a non-specialist encounters in studying mathematics, the attitude to a subsidiary study and the level of appreciation of and sympathy with the aims of the course. Some typical comments were:

> "Yes, I think it has been worthwhile because of what I have learned from it. It has not made me a mathematician or anything like that but just the fact that I have enjoyed it has made it worthwhile." (iv)
>
> "No, not really (worthwhile), because when it comes down to it, the crunch is getting it in on time. In a tutorial you may think next week I shall be a reformed character and spend a lot of time on it but when it comes to it you put your principal subjects first." (xi)
>
> "It seems to have got over what people were trying to get over. I have learned about new things, been given a broad picture, if those were the aims." (xx)
>
> "On the whole it has done quite well. It is a difficult subject to teach ... It is not done too badly it has got over the history, it has been reasonably successful (in presenting significant ideas)." (xviii)
>
> "Not at all (successful) really. I am quite sad about it to be honest. I went into it feeling quite optimistic, not so sure now. It has just been beyond my comprehension." (xvi) (May interviews)

The last quotation is illustrative of the ease with which a student of limited background may be overcome by the problems and find his interest declining. In the following examples, students were asked what they thought would be remembered of the course when they came to leave the University.

> "I do not know. That is difficult. I hope I will not have forgotten everything, the chances are I will. I do not know, I could not really say." (iv)
>
> "I think I will tend to remember the tutorials that were quite lively, some of the lectures as well. More maybe than what you have actually learned." (xviii)
>
> "I suppose my impression will be that it has changed my view of mathematics in that there is more to mathematics than just what one might call very boring subjects. There are some interesting things, like probability, so if anyone asked me what did I think of mathematics, I will be able to say, 'Well it could be interesting if you are that way inclined'." (x)
>
> "I think I shall remember a few things. Maybe if I came across them or read something it might spark off a memory." (vii) (May interviews)

Evidence was looked for that suggested students opinions would show an appreciation of the general aims of the course (see p. 35). This could be said to be the case in the last two replies but others indicated an image of a collection of facts which would gradually fade from the memory.

One conjecture which was mentioned in discussions with members of staff was that students' appreciation of the course may be more evident in the long term. Some evidence for this was obtained by distributing a questionnaire to a random sample of students who had just completed their final degree examinations in the summer of 1980. Thirty-six questionnaires were distributed and twenty replies were received.

Taking the means of the ratings given by the respondents, it was seen that there was no overall feeling for or against studying mathematics at undergraduate level or that the experience had changed their way of thinking. However, although the subject was rated difficult it was acknowledged as being an important part of our culture, that there was more involved than just calculation and the statement "the study of mathematics is not worth the effort", was rejected.

The overall tenor of the replies suggested that the attitudes displayed were not markedly different from those which the current students had expressed. For example, when students were asked to give more open-ended replies about broad-based studies, a majority (13 out of 20, 3 neutral, 3 against and one no reply) agreed that broad-based undergraduate courses were desirable but this was reversed (7 for, 13 against) when they were asked about the course being worthwhile. Statements about their remaining impressions of the course varied considerably, from the curtly dismissive to another which said that the experience was enjoyable and a positive gain, a valuable part of the curriculum. Overall, less favourable remarks were rather more in number than favourable ones. There was no relation between performing well on the course and having a more favourable opinion about it.

It must be emphasised that this evidence is limited and cannot be conclusive. The response rate of just over 50% left too many students whose views were unknown (see remarks in chapter 5 concerning the use of questionnaires). The major difficulty however was that these students were still very close to their involvement in the course and it was arguable that there had been little opportunity for their views to mature. Unfortunately, circumstances which required the investigation to remain internal to the University did not allow for the canvassing of post-graduates and this line of enquiry was not pursued further.

Before giving a brief summary, it is worthwhile to note the problems inherent in presenting a broad curriculum. A student studying a discipline which is far removed from his chosen field will not find it easy and this is likely to aggravate the fact that he may find difficulty in sustaining motivation to study something which does not appear to have particular relevance to his main interest. The task of the lecturers is much harder because it is so different from their normal experience. Also, students in their comments are always likely to view their work-load as slightly more than can be accomplished in the available time and are unlikely to admit to finding anything easy.

In nearly all cases, discussions revealed that students had, in some degree, come to view mathematics in a different way. It was encouraging to see that some students who had approached the course with no particular enthusiasm had found some parts had caught their interest. There was evidence of a more favourable attitude to the subject amongst many students although some students thought that it did not represent a big change and there were a few whose attitude appeared to be less favourable. Three factors were noted as determining why, in certain students, the value of the course was not judged highly. Information about the impressions the students would take away with them conveyed the idea that the subject was seen generally in terms of its usefulness and the relevance, or otherwise, of the mathematical techniques that were discussed.

The last three chapters have presented data about the students and their reactions to the course. The implications of these findings will be discussed in Chapter 11, but first, the following chapter looks at the course from the staff viewpoint and Chapter 10 discusses some issues of teaching method.

CHAPTER 9

THE COURSE AS VIEWED BY THOSE MEMBERS OF STAFF INVOLVED

Introduction

In this chapter, a picture is presented of the academic staff view of the Subsidiary Mathematics course. The evidence was collected during a series of informal discussions with the people concerned during the period from the summer of 1979 to the autumn of 1980. It was possible to talk to all members of staff involved with the course during that year, either as tutor, lecturer or both. In all, there were seven individuals, one acted only as a lecturer, one only as a tutor, the other five being involved in both capacities.

9.1 General View of the Broad-Based Curriculum

There was no disagreement with the philosophy of offering broadly-based undergraduate courses though one suggestion was that possibly it was necessary to attract the right kind of student. There was definite support for the idea that arts-based students should have the opportunity to study mathematics. One remark was:

> "People should know something about mathematics. Its importance is such that any person who considers himself educated should have some appreciation of the discipline."

Asked if such courses were best left to sixth-forms in the schools, tutors felt that this should not be the case. One reason given was that the more mature person would be capable of appreciating more than the school pupil. One lecturer felt there are problems because staff in universities may be interested in other things. One person rejected the idea of leaving it to schools by saying:

> "I think not, partly because even if one fails, it is something one has got to try ... There is no doubt that if you talk to someone who has gone through the course about some topic in mathematics or science, it is a very different exercise from talking to someone from a sixth-form who has done arts and that's all. At least you start off on common ground. I think the student is convinced that theoretically it is possible for them to communicate and to understand. That is perhaps a good thing in itself."

9.2 Experiences of Teaching the Course

There was almost unanimous agreement that the experience of teaching on a course of this nature was enjoyable and rewarding, although this did not imply that it was without difficulties. Typical of the general comments were that lecturing was a variety of experiences but it was a worthwhile thing to do at all levels. Another lecturer felt that he had learned a great deal about the problems facing the non-mathematician. Three people were of the opinion that teaching on such a course was more demanding than a principal subject. One remarked that this was because you could not assume anything. Another added more detail by saying that conventional courses were well documented, you just did what was being done in every other university in the land. This course was worthwhile whatever the problems. There was something of significance in the way it presented a picture of mathematics; even some principal mathematics students would get quite a lot out of it.

Clearly, teaching was seen as raising different problems from a more conventional course. This was illustrated by the following reply when one member of staff was asked if it was a difficult course to teach.

> "Certainly with a principal course it is quite clear what has to be done. Here one is often very doubtful about how far one can go. One lapses into saying "If only you had done a bit more mathematics you could appreciate this ..." I suppose you have deliberately to make an effort to put yourself in their position, discipline yourself about what you are allowed to talk about."

Another lecturer commented that it was quite stimulating and required a lot of reading and suggested that perhaps they slipped up on occasions by not bearing in mind the wide variety of experience to be accommodated.

There was evidence of the frustrations that can be encountered in teaching on a course of this nature. As can be seen from the material already referred to, the teaching environment was seen as one which was not so familiar to the university lecturer and presented its own problems. The low level of student understanding contributed to the specialist's problems, for he must continually guard against complicating the argument and so confounding his audience. Also it was possible that the degree of interest which some students brought to their studies was limited, the staff morale might be expected to suffer. One comment was that, despite the frustrations, which were many, the course was worthwhile. Another reply suggested that the problems were becoming more pressing with increasing numbers:

> "... I have become increasingly worried about the general atmosphere. Partly it is connected with the number of students, I certainly feel we have lost a grip of the situation, especially in lectures ... "

This was not a unanimous view, one lecturer found his particular section of the course quite satisfactory, but referred to difficulties if one were led to introduce formal mathematical language. His view was that the topic itself was interesting and he tried not to make it too formal but to talk at the students' level. Another replied as follows when asked to say how successful the present course was:

> "I think it is a good course. It is an improvement for this type of student on what was in the course previously. There is scope for putting various things into it. I think the general idea is good, certainly."

The variety of opinions that were expressed concerning the effectiveness of the course were not unconnected with the students' reactions to the different topics. Lecturers involved with topics that were difficult to teach, either conceptually or because of technical complexity, or which proved intractable to the students for other reasons, were those more willing to accept the need for improvement.

There was one factor that was seen to have a significant effect on the overall character of the course, and that was the reaction to a large group of students in a single lecture. This was referred to in 8.4 where the student reaction was examined. All but one of the lecturers spoken to were of the opinion that they found the atmosphere in the large lecture group not too congenial. The most often referred to reason was a lack of "feel", the lecturer lacked communication with the audience. This was despite quite definite attempts to make the presentation as informal as possible, for example, by removing the lecturer from the podium, putting him on the level of the students and using an overhead projector for displaying his material. Two lecturers felt that reducing the numbers in the room would produce an improvement in overall student attitude, but a third thought this would require a positive effort to improve the situation concerning some less-well-motivated students. However, it will be seen in the next section that some students were thought to display a commendable devotion to their studies, and staff reported a wide range of responses.

9.3 Staff Perception of Students' Attitudes

Staff comments on the overall level of student interest showed that there was a general feeling that a major part of the students on the course adopted a responsible attitude and this was evidenced by the substantial amount of time and effort they devoted to their work. Although some students did not feel highly motivated towards subsidiary courses, nevertheless, three tutors specifically mentioned what they felt was a high degree of interest displayed by a number of students and one pointed out that even students on principal courses were seen, on occasions, to lack motivation.

> "On the whole, considering what course it is, I am surprised how well-motivated they are, how they do

stick at the work. The majority do hand in work regularly, very good standard."

One lecturer was not disturbed by the concern that some of his colleagues felt about the overall effect of some less-well-motivated students. He did not consider that their interest was that low. On the evidence of his lectures, they were attentive and there was no concern with loss of motivation.

The suggestion that students on this course will always find the subject difficult attracted little support. Several of the staff interviewed spoke of the wide range of ability that was encountered. It was pointed out that some of the students could well have done A-level mathematics but chose to do other things. Nevertheless, there were always those who would find difficulty.

"It is a wide variety, you are continually aware of people to whom you explain a thing again and again and you realise will not understand it ever. There are some people really who just think it is a game with complicated rules that we make up as we go along, but there are quite a lot of bright mathematicians amongst the groups."

The staff were asked to say how they viewed the students' attitudes to studying a science subsidiary. It was interesting to note that the replies suggested an appreciation that students found a broad curriculum quite demanding.

"They have expressed the view that they are bad at science, I do not think they have said there should not be a broad-based curriculum. It is very easy to say 'Yes, I am in favour of this'. They must come to university and see it in practice."

A second replied that he had not heard anyone say directly that they did not like a science subsidiary but he had heard the criticism that some students found a broad-based course a bit fragmentary. Two other opinions were that a significant number of students would come to appreciate the course more fully after they had completed their studies and gained more experience elsewhere. One pointed to evidence of this from talking to final year students who previously studied subsidiary mathematics. (Further comments on this point appear on page 122.) Another felt there always seemed to be a small group of students who made little progress on this course due to a combination of some lack of mathematical skill and a not very positive approach to the study of a science.

It was generally agreed that some of the students were not well informed about the aims of the course.*

> "Some students' ideas are ridiculously wide of the mark. Some answers did suggest some idea of what the course was trying to get at."

A second lecturer commented that there were some students who found the course something of a shock because of their misconceptions and that possibly it was profitable to try and counteract this. Another quoted a student who remarked that the course "Contains too much mathematics". It was appreciated that this was unfortunate, if students had wrong ideas, the disillusion that followed would affect their motivation. Equally, the staff were aware that for some students,

* The methods of providing information about the course were outlined in 7.2.

mathematics was chosen as a subsidiary subject partly by eliminating less attractive alternatives. Not all felt that this was necessarily a bad thing. One commented that to choose it for this reason was as good as any. If they were required to do a science, it was a bonus that they discovered an interest, if they found that they could cope and achieve something.

Lecturers were asked if they felt school experience had any significant effect on the students' attitudes. Generally, although it was felt that many students may not have been taught very effectively, the influence it had on this course was not significant. One lecturer put it that since nothing was assumed anyway, it would have little bearing and he did not seem particularly concerned that they may have disliked the subject. In contrast, one person felt strongly about the effects of unsatisfactory teaching at school. He was of the opinion that one aim of the course should be to rectify a situation which he felt was cause for concern.

Finally, there was no sympathy expressed by any member of staff for the view put forward by certain students that the course might contain some "useful" mathematics, like the calculation of interest and mortgage repayments for example.

> "Can they explain what practical mathematics is? Do they mean engineering? Mathematicians would not accept filling in tax forms as mathematics. If that is what they want I think we would have to say go somewhere else."
>
> "Surely they do not expect this course to be about percentages and compound interest?"

However, in discussing various topics on the course, there were often references to the technical difficulties that were an ever-present pit-fall for the unskilled. More will be said of this in the following section in which the staff view of the objectives will be discussed.

9.4 Staff Views of the Objectives of the Course

All the staff accepted as a general aim of the course that it should present a broad view of the subject which would be of interest to a particular group of arts-based students. One lecturer pointed out that this gave the course an unusual feature in that there was a very wide field of topics. There was an underlying philosophy that the purpose of the course should not be solely to pass the examination at the end.

There were some interesting comments concerning the feature that the course should comprise a fair number of more or less independent topics. Although no-one who was questioned disagreed with the concept of a "fresh start", some complications were mentioned. One person noted that despite this intention, the course developed a structure of its own, there was a way in which it did fit together and was more than just a collection of topics.

> "One of the rather grandly-phrased aims is to give a view of the ideas and achievements of the subjects and we are probably unfair to the subject if we try and disjoint it too much. Some attention must be paid to the fact that all subjects are linked often in a very complicated way."

Another lecturer stated that he agreed with the concept of teaching a fair number of relatively independent topics. However, he added that there was a difficulty in that the amount of material covered may be large, not necessarily in absolute terms but in the amount of assimilation required. This question of balance was clearly in the minds of most people. One member of staff put it that an important objective in teaching his particular topic was an appreciation of a significant mathematical process but he was not sure if this impressed the students. In another reply, the lecturer concerned stressed what he felt was the importance of a good grounding and mentioned two topics in this respect:

> "I would like to give them a fairly good, general grounding in some things ... I would like to think that we could give them something fairly solid in basic understanding of co-ordinate geometry and calculus. I would always want this to be an important part of the course."

One person, commenting on the course design, speculated on the possibilities of student choice of topics. He noted the way the present course frequently changed topics and the advantage this gave for a fresh start:

> "... but academically as far as the course is concerned, if students got the kind of course, which first of all they chose of their own free will because they wanted to do it and were taught a piece of mathematics which hung together over a long period ... I think they might look back on it more favourably."

Returning to the idea of linking the various topics together, it was of interest to record reactions to the idea of topics being related in some way in order to give a less disjointed appearance. Certainly, some students had reacted by saying they viewed the course as too fragmentary. (See 8.3) All the staff involved saw that there were some quite definite links between different topics and accepted that there were advantages in exploiting them to help present an overall picture. One comment was that there were some quite definite links and cited the way Cartesian geometry leads to calculus. This view was supported by a similar comment that links were made when they were immediately obvious and suggested graphs and trigonometry as an example.

Links between subjects may be created at different levels. One is when certain material is pre-requisite for later studies, another is a more external relationship, where underlying concepts, or themes, appear in a diversity of topics. The latter form may help the student by creating interest and aiding the learning process. Examples of this were the various geometries as axiomatic systems or set algebra, propositional calculus and the algebra of switching circuits being isomorphic. Thus one lecturer commented in referring to geometry:

> "Certainly it is what I had in mind and it is what one or two others have in mind to unify the course. We wanted to think in terms of axiomatic systems and models of axiomatic systems and then applications to the world. We were certainly conscious of that."

Two people made comments that they would not like to see the idea of linking pursued without some caution. Both felt that there might be a danger of making the students more confused rather than less. One suggested that the links must be genuine, not artificial. The other felt it was a question of balance:

> "We must not tie the thing up too tightly. It is a question of balance ... we have to try and steer a delicate middle course."

Finally a point which arose from the fact that several lecturers were involved on this course. One remark suggested that sometimes the material presented by different lecturers did not link well together, the person concerned felt that he occasionally did not see a connection and he felt the students did not either. Obviously this is a possibility on any course where several members of staff are involved.

9.5 Opinions about the Course Aims

In this section the responses are presented to various suggestions concerning course aims. The staff were asked to express opinions about their relative importance. Not unnaturally, some objectives were seen as more important than others depending on the nature of the topic which was the interest of the particular lecturer. There was general agreement however that the course should generate some mathematical activity within the students and also that it should lead to an appreciation of mathematics as a precise way of reasoning.

One lecturer made it a definite aim in the presentation of his topic that students should appreciate its mathematical significance and he viewed it as very important to generate some worthwhile mathematical activity. Another said that activity was a vital component, the only course he could envisage where this was not the case would involve students with considerable previous experience. A third agreed that the course gave an indication of the precise way of reasoning characteristic of the subject and held the view that most of the students achieved some insight that they did not possess earlier; certainly, the work load was considered as demanding in its way as that of some principal courses.

There was a wide range of opinion on the emphasis that might be given to historical development. Two replies illustrated a difference in approach; in one, the lecturer found the historical development was one that the students could cope with, the other held the opinion that the historical aspect was not particularly helpful and did not pursue it. One comment seemed to summarize what was the generally held opinion:

> "No, I do not think it can ever be quite that (history of mathematics) because to do the history properly is really too difficult. Rather I would see it as presenting some basic mathematical ideas in the historical context."

When questioned about the possibility of a rigorous approach to the subject, no-one saw this as desirable. However, the answers showed that people were aware that if the precise nature of the subject were to be illustrated, one had to be as rigorous as possible and one did not want to present merely a jumble of formulae.

> "It may be that in geometry we do adopt a fairly rigorous approach but as far as the rest of it, not in the calculus obviously. As rigorous as possible I suppose; limited in what is desirable on a course like this."

One objective which was often put forward both in the literature and in student comments as of major importance in courses of this nature was to present the use of mathematics in the real world. The response of the staff to this idea was of particular interest. None argued with the suggestion that this was a possible source of interest for the non-specialist but more than half of those questioned thought difficulties arose in the degree of technical mastery necessary for a true appreciation. One lecturer said that this aspect came into his topic as an offshoot. Another put it that he would like to think his subject showed how mathematics could be used in the real world but felt that it was usually at that point (when it became too difficult) where you lost many students. The overall impression was illustrated by the following:

> "Certainly that was the intention to try and keep bringing it back to actual applications, but it is not easy ... (sometimes) you cannot do enough really to get a significant application."

Finally, one lecturer wondered if the "real" world was an appropriate word, were they not concerned with idealised situations? Were they not just as concerned with solving problems as with where the problem came from?

One aspect that was considered particularly was the extent to which the lecturers saw the course as being concerned with conjecture and proof and the cultivation of intuitive thought. Of the six people who made comments on this aspect, all mentioned the difficulty encountered by students of such limited background. This was generally expressed by suggesting that although this feature of mathematics might be seen as significant, there was some doubt about how much could be achieved. It was pointed out by one person that number systems were easier for these students than some other topics:

> "Looking at divisors, say, the material is simpler than looking at a Euclidean proof. It may be that what little mathematics they have done was largely numerical and is more familiar. Very often it is not that the material is not there, they do not know what is required in answer to something which is too vague."

Another lecturer, asked to comment on the students' apparent difficulty expressed what he felt was a major consequence of their lack of experience:

> "This is what we might expect of them. We adopt certain conventions and say that certain things are obvious, we know from experience what things are obvious. They do not have any intuitive feel. They think we make the rules up. I can see that when you come to a subject from outside it is really quite difficult to know what is required."

One member of staff was of the opinion that his particular topic was not suitable for a substantial involvement with proof. He remarked that his approach was more philosophical, he would convince them Aristotle's theory of motion was right, then show them it was wrong. A different point made by someone else was that although, mathematically, inductive processes were important, students were unwilling to adopt them.

Finally, the remarks of one lecturer returned to the difficulty presented by the limited experience of the students. He felt that often, this implied having to lead them carefully along the right path:

> "... Some of the things are really very easy and the difficulty is just understanding about axiomatic systems and how you provide axiomatic proofs ... if you sit down and start to do it, it is hard, you have got to feed that into them ... I am wondering if it would not be better to back-pedal slightly on the axiomatic approach and go rather for the geometry-co-ordinate geometry-calculus, that sort of area."

It appeared that opinions varied on the emphasis which could be given to introducing ideas based on conjecture and proof. The general impression was that material needed to be carefully tailored if the students' lack of background experience were not to make it too difficult. There was a feeling that in the more technically complex topics, this feature was likely to have limited application.

9.6 Staff Views of More General Aspects

As was seen earlier, staff experiences suggested that lecturing to such a large group was the source of some concern. In contrast, all those associated with the tutorials felt that, generally, these worked well and were crucial to the success of the course:

> "Once the atmosphere is set up, tutorials can be very fruitful to the student, even if the amount of ground covered may be limited ... The tutorial group of that sort of size is absolutely essential to make the course work. Without that it could be a disaster."

> "Basically I think the tutorial system is right, I

prefer slightly fewer numbers, you get the time to know the students."

All the members of staff involved were of the opinion that mathematics could be attractive to the non-specialist. A comment from one person was that although he thought it could be attractive, in the context where it was a subsidiary course and time was committed to other things, students may not see it as welcome. A second person said that the evidence from the course itself showed that it could be attractive, the course was successful and it was a very good course. The fact that students found it hard work may be a useful discipline. Another lecturer felt that although some people adopted a rather light-hearted approach, he thought a significant number of students got a reasonable amount from their studies; most of them did it fairly conscientiously. The general feeling, and the problems, are summed up in the following quotation:

> "I am convinced it can (be attractive). I have talked to people who are obviously bright and have no mathematical background or had mathematical blocks in the past and find they very quickly get interested in a number of topics ... the difficulty is the mass in the middle who end up on this course for one reason or another, the difficulty is in interesting them, we do not always seem to get that right."

Lecturers made various comments throughout the discussion, about action which they thought might make the course more effective. Opinions varied considerably about how necessary this might be. One person saw the course as being successful, a very good course and saw no reason for major changes.

Another saw it as a good course, the general idea was good certainly, and he thought it important that there was scope for introducing different topics into the programme.

Some of the comments referred to problems associated with the size of the student body. One member of staff put it that a few uninterested students did not seem to have much effect in a relatively small group but when the group exceeded some critical size, the same proportion made a bigger effect on the morale of the course. There was some discussion of the likely effect on the course of restricting it to a slightly smaller group of more-committed students, though staff pointed out certain practical problems in trying to assess an individual's level of interest on entry. One person thought that the present picture was not so attractive as it was a few years previously.

In one discussion, a lecturer speculated that, although he sympathised with the idea of providing opportunities for a fresh start, it might be an improvement to cut down on the number of topics and spend a little more time on each. Possibly, they were trying to do too much with all the separate units. A member of staff pointed out how easily one could make the material progressively more difficult over the years because of a growing familiarity. It was particularly difficult to judge this on a course of this nature and they had to be careful.

In the various discussions, several of those involved were moved to mention the academic level of a subsidiary course. This was an indication of the concern that a suitable balance should be struck between the appropriate demands of undergraduate study and realistic course aims in terms of the student's backgrounds. A second feature was that students went into the outside world with a degree listing mathematics as an undergraduate study. Although this was held to be beneficial, there was a concern that prospective employers may think in terms of more traditional mathematical skills than those represented by the aims of this course. As one lecturer put it:

> "We cannot shut our eyes to the fact that they are going out with a qualification in mathematics."

It was apparent from their remarks that the staff were aware that on this course, as on many others, there had to be a compromise between factors which were sometimes in conflict.

9.7 Summary

There was unanimous agreement with the philosophy of broad-based undergraduate study and support for the non-vocational course in mathematics at this level. Two problems were noted that could arise in adopting this approach. Lecturing and particularly acting as a tutor were held by all concerned to be rewarding and often enjoyable. Most people found the different lecturing approach that was required was very demanding in its own way. There were mixed views about the level of success of the existing course. One factor which was a great influence was the large numbers of students enrolled; this was particularly evident in the character imparted to the lectures.

The staff were of the opinion that some students were not well informed about the real aims of the course and this was likely to affect their attitude to what was actually achieved. There was a definite opinion that many students were well-motivated and that based on the quality of the work they submitted, they had expended a fair amount of effort. What was not quite so encouraging was a group of students who did not display any great interest and may not have been very positive about their choice of science option. It was generally agreed that not all students found the subject difficult, there was a wide range of ability and some students could do very well.

School experience was not thought to have any great influence on the way students reacted to the course. No particular merit was seen in a course which concerned itself with "practical" mathematics.

There was unanimous agreement that two general aims should be to appreciate the significance of mathematics and its precise way of reasoning and to generate some genuine mathematical activity for the learner. Presenting a broad view of the subject entailed teaching a fairly large number of more or less independent topics. There was recognition of the balance required between presenting this broad view and attempting to study each topic in sufficient depth to bring out its significance. There were practical details of co-ordination when several lecturers were involved.

There were varied reactions to more specific objectives, for example, the applications of mathematics in the real world. In commenting on the role of conjecture and proof, staff were aware of the, not unexpected, difficulties for students with limited background.

In discussions of more general aspects, there was unanimous support for the tutorial system which was seen as an important and fruitful component of the course. The large numbers attracted to the course were thought to make the task of the academic staff disproportionately more difficult because of a change in the general atmosphere. All members of staff were of the opinion that some aspects of their subject could be attractive to the non-specialist although it was acknowledged that students' levels of interest showed a wide variation. There was a genuine concern with establishing what was accepted as an appropriate academic level for a subsidiary course and striving to see that this was achieved.

Different views were aired when discussion turned to the effectiveness of the course and the possibility of some improvement. Some thought that the course was successful enough to require no major changes, others that with developments over the years, some reappraisal was necessary. Certain areas of the course were judged to be satisfactory but it was natural to find that some people saw room for changes. Comments showed an awareness of students' problems in studying mathematics and the difficulties inherent in a lecturing task that was different from that normally encountered. It was noticed that the staff attitude demonstrated a flexibility in approach which allowed for continual development. This point will be considered more fully in Chapter 12, where a more recent change in course structure is described.

CHAPTER 10

THE DEVELOPMENT OF COURSE MATERIAL

One aspect of the research was to investigate the design and use of booklets prepared from the course material. Booklets were written for two topics and their effectiveness with respect to certain specified objectives was considered. More general aspects of instructional design are considered next and the results of discussions with various staff members are given.

10.1 Reasons for the use of Booklets

It was anticipated that the booklets would be useful for four reasons: firstly by acting as a summary of the lecture material, secondly by offering an alternative form of study to the lectures, thirdly by being able to offer extra material for interest and enrichment and lastly by being a useful focus for revision of the particular topic. To test these assumptions, booklets were prepared for the topics of Flow Charts and Boolean Algebra. The content of these booklets can be seen in Appendix 5.

Students sometimes remarked that they were uneasy about whether their own notes were an adequate and reliable record of the material covered in the lectures. They felt that they might miss taking note of some vital step and that mistakes were all too easy to include. Hastily taken notes may not have been the best of help in a student's attempt to master the subject. In the absence of a text-book, it was conjectured that a booklet, giving an accurate record of the topic, would be an advantage and allow them to listen to the lecture undistracted by extensive note-taking.

The amount of help that students perceived that they received from the lectures varied considerably, on occasions, some students felt they had become quite lost. It was felt that a booklet would be helpful in these circumstances since it provided an alternative mode of study. For this reason, the booklets were designed to include exercises and answers as a guide to individual study. It was conjectured that students would be prompted to work through systematically, checking their progress as they went. The booklet was thought of as being parallel to, yet independent of the lectures.

Many authors have pointed out the value of good, illustrative examples in teaching mathematics, including Avital and Shettleworth (1968), Courant and Robbins (1941) and Kline (1977). For example, Avital and Shettleworth said this:

> "By its very nature, mathematics is the study of relationships. Such relationships are ultimately abstracted and made independent from the nature of the elements among which they are established. However, the comprehension of mathematical relationships is certainly enhanced by pointing out their existence in a large variety of models taken from the environment."
> (Avital and Shettleworth, 1968, p.37)

For this reason it was decided to include in the booklets some extra examples that were not essential for the development of the argument but added interest by showing more significant applications of the principles involved. These were usually rather more complex and were included as worked examples. They were marked by a suitable symbol as being optional material. It was thought that better students would demonstrate their interest by working through them in some detail. The weaker students would at least be moved to read through them and gain some interest by being aware of some significant applications.

That students would find booklets of some benefit for revision and review of the topic was a function of the feature mentioned earlier, that is a booklet was an accurate and relatively compact record of the material covered.

A factor that became clear during the collection of data was that almost without exception, students made little attempt to refer extensively to suggested reading material. Students would mention referring to a book if this proved necessary for the writing of an essay, but it was common to discover that between one interview and the next, students had not found it necessary to look at a text book at all. This pattern was confirmed by responses to questionnaires when participants were asked to record the number of times they had consulted a text-book recently. By far the largest response was "not at all". In view of this attitude, it was thought that a booklet which was closely linked to the material of the lecture programme would be a valuable document for the average student.

The most valid data about how students used these booklets would be obtained by direct investigation of individual study patterns. However, it was clear that the requirements of time and effort to do this effectively were too great to make it a feasible proposition for the investigator. It was decided to use the programme of interviews to collect information on the use of the booklets. However, as mentioned in Chapter 5, interviews have some disadvantages as well as advantages. The information they provide can only be partially representative and is "filtered" by the choice of sample. Thus it was hoped that students' remarks could be confirmed by a limited use of a questionnaire plus comments from the lecturers on how effective they thought the booklets had been.

10.2 Booklet Design

When the design of the booklets was considered, the same theoretical framework was adopted as that used in developing the teaching material in the second study which is part of this thesis (details of this are given in 14.1). Basically, the material was categorized in terms of information to be acquired, techniques to be developed and applications to be demonstrated. Also, in this case, it was decided to construct the booklets so that they would be complementary to the lectures and yet be a useful guide to private study, hence the material in the booklet would be the same as that of the lecture programme.

The first task was to discuss the areas of study with the lecturers concerned. The two topics were chosen mainly on the basis of being familiar to the author. It was important to identify the objectives that each lecturer had in mind when choosing his material so that the proper emphasis could be written into the appropriate sections of the booklets.

Suitable examples and exercises were incorporated in the text for use in tutorial sessions and as coursework. Two different patterns were used in the siting of the solutions to the various exercises. In one they were embedded in clearly marked sections within the text; in the other, hints and solutions were in a separate Appendix. It was hoped to discover if either method had a clear advantage. (See, for example, Continuing Mathematics, 1976 and Mathematics Applicable, 1976.)

A draft booklet was prepared for each topic and discussed with the appropriate lecturer. After suitable amendments, the booklets were photo-copied in A5 format and stapled to make a conveniently sized document.

In the first instance booklets were prepared for the 1978/79 academic session. After the consultations with the lecturer it was felt necessary to modify the proposed treatment for Boolean algebra so that the presentation was more in accord with the ideas held by the lecturer when he prepared the examination question for that year. An unexpected rearrangement of the timetable resulted in the booklets only being available after the corresponding lectures had been concluded. Because of this, it was felt necessary to give greater emphasis to the data on this aspect of the work which was collected in the following year.

10.3 The Flow Chart Booklet

The part of the syllabus to be covered by the booklet was dealt with in one week of the course, involving two lectures and a tutorial. It was important not to swamp the student with too much material. The programme of lectures was concerned with calculating procedures and the study of flow-charts led on to a consideration of numerical approximations to $\sqrt{2}$, π etc. However, for the non-specialist it was felt that a blend of numeric and non-numeric examples would best develop an appreciation of the principles involved. The discussions with the lecturer identified the following overall aim together with specific objectives:

(i) to develop an appreciation of precise reasoning in operational form,

(ii) to present the information about the necessary components used in the construction of flow-charts,

(iii) to develop algorithm design, showing the function of decision boxes, loops, exit points etc.,

(iv) to apply the technique to both non-numeric and numeric examples, and in a later development,

(v) to enhance appreciation of the process by illustrating the use of flow-charts in a more complex problem.

In the version of the booklet issued in November, 1978, the first examples involved crossing the road safely, deciding if three given rods could form a triangle, the computation of N! and of $\sqrt{2}$ and an algorithm for the issue of a theatre ticket. A more complex example involved sorting a list of numbers and introduced the idea of a sub-routine. Two sets of examples were included in the text, with solutions given in boxes immediately following. In addition, a page of examples was included at the end, without answers, that could be used for homework etc.

In the light of the experience gained by using the booklet in 1978, a modification was undertaken in the following year. The lecturer was happy with the way the booklet had followed his development in the lectures and tutorials. He thought it would be useful if the booklets could be given out before the topic was covered in the lectures so that students could come prepared. Further discussion took place on the balance of non-numeric and numeric examples. This resulted in some replacements in the worked examples and students exercises. An optional section was added which showed how a complex tax rule could be presented as a relatively straight-forward flow chart. The section on sorting was removed as this would not form part of the lecture course. Two sets of exercises, with answers, were retained, with the addition of a non-numeric question, requiring the student to write a section of the highway code as a flow chart. (See Appendix 5.)

10.4 Student Reactions

In an interview with a group of five students on 2nd February 1979, they were asked about their reactions to the booklet which they received at the end of the previous term. Asked if the idea of using a booklet was of value, two of the students did not get the booklet at the time of the lectures and did not venture an opinion. However in the following discussion, one did say that she enjoyed the first part of the booklet because it showed the idea of breaking down everyday things and this was of interest to an arts student. The other student remarked that one feature of flow charts was that it made you think before putting things down. The remaining three students thought having a booklet was useful. The main reason advanced for this was the inclusion of exercises which could be studied in one's own time.

> "A booklet is better than duplicated sheets. Often thought having a text-book to look up would be helpful ... The idea of exercises is good. Doing it yourself is the only way. It looks o.k. on paper until you come to do it. I think you need someone to go over it and point out your mistakes." (vii)

> "Good idea (having a booklet) you have the opportunity to read it, stops the topic being an absolute shock." (iv)

Another comment was that more detail would be useful and asked for more examples showing applications to necessary rather than pointless things. (From previous remarks, this student identified N! as something of little importance.)

Questions about the exercises in the text revealed that those students interviewed were far happier about the non-numeric examples than the numeric. Three mentioned problems with following the procedure for evaluating $\sqrt{2}$. Although the idea of introducing exercises was not queried, there was no evidence that the students had worked through the material systematically. Many remarks were indicative that only those sections directly concerned with lectures or tutorials were looked at in detail.

> "I cannot quite remember which ones we did (on page 10). We went through the booklet when we got it. I did not do any of the top part. We did the first question of the homework, which was all right." (vi)

On the basis of this limited evidence it appeared that the booklet was a useful source of lecture notes and examples but was not used extensively as a means of independent study.

In November 1979, it was possible to interview a group of five students at the time they were actually working with the booklet, which had been distributed before the corresponding lectures began. Three students had read some part of it before the first lecture. Two students worked through it systematically and tried most of the exercises, one student worked through one of the two sets of exercises and one tried some of them but added that she did not spend too much time on it, although she had read it right through. The final member of the group had concentrated on the parts required for the tutorials. He remarked that he did not have time to do more, it was very difficult to find the time. (This student had a weak mathematical background and may well have been slower than some of his colleagues.) In view of this it was interesting to record that he thought the booklet useful since it gave you the precise information you required. He also thought the booklet would be useful for revision.

> "I thought it was useful. It was clear on the whole. It is useful because it gives you the precise information that is needed. In lectures I find I do not know what to take down and what not to take down. This gives you everything you require. I did not work through it systematically ... I think I will go through them (exercises) for revision. Very difficult finding the time." (xvi)

> "I found it really useful. In lectures you did not have to concentrate on two things, taking notes and trying to understand. Yes it was a useful complement." (xviii)

Altogether, all five students thought the booklet was "useful" or "very useful". As reasons they gave that they did not have to worry about note-taking and could concentrate on the lectures (two), that it was a more reliable source of information than notes (two) and that the booklet was seen as a useful complement to the lectures (two). On a less favourable note, there were two comments by different students during separate interviews that suggested the booklets would be of limited value to some students because they might not give them sufficient careful attention.

10.5 The Boolean Algebra Booklet

Two weeks of the course were devoted to this topic. Discussions with the lecturer concerned identified the aspects deemed important to be:

(i) the appreciation of an algebraic system where the elements concerned were not numbers,

(ii) the demonstration of the application to propositional calculus and switching networks,

(iii) to show how the algebra of subsets, propositional calculus and the algebra of switching networks were essentially the same, or isomorphic,

(iv) to demonstrate how the basic structure could be represented axiomatically and the concepts generalised to an algebra containing more than two elements.

The order of development was to start with the algebra of sets, since sets had been discussed earlier in the course, proceed to propositional calculus and then to switching networks. Then Boolean algebra would be presented in terms of a set of axioms and finally, there was a simple introduction to Boolean algebra with more than two elements. Six sets of exercises were included to which answers were given. A set of examples, without answers, were included for use in tutorials and as homework. The final section contained hints and solutions to exercises, with references in the text. (The booklet can be seen in Appendix 5.) This last feature was different from the layout adopted in the flow chart booklet in that answers to exercises were not embedded in the text. The disadvantage was that looking up the answers could result in a lot of frustrating page turning but on the other hand, students were not tempted to take premature "peeks" at the solutions. Since it was decided to include hints to the main text as an appendix, the opportunity was taken to include the solutions to the exercises in this appendix and to ask the students which layout seemed better. Four pieces of optional material were included, suitably marked by a recognizable symbol. These illustrated more complex problems in propositional calculus and switching networks and also an exercise involving a system that looked like a Boolean algebra but was not (the addition and multiplication of odd and even numbers). This form of the booklet was used in May 1980. (As mentioned earlier, the version used in 1979 was slightly modified; there were fewer examples and the order of material was rather different.)

10.6 Student Reactions to Boolean Algebra Booklet

For the reason given, the booklet prepared for May 1979 did not provide much useful data. It did however, provide useful practice in booklet writing. In the interviews conducted in May 1979, three interviewees had received copies of the booklet at that time. In all three cases the comments were favourable. One gave as a reason that it contained exercises and solutions, another mentioned that it was much better for reference and since there was no suitable text-book would be much better for revision. The last one confessed to having read the booklet only a short while ago and not having studied it in detail. However he liked the idea of having exercises with answers so that one could check one's progress. He remarked that by the time you had received back your homework and checked the corrections, it was too remote from the topic to have much relevance.

> "Yes, when it gives exercises and that sort of thing (it is helpful). We did a couple in the lectures but I did not do any on my own. I think if there are hint-type things they should be incorporated into the text as it went along, whether this would defeat the object of the exercise I don't know. I suppose it could be difficult having it all together but it is a good idea that it is there anyway." (iv)

In May 1980, five students were interviewed at the time they were studying Boolean algebra and had been given the booklets. All five welcomed the booklets, two mentioning specifically the opportunity to go over the work at your own pace. They felt that the work was clearly set out and commented that this should help with revision. One noteworthy feature was that, at the time of the interview, no-one had worked through any of the optional material, although two were sure that they would do this when they came to revise. Only one student looked at any of the booklet before the first lecture. Two said that they had not received the booklet beforehand but owned that even given the opportunity they doubted if they would have read any of it.

> "Yes, for revision purposes, excellent really because you can work through it. I think they should do this for every topic, perhaps half-way through the lectures, not straightaway at the beginning. Then if you need it for revision, that's excellent." (xix)
>
> "Yes (I like booklets). I think that is why I understand Boolean algebra ... The thing I like is that you know you have got all the important facts you need. When I look back at my notes I do not understand them anyway." (xvi)

(Is optional material useful?) "Well, I suppose for people who are into it a bit more. I might be tempted to leave it if it is optional." (xx)

There was no evidence from the replies that any of the students had attempted to work systematically through the booklet. One student thought that having looked briefly at the booklet beforehand, it was better to go through the booklet after the lectures and look at the difficult points. Another student admitted to finding the lectures difficult, although the tutorials were all right and she had decided to work through the booklet the following week. No student found difficulty with the layout having hints and answers at the back. One did mention that it stopped you accidentally seeing the answers and so he preferred this arrangement.

Since not much useful information had been obtained in May 1979 about the use of this booklet there was further reason to supplement the data from the interviews by distributing a questionnaire to some tutorial groups in May 1980. In this way, nineteen replies were forthcoming, representing a 100% response from those who were present.

Twelve students thought booklets helpful in their studies and eleven felt they were helpful in following the lectures. There were four who attempted to work through the booklet systematically. The layout of the booklet was generally reported as being 'clear' or 'reasonably clear'; three students thought it 'not really clear'. There was little use made of the optional material, four worked through some of it, three made 'not much use' and twelve reported 'none at all'. Only three students read any of the booklet before the first lecture. One commented that he had not received the booklet beforehand, so there may have been others in a similar position.

Only one student commented adversely on the arrangement with hints and answers at the end, the others being neutral or felt it a convenient system in about equal numbers. Asked if the booklet would help in revision, three felt it would be of 'little help', four of 'some help', with eleven saying that it would be 'helpful' or 'very helpful'. When asked if they thought booklets could be useful in other topics, thirteen were in favour, five were undecided and one against.

Students were asked to add any appropriate comments. Eight replies were forthcoming. Most were concerned with difficulties with either the topic or in following lectures. Two made complimentary comments about the booklet. Others mentioned there were not enough easy stages in the presentation (one), that the lectures were difficult to follow (two) and that Boolean algebra was not a topic they particularly liked (three). Typical remarks were as follows:

> "Not enough easy stages, complicated too soon."
> "Lectures difficult to understand but the tutorials and booklet clarified everything."
> "As I do not really understand Boolean algebra my answers are bound to be biased."

10.7 Conclusions Concerning the Use of Booklets

The evidence from the questionnaire generally supports the opinions expressed during the interviews. The one slight difference worth noting was that whereas the interviewees were unanimous in saying that booklets were helpful, two answers to the questionnaires thought they were not much help, particularly in following the lectures.

It was concluded that the booklets served a useful purpose and that the students, in general, found them an advantage. The evidence supported the conjecture that booklets were a useful complement to the lecture. The major reasons were that the booklet was a reliable reference compared to hastily written notes and also that the necessity for making lecture notes was removed, allowing more attention to be given to the lecture.

The second conjecture, that the booklet would be used as an alternative form of study to the lecture was not so well supported. Students on the whole did not declare that they worked systematically through the booklet, although there were exceptions. One reason for this was possibly that the students did not find either of these topics of particular difficulty and so were not motivated to study in this way. It was significant that students welcomed the availability of exercises with answers which could be used in private study.

Only a minority of students made any use of the optional material in their studies. The conjecture that they would at least read through such examples and thereby find the topic more interesting was not well supported. The most typical comment was that if the material was known to be optional, one would be inclined to leave it out.

There was a majority opinion that it would be of value as a useful focus for revision, one reason being that it was seen as a reliable source of information. The availability of worked examples and exercises with solutions was also seen as a useful aspect. However, since the evidence was gathered before the students revised for the final examination, this represented an intention rather than a fact. It was possible that these intentions were not carried out but it was felt that the effort required to provide evidence of this would have been large in relation to the value of the resulting data. It was considered that since the students themselves had reported the booklets to be clear and useful for reference, there was every reason to believe that they would be used in the way described.

The information concerning the siting of the answers to exercises (in boxes within the text or in an appendix) was not clearly in favour of either method. It was concluded that if hints were to be included at the end of the text, then the answers fitted neatly into the same appendix. This also reduced the temptation to look prematurely at the solutions but if there were many short answer questions in the text, it might result in a frustrating amount of page-turning.

The availability of a booklet was considered to be of value, particularly in the absence of a directly relevant textbook. The design as a self-study document was also seen as helpful, both in complementing the lecture programme and as a source of revision material. In the light of this experiment, it is of interest to look at the subsequent development of the course which was redesigned to allow for fewer lectures and a major part of the teaching carried out using seminars with emphasis on the provision of fairly detailed study notes. Outline details of this can be found in Chapter 12.

10.8 General Aspects of the Teaching Approach

In addition to the discussions with the academic staff about the use of booklets, there was also some conversation with them concerning other course content and its presentation. Two cases will be reported. The first of these involved the lectures on counting and measuring which were part of the programme on number systems given in the first term, the second discussions concerned the restructuring of the calculus lectures in the 1979/80 session.

The task of formulating suitable teaching objectives that were commensurate with the students' experience and understanding can be illustrated by considering the development of the section called counting and measuring within the theme of number systems. Having considered the rational numbers, the objective was to progress to irrational numbers. The first step was to prove that some numbers were not rationals (e.g. $\sqrt{2}$). This required a proof by reductio ad absurdum. Repeated subdivisions of the number line for approximations to $\sqrt{2}$ led to the ideas of a limit, convergence and series. To show that these concepts were a problem in Greek mathematics, reference was made to the Zeno paradoxes. Summation of a series was illustrated by considering the partial sums of arithmetic and geometric series. Tutorial work on the summation of series required proving summation formulae by the method of induction.

It can be seen that the students, in a relatively short space of time, were introduced to some new and in certain cases, quite difficult concepts. It was not surprising that some students had reported, during the interviews, that they had experienced problems with this feature of the course. What was clearly illustrated here was the way in which the major objective (extension of rational to real number systems) had generated so many side issues, including the concept of the convergence of series and two quite different kinds of proof. Similar problems were evident in the discussions that took place concerning the section in calculus. The topic had been remarked upon as giving rise to difficulties for many students and the Department took the opportunity of devoting more time to this subject in the programme of 1979/80. One feature that was given some consideration was the idea of using some significant examples as an introduction to the subject and to develop calculus as a way of solving the problem. One example of this nature was to ask if manufacturers used tin-cans which used the least amount of material for a given volume. In this instance, the lecturer provided examples which were placed on the desk. It was noticeable that students could remember this, as they did the fact that tennis balls had been used in the lectures on motion. Was this interest too superficial? Probably not, one student put it that people were actually taking notice. Their interest had been aroused and their thoughts had something to latch on to. Two points were worth noting. Several of the staff had mentioned the value of good, rich examples in developing a topic. Also, the value of simple visual props was evident in the way in which these had succeeded in focussing the students' attention.

The main issue that emerged during these discussions was the problem of balancing the various factors which confronted the individual lecturer. There was a concern to develop the subject in a meaningful way within the overall aims of the course, there was the need to appreciate the level of understanding of the average student on this course and for the lecturer to formulate teaching objectives at each stage and communicate these to the student who would know where he was going and why. The difficulty of communication must not be underrated. You could not explain to students in terms they did not understand. In an effort to take them into one's confidence, they must not be confused by the jargon involved.

In conclusion, it was evident that the lecturers had given considerable time and effort to the preparation of their teaching material and several successful features were identified. It was noteworthy that the academic staff were willing to give of their time for these discussions with the investigator and were happy to discuss their teaching methods. This is by no means a common experience in higher education (see also Chapter 9). Without doubt, a course of this nature was recognized as presenting unusual problems for a university lecturer. These were identified as stemming from the need to formulate and attain realistic and significant teaching objectives given the experience of the average student. In the following chapter, some consideration is given to possible alternative syllabus material and the associated teaching objectives.

CHAPTER 11

SUMMARY AND CONCLUSIONS: A SUBSIDIARY MATHEMATICS COURSE FOR ARTS UNDERGRADUATES

A summary is presented of the data in the previous five chapters in the form of student and staff profiles. A summary is given of the major points concerning the existing course structure and course content and possible developments are discussed.

11.1 The Student Profile

In chapters 6, 7 and 8, the information about the students was documented in detail. This is summarized in the form of a student profile, some indication is given where characteristics were likely to be common or show some variation. It should be remembered that this information is based mainly on the results of interviews and supportive evidence as described earlier.

Considering the mathematical background, students were most likely to have given up the study of mathematics after obtaining an O-level grade. They were about equally divided in that those who found it difficult were balanced by others who coped reasonably well. Attitudes to the subject were quite varied, many students being better disposed towards it than might have been expected in arts students. The influence of the teacher had been significant in many cases, sometimes for the good but not always so. Coming up to university, the student was rather more likely to be positively in favour of broad-based undergraduate study and this was reflected in the attitude to a subsidiary science course for arts students.

The study of mathematics was viewed as important, mainly reflecting an opinion about its importance in a technological age. There was no clear idea about how this was translated into the level of study or which aspects needed to be studied. There was a fairly equal division between favouring some serious study of the subject for a non-specialist and feeling that this was best left to those with a special aptitude. The significant aspects of mathematics were seen as arising from its usefulness. The appreciation of the place of mathematics in our culture was not viewed as being of particular importance. On occasions, the subject was found to be frustrating, but when topics were fully understood and the required results obtained, a considerable degree of satisfaction was experienced. The frustration was often associated with the precise way of reasoning of mathematics; the subject was viewed very much in terms of black and white. Experience on the course modified this opinion and other features became more apparent, for example, the idea that all the arguments were based on axioms.

Expectations of the course comprised a combination of an increase in mathematical knowledge, a broadening of outlook and a change in attitude to mathematics, the first being the most likely and different individuals giving different emphasis to each aspect. Reasons for choosing a mathematical option were varied. Although it was possible that the individual had an interest in the subject or saw the opportunity to study something that was thought to be important, there was also those who had little particular interest in mathematics and chose the option because other options appeared less attractive for various reasons. Ideas about what the course was designed to achieve were sometimes imprecise or only partially true, based on information passed on by former students. Students who were more enthusiastic about the course were likely to display this in their better appreciation of the aims.

When asked to propose what he thought were reasonable aims for this course, the typical student talked about usefulness and practical value without displaying clear ideas about what this implied. However, topics which could be understood and were a focus of interest or satisfaction were viewed very favourably. Lack of wider experience made attempts to formulate precise objectives rather unproductive and the impression was given that it was a question, on some occasions, of trying to define something that was simply different from current experience. However the responses to a suggested list of course aims gave reason to believe that a broader view was taken than at first might be thought. Although aims like getting right answers to the exercises and passing the course had their appeal, importance was given to such aims as the development of independent thinking.

Virtually all topics in the syllabus were likely to be sources of either particular satisfaction or frustration to individuals. It was probable that the first term would pass without too much difficulty arising and that at least one aspect of the different geometries would cause some concern. It was considered a welcome feature that topics retained sufficient independence to provide opportunities for a fresh start and it was not generally felt that the course was too disjointed as a result. The more obvious connections between certain topics were recognized and understood by the majority.

The lectures were a source of some unease. Contributory factors were seen to be the size of the lecture group and the large lecture theatre in which the lectures took place creating an impersonal atmosphere which was not relished. Some of the lectures were found difficult to follow, an unfamiliarity with the style and lack of practice in following mathematical arguments were other contributory factors. There was less inclination for such students than for specialists to spend time pondering over the awkward points. The attitude adopted towards and interest in a subsidiary subject had an influence in this respect. The tutorials were described as the most satisfactory feature of the course. They were stated to be the major source of learning and were often described as enjoyable. The contrast between the reaction to the small, more intimate climate of the tutorial and the large lecture was quite marked.

Generally, the study of mathematics as experienced on this course was found to be difficult. The ideas of conjecture and proof were often the cause of some bewilderment. Limited experience meant that many students were without the necessary framework within which to construct their proofs. Different forms of proof were not fully appreciated and a grasp of what were the starting points proved elusive. Despite having to grapple quite hard with this subject, students made little attempt to read extensively about it; interest did not extend far beyond the lecture handouts and the tutorial exercises.

As far as the effect of the course was concerned, a different attitude to the subject was engendered. Mathematics was not seen as a rigid body of knowledge but it was appreciated more in terms of its ways of thought and it was likely to be acknowledged that communication between the disciplines was possible. It was accepted that the course was successful in pursuing the aims chosen by the staff and the students were conscious of the time and effort that must be devoted to the lectures and tutorials. However, some students felt these aims were not entirely appropriate and at the completion of the course, a student might feel that he had achieved little success in the study of certain topics and that the knowledge gained was not particularly relevant to what he proposed doing in his future life.

A little evidence was obtained on whether students' appreciation of the course may be more evident in the long term but this was insufficient to provide firm conclusions. (See p. 122.)

11.2 The Staff Profile

The details of chapter 9 may similarly be drawn together in conclusions which present a typical view of the staff involved with the subsidiary mathematics course.

Turning first to the general attitude, it was thought important that undergraduate courses should be broadly-based, although it was also recognized that not all members of an academic staff were attracted by lecturing to subsidiary courses for non-specialists. The idea that arts-based students should be given the opportunity to study some mathematics was strongly supported. Teaching on this course was held to be a rewarding and sometimes enjoyable undertaking but it was by no means an easy task and was as demanding as, if not more so than, a specialist course. Staff realised that putting oneself in the place of the students was not easy when their backgrounds were so different from those normally encountered on specialist courses. It was appreciated that not all the students were strongly motivated, that interest might be low and the lecturer or tutor might feel some lack of achievement.

Opinion was about equally divided between feeling that the course was satisfactory more or less as it was and wishing to see a certain degree of change. Probably a significant influence was which of the course topics were the individual lecturer's major concern. Some topics were viewed as more easily assimilated by the students than were others and student responses were acknowledged to vary widely. Lecturing to a large group, in a large, rather formal lecture theatre was the source of almost universal concern, because of the impersonal nature of the surroundings and the lack of "feel" that this imparted. The popularity of the course and the increase in the number of students as the years had gone by were felt to increase the pressures on those involved and were seen as factors tending to make initially minor problems that much more acute.

Staff realised that some students were only partially aware of the real aims of the course and that a number might have chosen this option as the least of possible evils. This was not universally seen as a drawback, for whatever the reasons for the students being there, this was an opportunity to teach them something worthwhile. Several gave their considered opinion that after their experience of the course, the majority of students would have a better appreciation of the subject even if this did not become obvious until some time after the course was completed. It was acknowledged that many students must have put in a great deal of effort to produce the amount of homework that was handed in. Some students were seen to be very competent and did quite well, displaying a good deal of interest but on the whole, it was felt that the majority were always going to find mathematics a difficult subject to study.

The major aim of the course was seen as presenting a broad view of the subject. There was no sympathy for the view that the course should be concerned with "useful" mathematics. The value in trying to keep topics independent was appreciated but it was also felt that there was value in exploiting common features which linked some topics together and gave a more integrated picture. The objectives that were given general prominence were to cultivate an appreciation of the mathematical way of precise reasoning and the generation of some genuine mathematical activity. In what ways these were best achieved was the subject of some debate; as before, a significant influence was the extent of the individual's concern with a particular topic. It was clear from the discussions and comments that a considerable amount of time and thought had been devoted to the design of lecture and tutorial material and its presentation.

In general discussion, a genuine concern was revealed in trying to ensure a proper balance in a course which was a subsidiary study for the non-specialist yet contributed to a degree award and therefore was required to have an academic challenge. It was felt that the non-specialist was undertaking a very worthwhile activity in studying mathematics and it was the task of the lecturers concerned to create something of value for the student. The tutorials were said to be fruitful and work well, being rewarding to both staff and students. This was in contrast to opinions about the lectures which were mentioned earlier. With the increase in the number of students, there was a fairly general feeling that the character of the course had changed and that those involved, both staff and students, were not finding it such a worthwhile experience as previously, giving rise to the view that some reappraisal was likely to repay the effort involved.

11.3 Factors Involved in the Interpretation of the Evidence Collected

In sections 5.5 to 5.7 there was a discussion of the nature and interpretation of the data required in the undertaking of this investigation. There was, for example, the point made that in the two previous sections, (which were essentially summaries of descriptive surveys) it was not reasonable to produce a conclusion in quantitative form from that kind of evidence. A summary is presented here of the major points that determined the kind of picture that was presented.

Considering the data collected from the interviewees, the first notable point was that judging by the grades achieved on this course the students displayed a wide range of ability. They were by no means all good performers as might have been suspected from a group of volunteers, (although, perhaps, poor performers might equally wish to present their views). They were found to be studying a wide variety of principal subjects and were quite varied in their initial attitudes to the subject. The interviewer was known not to be a member of the academic staff of the university and although the interviews were recorded on tape it was stressed that anonymity was to be preserved.

Since the majority of interviewees were seen more than once on occasions which were spread throughout a whole academic year, opportunities were present to check any inconsistencies, but care had to be taken not to confuse this with genuine changes in attitude. The confidence of the students was also gained as the number of interviews increased. They were familiar with the interviewer and knew what to expect in the interview. This made them more relaxed, requiring less prompting to make useful comments. One of the most note-worthy features with regard to the validity of the information was that the students seemed quite happy to make comments which could be construed as detrimental to their own image. Thus, one possible reason for not wanting to be strictly accurate did not seem to operate in this case.

Corroborative evidence was sought using questionnaires given to samples throughout the year. As reported in chapter 5, there was no purely quantitative comparison with respect to the returns. Generally, the conclusions were drawn as being the most likely interpretation when interview and questionnaire were combined. A third source of data was also used to provide another means of comparison. This came from the discussions and informal conversations with the members of staff involved with the course. In all these activities, care was taken to ensure that comments and replies were anonymous as far as the individual was concerned. Finally, the willing co-operation of the staff involved allowed a limited amount of observation of actual lectures and tutorials. This also supplied valuable information on how the course functioned which could be compared with the stated views of the students and staff. (See p. 112.)

11.4 Conclusions: the Course and its Operation

This summary begins with the listing of seven factors which were identified as significant in shaping the course and the way it functioned. Conclusions are drawn concerning the achievement of aims and lastly, a statement is made about what were thought to be the main characteristics of the course.

Firstly, there was the philosophy of the University itself, with an emphasis on broad-based undergraduate study and the requirement of a science subsidiary for arts-based students. This led to the design of a course to meet the post O-level experience of the prospective student and yet to introduce something of academic significance with a more cultural bias than was usual in traditional service courses. This required the staff involved to face problems which were rarely met, if not unique, in the university context in this country. In the event suitable aims were identified and a syllabus and course were constructed to pursue them. One notable aspect was that the content remained flexible, the staff involved proved willing to learn by experience and the course underwent changes even during the course of this investigation.

The second factor was the aims of the course as formulated by the course designers. The decision to present the development of the subject with stress on its ideas and achievements meant that the syllabus content would be broad-based with a relatively large number of more or less independent topics. This presented its own problem of balance, and of how to develop ideas and achievements without going outside the extent of the students' mastery of technical detail. This question was never fully resolved, perhaps not surprisingly; there is some discussion of this later in this chapter, but with changes in the staff involved and the differences in student groups, the discussion is likely to continue. One aspect which made a genuine positive contribution was a group of staff who showed a great deal of commitment to this course and clearly devoted a significant amount of time and effort to its operation.

A third factor was that the course represented an academic task very different from that traditionally associated with undergraduate teaching. It was clear that the lecturing and tutoring was a difficult task, more so than some principal courses, which posed more familiar problems. It was appreciated that service courses generally, let alone a course of this unusual nature, were not to every lecturer's liking. Those involved realised that it was difficult for them fully to appreciate the viewpoint of the student and to foresee possible difficulties. The fact that, almost without reservation, the staff had felt that the course was worthwhile and the source of real satisfaction was a factor which made a valuable contribution to the achievements of the course. One also felt that course designers were to be commended in allowing students to be credited for effort. This was very much in keeping with the spirit and aims, for students would be encouraged to get involved in mathematical activity if they were to feel the stimulus of reward rather than frustration as the fruit of their labours.

The next two factors that were identified concerned the students. The first of these was their attitude to broad-based courses and a science subsidiary in particular. Some students acknowledged when interviewed that a broad undergraduate education was what they were seeking; there was evidence that some students were keen to show some progress in the study of mathematics and accepted the opportunity for yet another try at a subject which they felt was difficult but important. On the other hand there were some students who were not so committed to the general philosophy on which such courses were based. It was possible that this effect was more noticeable as a consequence of increased numbers on the course, particularly as any institution will contain some students not particularly committed to the kind of study they are going to experience.

The other factor which concerned the students themselves was the nature of their mathematical background. Although many of them did reasonably well at school up to O-level standard (and there was a surprisingly high proportion amongst the interviewees who claimed to have enjoyed the subject), the average student could not be assumed to have more than a modest pass grade. There were two consequences of this limited background. The first of these was the difficulty it gave the student in appreciating a broad view. When struggling with the detail, it was difficult for the student to step back and take in the overall picture. Skemp (1976) has described the process of solving a mathematical problem in terms of a constant re-focussing; looking in to cope with the minutiae of the technical manipulation, standing back to appreciate what has been achieved. This is a process which may hold problems even for the specialist. The second consequence was that their limited concept of the nature of mathematics left them in a poor position to assess what might be worthwhile aims for the course, or even of recognizing these if put forward. Students may not always be the best judges of what is appropriate for them and some were reluctant to accept or found difficulty in understanding arguments in support of the chosen aims.

The sixth factor which had a significant effect on the nature of the course was the number of students choosing this option. This had grown considerably over the years, so that in this investigation, the entry per year was of the order of one hundred and forty students. There was strong indication from the staff that the general atmosphere had altered as the numbers had grown. There was some feeling that the effect of less-motivated students had become more noticeable as the numbers had risen. Whether this was due to the larger numbers per se or that the change in numbers was accompanied by a less sympathetic feeling towards science subsidiary courses was not clear. What was discernible was that a number of students had misconceptions about what the course was trying to do and this was a source of some disillusion.

The seventh factor is linked with the last one and concerns the system of instruction, that is lectures to the whole group and tutorials involving small groups of students during each week. The large numbers made lecturing difficult for the lecturers and a less than satisfactory experience for many students, due to the rather impersonal atmosphere that was engendered. On the other hand, the students were unanimous in their appreciation of the tutorial periods and almost all were willing to acknowledge they were a worthwhile experience. The evidence from all sources was that this was definitely a successful feature of the current course.

In section 4.3 the aims of the course were identified. It is now possible to consider the extent to which these were achieved. First were those features which were mentioned in the prospectus, an emphasis on the development of the subject and on its ideas and achievements. These were reflected in the design of the course. A broad view was presented, with a great deal of ground covered. Students were introduced to a variety of topics, many of which would have been unfamiliar to anyone who had followed a traditional school curriculum. Next, a genuine effort was made to involve the student in some worthwhile mathematical activity. This often involved devising problems and examples which avoided too much algebraic manipulation, thus minimising the possibility of students becoming bogged down. A great deal of effort had been devoted to the preparation of tutorial sheets containing examples which the student could work through. Often these were designed to lead the student through various stages to a generalisation or maybe a simple proof. These were of benefit to many students and allowed them to feel some real satisfaction in their mathematical activities. (A good example of this was the set of exercises associated with number theory which can be seen in Appendix 1.) With reference to the aim of "an appreciation of mathematical reasoning", students were definitely aware that the subject was concerned with a way of reasoning quite different from that which they were familiar with in arts subjects. There was a real desire on the part of the lecturers to avoid presenting the subject as a collection of specific methods for performing precise tasks. The students were aware of this fact and the evidence of lectures and tutorials showed that this was successfully avoided. In certain topics, where this was appropriate, the use of mathematics in the real world and the history of the subject were used to focus the interest of the students. Again, from the evidence of comments that were made, this had been achieved, as, for example, may be concluded from the general response to the historical development in term one. A drawback was that lecturers felt that some of the more interesting applications took them beyond the scope of the average student. The other feature that discussions with staff made clear was their rejection of any aim for the course based on a narrowly "practical" value given to the subject.

The students generally felt that the course was reasonably successful in its aim of presenting a broad view of the subject. On the other hand not all felt that overall, it had been a worthwhile experience. Some also felt that the subject had proved rather difficult and this had left little room for a sense of genuine achievement. Looking at the whole picture, it was concluded that the following points were positive achievements.

The course designers had been able to identify some worthwhile aims for a programme which had few antecedents. They had constructed a syllabus which genuinely matched these aims and had been able to call upon the services of lecturers and tutors who had taken a real interest in the course and generally saw it as a worthwhile part of their teaching activity. Considerable thought had been given to the preparation of course material. There was much that a good student would find to be of interest. The syllabus had remained flexible and there had been a willingness to learn by experience. Even allowing for the fact that some students were not entirely clear about the course aims, the fact that student numbers on this option had steadily increased over the years was an indication of what had been achieved. The tutorial system that was developed for this course, with groups of students spending one hour per week with a tutor, was seen to be highly popular and successful. This must be attributed in no small measure to the work and enthusiasm of the tutors involved. The smaller grouping and the opportunity to communicate more easily were features which the students welcomed and the atmosphere seemed to suit their temperament so much better. There was evidence that students' attitudes to mathematics underwent some changes as a result of the course. (See section 8.5.) Many accepted that the idea of communicating with the scientist was not viewed in the same light as previously. (Staff reported noting a difference in students' approach after completing this course, see, for example p.127.) The subject itself was seen as quite different from the subject that was taught at school. Even students who thought the course was not particularly worthwhile were clearly going to adopt a different attitude to the subject as a consequence of their experience. It was evident that students had seen a side of mathematics that at school they did not appreciate existed and in view of the fact that this gave them a broader view of the span of human intellect, it was concluded that the course fulfilled one role for which it was conceived.

Some elements that were less desirable were also identified. There were some students less well motivated by the prospect of a broad educational experience or by a science subsidiary course in particular. There was, in some cases, a mismatch between student expectations about the course and what the lecturers had in mind. This seemed to lead to some measure of disillusion and probably to a greater readiness to be critical of what took place. Increasing student numbers had tended to bring some problems into sharper focus; for example, the lecture environment was adversely affected. Some sections of the syllabus had proved to be quite difficult to some students and this had resulted in a certain amount of frustration being experienced.

There would be an obvious bonus if the attitude engendered by the tutorials could be built upon and the rather negative response to the lectures diminished. (See Chapter 12.) Perhaps if the apparent stress laid on the lectures, as viewed by the students, as the major instrument of learning was reduced, then their general attitude might be improved. This might be achieved by providing instructional booklets or other written material which could be used in parallel with the lecture programme. One other problem associated with this area was the difficulty experienced by the students in appreciating why they were studying a certain topic or even why certain material was being covered in specific lectures. It was acknowledged that this was not simply a question of telling them what was going to happen. With limited experience, they may not be in a position to understand the terms used, and may end up being more confused than before. However, it must be an ever present concern when lecturing at this level to make it as clear as possible to the students what they are doing, why they are doing it and then to summarize and recapitulate such information at suitable points in the instruction. The students need to be taken into the lecturer's confidence.

There would be much benefit if the students were made more fully aware of the existing aims of the course. This could possibly lead to a few people deciding that some other option would be better for them and also reduce the risk that any students become disillusioned by their experiences on the course. How to achieve this is not obvious, particularly in a situation where students are permitted complete freedom of choice without course "entrance" requirements. Some students made rather poor use of existing information sources. Perhaps the Department should consider a more aggressive dissemination programme or a novel way of presenting the course aims. For example one might distribute a self-marked questionnaire or a flow-chart conveying information about the course aims which students might be prompted to work through. One last consideration was how the students might be made more conscious of the value of what they are doing. Two points were thought to be relevant. Firstly, as suggested above, if the students were better acquainted with the true nature of the course and saw that lectures and tutorials followed a pattern, there would be a more positive attitude to the whole experience. Secondly, it was noted that students were less likely to question the reason for doing topics which they understood well and felt they enjoyed doing. This also reduced any feelings of frustration. For this reason it was concluded that another positive factor was to build into the system the opportunity for an average student to achieve a reasonable level of mastery. This required careful preparation and review of the tutorial material and the exercises used as part of the course. Again, this was no simple task, there had to be enough straight-forward exercises for students to gain satisfaction by mastering them, yet there was a need to cater for a wide range of ability and to extend the more able student.

An approach which can accommodate a variety of student expectations is one where several parallel courses are offered, each of which develop different major themes. For example, courses might be offered where emphasis was given to the history of mathematics, recreational mathematics and mathematics in the real world. Since there would be smaller numbers choosing each option, this would avoid problems with large group size. However, there would be a considerable increase in staff commitment to cater for the extra instruction. Additionally, the problem of students being unsure of the aims of the course would still remain to be dealt with; it would be no help for a student to study the history of mathematics thinking it to be a description of the lives of prominent mathematicians if the course designers wished to present a development of significant ideas.

11.5 Conclusions Concerning Course Content

The problems of syllabus construction were discussed in detail in chapter 3. The feature common to all authors was a desire to present the subject in a way that aroused the interest of the students in what was seen as an area of significant human endeavour. The interests and opinions of the individuals involved had a considerable influence on the course content they proposed. Two aspects which were important in this particular case were the relatively large number of topics chosen in a broad-based course and the narrow experience of the students in mathematics. It was discovered in this investigation that there was a great deal of individual variation in the degrees of difficulty experienced in studying the various topics on this course. Some students reported no difficulties with topics which others found quite intractable. Where topics were mastered, there was no doubt about the satisfaction this generated and the corresponding motivation this produced. A brief review of the existing syllabus identified the following features (see appendix 2 for the syllabus).

The development of numbers and number systems, which comprised the backbone of the work in term one, used the history of the subject to good effect. Some well-prepared examples provided the opportunity for the students to get to the point via some guided steps. The subject required no particular technical skill. The problems arose with the possibility of the students becoming confused by the introduction of several new concepts in a short space of time, for example convergence of series and sums to infinity and various forms of mathematical proof. Flowcharts was interesting to the arts student who appreciated how this form of reasoning could be extended to non-numerical topics.

The various geometries which were studied at the beginning of the second term were the source of some confusion to many students. They were introduced to four different geometrical systems within the space of four weeks and in addition were confronted with the concept that the whole subject was based on sets of axioms. It is probably true to say that geometry was the least familiar of the mathematical structures to modern O-level students. Although students became aware that they were discussing axiomatic systems and their consequences, it was also seen that many students found difficulty with at least some part of the geometry programme. Calculus always presents technical difficulties because of the manipulation involved; further, the non-specialist is not always impressed by the mathematician's passion for the limiting process. There was evidence that the extra time devoted to this topic during the later years of this investigation resulted in better understanding by the students. This was one area where significant problems requiring the development of the mathematics could be pointed out. The links with the understanding of dynamic systems and theories of motion in particular were obvious.

The evidence suggested that a majority of students found theories of motion a part of the curriculum which was both interesting and relatively easy to understand. There appeared to be a good balance between using the history of the subject to illustrate its development and the avoidance of too much technical complexity and the interest of the students was obviously aroused. Sets, groups and Boolean algebra comprised another section of the syllabus which was accessible to the majority of students and was well received. Most seemed to appreciate the way Boolean algebra represented the abstraction of the three examples of the algebra of subsets, propositional calculus and switching circuits. No information was collected on the topic of models in mathematics due to its lateness in the term, and its proximity to the examination precluded the arranging of further interviews.

The major problem in choosing topics for this type of syllabus lies in trying to reconcile the demands for material that the student can master (and thereby experience satisfaction), that does not demand the development of technical skills to an overwhelming degree yet illustrates significant mathematical ideas and is the source of some worthwhile activity. This was recognised as not always an easy task, for example, choosing exercises which involved conjecture and proof in calculus was far different from doing the same thing in number systems. A motivating factor which has been mentioned was the use of mathematics in the real world but there were sometimes problems created by too much technical complexity.

One issue that was identified in the way the course was presented was the possible conflict in trying to present a series of independent topics yet not allowing the subject to become too disjointed and indeed, fail to show the way in which generalised concepts can be formulated in mathematics. It was concluded that a fruitful approach here was the identification of themes that loosely tied the material together but most importantly, could be exploited to give some purpose to what was being presented at the level of the lecture. As examples of this can be quoted the development of number systems which ran through virtually the whole first term's work and the idea of an axiomatic system which was the basis for the development of the geometry in term two. If such themes were identified and communicated to the students it could help to unify the material and ease the transition from one topic to another. Possibly, this might be achieved by including thematic references in the handouts used for other course material so that students knew beforehand what were the objectives of a particular group of lectures and tutorials. Also, some small allocation of class time might be devoted to periodic overviews which could be used to lay stress on the underlying theme and to tie various strands together.

It has already been mentioned that central to a consideration of the syllabus was the aim of stimulating the interest of the student. How this was best done was a matter of discussion, depending on the person's own interest and the particular topic involved. Three strategies were noted on this particular course. Since the development of the subject was of prime interest, a major emphasis with certain topics was their historical development. Students were reassured, for example, to find that the Greeks had problems too. It was possible to convince the students that a theory was plausible, then show them it was wrong. (See p.140.) A second technique was to pose an interesting problem and then show how a mathematical technique may be used to provide an answer. This was so, for example, in showing that calculus was the instrument by which to find the minimum of a function or the area beneath a curve. A third method involved deriving a general structure from a concrete example. This approach was adopted when the algebra of switching circuits was shown to be a Boolean algebra and to be applicable to other systems also. What perhaps was missing was an awareness by the student of <u>why</u> various topics were being studied. They were not, for instance, studying calculus to learn how to use rules for differentiation, although clearly this was something that might result from their efforts.

Following consideration of the use of methods to stimulate the interests of the students, it is of interest to consider alternative topics which might be incorporated, compatible with the course aims that are known to apply.

Firstly, consideration has been given to exploiting the mathematics of the real world. Does this not refer to concepts of mathematical modelling? It has already been noted that some models require a high level of specialist knowledge to understand; the calculus comes easily to mind. However, some interesting examples of modelling can be derived using at most a quadratic function. There are many applications of linear programming which can be solved graphically and yet are of interest to the non-specialist. The series of books on

Mathematics Applicable (Ormell ed, 1976) contain examples where the models are quadratic functions. Some of these could be used effectively to illustrate the essential features of problem-solving, i.e. to formulate the model, solve the mathematical problem and finally to interpret the results.

One of the objectives of talking about many different geometric systems was to demonstrate the point that the structure arose from the basic axioms. If students were confused at the resultant complexity, one solution might be to use a less complicated geometry. An example of this is the so-called taxi-cab geometry based on movement along a grid pattern of equally-spaced horizontal and vertical lines (an abstraction of an American City street plan). In this the ideas are simple enough for students to be able to establish many properties themselves, for example the taxi-cab circle, yet the only axiom different from conventional plane geometry was that of Euclidean distance. (Krause, 1975.)

Another aspect which has received attention is the exploitation of a significant idea or application. An example of this at the thematic level might be the various applications of the concept of the vector. The two-dimensional vector could be introduced in Cartesian geometry, be used in differential calculus and occur again in a discussion of motion. Often, an intriguing problem can be used to stir the imagination. This was a technique employed in a book written by Hunkins and Pernot, (1977). Each chapter began by posing an interesting problem, with the chapter then developing the necessary mathematics and finally producing the solution. This approach has already been used with some success on this course and it is one which might repay further consideration. The authors mentioned above claim that it has been effective in sustaining interest.

Concerning mathematical proof, there was little doubt that the evidence revealed that non-specialists were sometimes confused by the various forms encountered, were unsure of their starting ground and puzzled by the mathematician's insistence on the importance of this aspect. Students would mention that a great deal of time and effort was spent on proving things that they thought were obvious. One problem was the relatively short space of time for the assimilation of the ideas and strategies that were employed. Perhaps if some specific time could be allocated to this topic, the issues could be made clearer. Examples may be used which relied on very simple structures which were easily understood yet allowed for genuine mathematical reasoning. Some exercises of this nature are to be seen in the first few chapters of "Mathematics A Human Endeavour" (Jacobs, 1970). The purpose is to generate problems involving conjecture and proof using simple structures like the paths of a billiard ball, the chords of a circle and painted cubes. Another recent book which provides many similar examples is "Mathematics: Problem Solving Through Recreational Mathematics" (Averbach and Chein, 1980).

Taking these points into account, it is possible to speculate on a complete alternative syllabus for subsidiary mathematics. It must be noted that this could in no way be taken as definitive, every author will give a different emphasis to the aims and objectives. In this case the topics have been compiled bearing in mind the development and background of the existing course and its particular aims and philosophy. Since the aims have already been discussed in detail, each topic will only be given a brief description which tries to indicate the development and the teaching objectives. Some approximate duration is given for the individual topics.

Conjecture and Proof (1 week) What constitutes a proof, why it is seen as important. Examples illustrating different strategies. Some steps in problem-solving.

Number Systems (4 weeks) Historical development showing how the systems expanded to meet different needs. Opportunity for more exercises in conjecture and proof, particularly in number theory.

Flowcharts (1 week) Shows that mathematics can be concerned with problem analysis. Many interesting non-numerical applications.

Network Analysis (2 weeks) From the Konigsberg bridge problem, show how a new branch of study can develop from a "recreational" interest. Eulerian paths etc. Many recent applications can be cited.

Geometry (4 weeks) To show how the basic axioms decide the nature of the system. Is Euclidean geometry about the real world? Brief treatment of the history of the emergence of non-Euclidean systems. Taxi-cab geometry, exercises in a simple non-Euclidean geometry.
Cartesian geometry, graphs of polynomials. The idea of a vector. Maximum and minimum points, thinking ahead to the calculus.

Mathematical Modelling (1 week) Illustrating the use of algebraic representation. Problems involving quadratic functions. Solutions of quadratic equations. Optimum points found graphically.

Calculus (4 weeks) A most significant topic in "applied" mathematics. Calculus as a technique for solving problems e.g. maxima and area under the curve.

Theories of Motion (2 weeks) Historical background, leading to modern theory. Relative velocity, exercises involving simple vector addition. Links with Cartesian geometry and calculus.

Boolean Algebra (2 weeks) An illustration of another branch of the subject which is not concerned with numbers. Develop the isomorphism between the algebra of sets, propositional calculus and switching networks.

either:

Probability (2 weeks) Does not tie in easily with the other topics but is a subject which many non-specialists find interesting and think of as highly practical.

or:

Linear Programming (2 weeks) Application of simple algebra which generates interesting exercises. Links with geometry if a graphical technique is used to solve two-variable problems. Shows mathematics is not always concerned with equalities.

The preceding chapters have presented an extensive picture of the subsidiary mathematics course and this has been summarized in this chapter. In 5.4, seven objectives of the research were identified. Referring to these in turn, it may be said that:

(i) the purposes of a cultural course have been identified and the bearing these have on the consideration of course content and the emphasis in presentation,

(ii) a profile of the student involved has been compiled from the evidence of interviews and other contributory sources,

(iii) the salient features of the course as perceived by the student have been identified,

(iv) it was possible to compile a profile of the views and aims of the lecturers and tutors involved in the course,

(v) the responses of the students to their experience of the course have been established,

(vi) topics were identified which were fruitful starting points in discussions of further developments,

(vii) the design and use of study booklets was investigated and students' reactions sampled.

Various factors were noted which were seen to have a significant bearing on the course structure, the way in which it functioned and the responses it aroused. Points have been listed which were thought to be both of positive and negative influence. Some conclusions have been made about the considerations required in syllabus construction and the teaching approach.

A significant change was made to this course for the session of 1980/81. The teaching method was adapted to exploit the advantages of smaller groups for instructional purposes and the following chapter briefly describes the innovation.

CHAPTER 12

POSTSCRIPT

12.1 A Report on Changes in the Course during 1980/81

Before leaving the subject of the previous chapters, it is of interest to record some changes that were introduced into the subsidiary mathematics course for the session of 1980/81 and during the time that this work was in preparation. The staff who were involved held discussions in which it was decided to change the teaching method and to make some modifications to the syllabus.

It was decided that teaching would be by seminars and lectures. The seminars involved groups of students who met twice weekly and in each term had one different member of staff as tutor. The seminars were devoted to exposition and discussion of the more technical detail and manipulative aspects of the course and to the setting and discussion of exercises attempted by the students as coursework. The tutor worked from duplicated material in booklet or other form, which was common to all seminars, and he made his own choice on the nature and timing of the available exercises. The lectures were generally once weekly to the entire class (approximately 130 students). They were devoted to the more discursive aspects, such as historical background and development or applications and were intended to follow up rather than precede the material dealt with in the seminars.

The time available was divided up, as in previous years, to allocate a number of weeks to the study of individual topics. A lecturer was nominated for each topic and he was responsible for the preparation of the material for that group of seminars and for giving the relevant lectures. The seminar material included both expository text and exercises from which tutors could choose material. Lecturers were not obliged to lecture every week; the programme of lectures was co-ordinated by the course supervisor and announced in advance on the appropriate noticeboard.

It was observed that, compared with the previous study scheme, the student commitment remained at three hours class-contact per week, comprising two seminars and one lecture as opposed to two lectures and one tutorial. Likewise, the staff commitment remained more or less the same, for instead of two tutorials taught for one hour each per week, one seminar was taught for two hours per week. The total load was balanced by arranging for increased numbers in seminar groups.

12.2 The Rationale for the Changes

From discussions with some of the staff, the rationale for the adoption of the new structure was established. A major stimulus was a concern felt about the effectiveness of the old lectures given to the whole class of some one hundred and forty students. A feature was the impersonal atmosphere of the large lecture theatre and the corresponding feelings of remoteness in class and lecturer. The tutors also felt that on the evidence of questions and responses from the students in the tutorials, there was reason to suppose that the lectures were not particularly successful in getting over some of the detailed points to the students. On the other hand, it was thought that the tutorials made a major contribution to the learning process and were generally popular with both staff and students.

Consequently, it was decided by the lecturing staff to evolve a new structure which gave less emphasis on the expository lecture programme and exploited the success of the smaller-group teaching. A simple exchange of tutorial for lecture as on the old scheme was impractical because of the considerable resulting increase in staff requirement. The function of the lectures was now deemed to be different, with a move towards the dissemination of material in the smaller teaching unit of the seminar. A consequence of this was that the system was more like six parallel courses being given by six different people. One possible difficulty with this was that individual tutors could apply differences in emphasis when dealing with the individual topics. This was countered by basing the course on extended lecture notes, which were duplicated and distributed to everyone involved. Thus the tutors were aware of the detail that had to be covered and students had detailed notes which not only removed the pressure of note-taking from lecture and seminar but gave the opportunity to pursue more guided individual study.

Since there was a decision to re-write the course handouts, it followed that there was an opportunity to take a further look at the course material. It was remarked upon by more than one lecturer that over the years, there had been a tendency to reduce the number of topics covered. A study of the syllabus showed that in this case, the number of topics had not altered significantly from the previous year. On the other hand, a closer study revealed that with some of the topics, the amount of detail covered was rather less. Thus, for example, in the section on geometry in the Spring term, it was observed that there was no mention of finite geometry and that non-Euclidean geometry was introduced as illustrative lecture material when talking about geometry as an axiomatic system and what happens when the axioms of Euclidean geometry are changed.

One feature retained from the old scheme was the use of a lecture given during the first class period to the whole group in which the nature and objectives of the course were explained. In the light of comments suggesting an apparent lack of appreciation of what the course was about, for some students a lecture of this nature has much to recommend it. One hazard was remarked on by the course supervisor; because of the way in which the choice of subsidiary courses operates, it was possible that some students had not enrolled by the time of this first lecture. For this reason, it might be better to give this lecture at a later date, but this could be disruptive to the programme of seminars.

12.3 Staff Responses to the Changes

It was not possible to allocate time for an extensive programme of interviews and discussions. However, it was possible to seek the views of four out of the six members of staff who were involved with the course and also to interview an equal number of students. With such a small amount of data it would be inappropriate to try and draw firm conclusions, nevertheless it was not unreasonable to look for certain indications.

Looking first then at the reactions of the staff involved, those who were questioned were unanimous in the opinion that the new structure was an improvement. Comments included the suggestions that the scheme worked much more smoothly and that the extra contact in the seminars with the students was a real advantage. It was clear from all the remarks that they were happier with the seminar arrangements than the previous lecture programme. It was interesting that although lectures still existed in the scheme, no-one volunteered any comments that they were felt to be unsatisfactory. All those questioned agreed that their lectures had been designed to serve a different function; "rather like icing on the cake", as one put it. One lecturer pointed to a slight problem arising from this; when students became aware that the lectures were not essential, they were tempted to miss them out.

One point of considerable interest was the responses to the task of preparing detailed seminar notes. Three of the four lecturers specifically mentioned how salutary this had been as a reminder of how formidable were the problems that confronted the student with no more than a limited experience of the subject. One lecturer particularly made comment on how time-consuming and difficult this rewriting had been, requiring as it did, considerable liaison with his colleagues. Another lecturer also commented that the new arrangement was so much better in that students seemed more aware of what was going on. He was of the opinion that another factor in sponsoring this feeling was the much better communications between staff and students.

When looking at the course material itself, it was seen from the replies that there had been a definite attempt to reduce the extent of the coverage. It would have been remarkable if all the modifications had proved totally successful, so it was no surprise that lecturers saw the need for further developments. For example, one lecturer mentioned that although the reduction in material had eased the difficulties somewhat, this in his opinion had been countered by a rather more involved treatment of some of the remaining topics. He pointed specifically to some of the examples in the section on calculus. It was satisfying to note that the booklets prepared and evaluated as part of the research programme as reported in chapter 10 were incorporated as study notes in the revised course. The lecturer concerned proved to be quite happy with the style and layout of these works and felt that they were well suited to the new teaching approach.

12.4 Comments from Students

Ten students volunteered their services as interviewees but in the event only four of these actually turned up at the due times, three in February and one in early May. Unfortunately, because of constraints of time and distance, alternative interviews could not be arranged. The remarks that are reported below are therefore deemed to be, at most, illustrative. The four students concerned were all pursuing different programmes of principal subjects; Computer Science and Psychology, Philosophy with Politics and Economics, English and History, and finally History and American Studies. One fact however, which would suggest the sample was not typical was that two of the students were in their mid-twenties.

There were no obvious differences in the mathematical backgrounds of these students compared with previous interviewees. All had O-level passes although one gained this award after failing at school and going on to Further Education. At that stage he then felt quite confident and enjoyed it as did one of his colleagues, who was reasonably successful at school. The two remaining students did not enjoy it, one saying that he did not have any confidence, the other that he was "frightened stiff". Generally, these attitudes were what might have been expected on past experience.

When asked about the objectives of the lecturers, three of the four students made replies that showed some perception of the idea that promoting mathematical thought rather than dispensing information was the real aim. There was some difference between them in the degree to which this was thought to be desirable. The fourth student said that he saw the aims as firstly to create an interest in the subject and secondly to get over the common fear that most people had.

Clearly, it was not possible to ask students to compare the old and new teaching methods. However, it was noteworthy that two of the students said that the lectures were very good (the mature students), the third that some of them were quite interesting and the fourth felt that they were not very successful, being rather repetitive of the seminar material. One of the students in fact suggested that the course might be restructured to include more of the sort of material that could be included in an extended lecture programme. As a group, these students felt that the function of the seminars was inhibited by the reluctance of many students to join in the discussions.

When asked to comment about getting lost in lectures and seminars, two replied that there was no real problem, a third said that she became slightly lost on some occasions and one of the mature students owned to becoming terribly lost.

Asked if they felt their attitude to mathematics had changed in any way, they were unanimous in the opinion that this was the case. Three held that they were more favourably inclined, one that he felt more hostile. Reasons given for the change were various, one student pointed out that a whole new aspect of the subject had been opened up, it was not so dry and more pleasant than she thought it would be. The other two favourable replies were both influenced by an increase in confidence, one saying that when he saw a complicated formula he no longer immediately cracked up. The fourth student thought his less favourable attitude was because, after struggling to gain his O-level, he wanted to enjoy mathematics and make progress and this he found had not happened.

There was some discussion of the availability of information about the aims of the course. This suggested, as had previous interviews, that there was some misunderstanding on the part of the students about the real purposes. One student felt that more could be done by the Department to supply more detail, although his colleague pointed out the difficulty of trying to explain something if the terms cannot be understood in the first place. A third reply illustrated the difficulties of interpretation. The student thought that since the course was said to be "cultural", this implied it would deal with the historical and philosophical background and he was disconcerted by the approach that was adopted.

12.5 Summary

Although no attempt is made to draw conclusions from the above, there are several points which are worthy of mention. The first was that the academic staff had demonstrated a positive response to certain identified problems. They had exploited the strength of the course by developing the smaller study group. By providing the extended lecture/seminar notes, it was possible to counter, in some measure, the problem of getting lost in the formal sessions by providing an alternative method of study. On the evidence of the tutors who were consulted, it seemed reasonable to suppose that the new teaching method was to be preferred and had engendered a much better learning environment. The evidence from the students suggested that the response, at least, had not been less favourable compared with previously, and might have improved in the case of the lectures themselves.

Conversations concerning the course material showed that discussions on the content, purposes and methods of teaching amongst the staff would continue. It was noted that individual interests and interpretations, not unnaturally, tended to be different. The possible implication of this was to stress the requirement on courses of this nature to consider the purposes of teaching particular topics as well as the topics themselves and to design the teaching approach accordingly.

It was not possible to make an objective comparison of examination results between the old and the new scheme, but it was the opinion of the lecturers concerned that there had been no obvious sign that student performance had deteriorated.

There was some feeling that the results, together with subjective judgments about performance in seminars, indicated that some topics were more readily understood than others and that there was room for continuing reappraisal of the depth of treatment of the individual parts of the syllabus.

From the point of view of evaluation, this brief survey could not produce any firm indications. However, it seemed reasonable to record that the Department had made worthwhile changes which had been compatible with the identified strengths and weaknesses and that the result fully justified the resolve of the staff to continue with this line of development. In view of the evaluation involving study booklets that was referred to earlier, it was interesting to record the adoption of a system of extended seminar notes as a major feature of the teaching scheme. Demonstrating a notably flexible attitude, the lecturers proved willing to undertake a major review of the course and to continue the debate about course content and teaching method.

CHAPTER 13

A QUANTITATIVE METHODS COURSE FOR B.A. ECONOMICS UNDERGRADUATES

The Background and the Research Objectives

13.1 Introduction

The following chapters contain an account of a study of the teaching of mathematics to first year undergraduates reading for a B.A. degree in Economics at a Polytechnic in the United Kingdom. The major part of the data collection occupied the period from September 1977 to June 1980. The report contains details of the development of a teaching method designed with the specific requirements of this course in mind and of the formative evaluation that took place over the period. An account is given of the assessment of the role of mathematics as perceived by the students and the influence that this had on their attitude to the subject. Another interest was to discover the views and attitudes of specialist economics lecturers concerning mathematics and to assess the effects these were likely to have on the students' attitudes.

13.2 The Background of the Quantitative Methods Course

Students studied this subject as part of a three year, full-time undergraduate course leading to the award of a C.N.A.A. degree in Economics. The degree was administered by the Department of Economics and Social Studies and service work was carried out by the Departments of Business Studies, Legal Studies, Arts and Computing and Mathematical Sciences.

In part I, which is the first year of the course, the students read five compulsory subjects, one of which was Quantitative Methods (see table 1). In part II, comprising the second and third years, each student read three compulsory subjects and additionally chose two (degree) or three (honours) options. Of the three compulsory subjects, one was split into Economic Statistics, which was studied in the second year and a General Paper, which was studied in the third. The optional subjects contained what were deemed to be mathematical subjects in the form of Econometrics and Mathematical Economics. It will be seen that the first year contained the only formal study of mathematics and that this was therefore the foundation for the mathematics used on the whole course. It was also noted in a study of the curriculum that the second year syllabuses in Macroeconomics and Microeconomics contained references to such techniques as constrained utility maximisation, linear homogeneous functions, problems of existence, uniqueness and stability, and aggregation and pitfalls in predictive models.

Part I	Part II	
First Year	Second Year	Third Year
Macroeconomics	Macroeconomics	Macroeconomics
Microeconomics	Microeconomics	Microeconomics
Quantitative Methods	Economic Statistics	General Paper
Social Frameworks of Modern Britain	PLUS	PLUS
Contextual Studies	2 or 3 Options	2 Options (degree) or 3 Options + Dissertation (Honours)

Options:

Group A	Group B
Accounting	Econometrics
Business Law	Economics of Labour
Applied Statistics	Economic Analysis of Business decisions
Political Science	Environmental and Urban Economics
Behavioural Science	History of Economic Thought
Economic History	Economics of Growth and Development
	Mathematical Economics
	Public Finance

Candidates for a degree select not more than one from Group A, candidates for honours, not more than two.

Table 1. B.A. Economics Degree Structure

The Quantitative Methods syllabus contained mathematics and statistics in roughly equal quantities. The class contact for this subject was six hours per week, split equally between mathematics and statistics. Separate lecturers were allocated to the two topics. The class contact was provision for both lectures and tutorials and traditionally this time was divided between two, one-hour lectures and one tutorial. If class size was large, the tutorials were duplicated to keep the group size to a maximum of about twenty students. Assessment was carried out in this subject by combining the results of a written, unseen, examination at the end of the session with the best two results from three one-hour assessment tests carried out at roughly equal intervals throughout the year and the results of a mid-sessional test. The final examination paper in Quantitative Methods had equal numbers of mathematical and statistical questions in two sections. Five questions were to be attempted, allowing a choice of two or three from each section. Separate one-hour tests were used in each topic for the assessment of coursework. For the final grade, the marks were summed; there was no minimum requirement for the mathematics and statistics separately.

The syllabus of mathematics topics contained in the Quantitative Methods course conveniently divided into four sections. The first part, referred to by the title Variables, Functions and Graphs, contained work involving polynomial, logarithmic and exponential functions, graphical techniques, geometric series and applications to problems in economics. The second part, Rates of Change and Differentiation, involved the basic techniques of differential calculus and their application to such problems as finding function maxima and minima. The third part was entitled Matrix Algebra and took the subject as far as the technique of finding the matrix inverse and the fourth part, Functions of Two Variables, covered the basic technique of partial differentiation and finding the optimum points of such functions, with and without a linear constraint. (The full syllabus is given in Appendix 8.) The periods of time allocated to each part were respectively six weeks, six weeks, five weeks and four weeks but in practice, these may have been extended to accommodate assessment tests. The teaching year was thirty weeks, so a certain amount of time was available for revision before the final examination, even allowing for the inclusion of formal assessments.

A strong influence was the stress laid by the Department of Economics and Social Studies on the importance of mathematical aptitude in the study of economics. Referring to the aims of the course, the submission to the C.N.A.A.* stated: "The teaching method and the proposed methods of assessment both seek to develop students' ability to think logically, to criticize and to communicate. Reflecting developments in the discipline of Economics, an emphasis throughout is on mathematical modes of expression." Later in the same document, the Chief Examiner noted that: "... the Polytechnic has set its sights at a fairly high standard, particularly in so far as the mathematical/statistical competence of its students are concerned ... I note with interest that compulsory mathematics and statistics has become exclusively a Part I subject and hope that it will not prove too formidable a hurdle for the first year students."

The minimum entry requirements for the course included an O-level pass in Mathematics and this was normal for most students. Some students possessed an A-level in the subject and others may have continued their study in the sixth form or further education. Occasionally there was someone who had studied for a National Certificate or Diploma which contained some measure of mathematics or statistics. Taking aggregate figures for the two entries in 1978 and 1979 as indicative, 64% had an O-level qualification (or overseas equivalent), 21% had an A-level in Mathematics or Mathematics with Statistics and 8% held an Ordinary or Higher National Award with some mathematical content, the remainder were mature students with non-standard entry qualifications.

* Document submitted to the Council for National Academic Awards (C.N.A.A.) by the Polytechnic when seeking re-approval to offer the degree.

Some students did not gain a particularly good grade in their O-level and consequently, a significant feature was the wide range of mathematical ability that existed in each first-year group. Since the mathematics comprised what is generally accepted as a service course, a major concern was that the parent department required a mastery of the required skills and that the students should be able to apply their mathematical ability in the realm of economics. This last was seen as requiring rather more than simple manipulative skill. More is said about this in the following section, where the aims of the mathematics instruction are identified.

13.3 The Aims of the Course

Some of the problems encountered in traditional service courses were listed by MacDonald-Ross and Rees (1972), who pointed out that often, only a satisfactory terminal behaviour was demanded in such cases. The main problem was identified as one of relevance. This same point was taken up by Romiszowski, Bajpai and Lewis (1976). They identified difficulties arising from three sources; a lack of motivation because the subject appeared irrelevant, lack of co-ordination between mathematician and subject specialists and the fact that the student may have chosen his main subject partly to avoid mathematics. They also noted that objectives may vary considerably depending on the point of view; the subject specialist may see the task as achieving a simple facility, whereas the service teacher may be much more concerned with mathematical development. Allanson (1968), writing in the Journal of Biological Education and referring to the mathematical education of the non-specialist undergraduate, claimed that the frustrations often reported in this context may be countered by concentrating on how mathematics is used in the real world and he advocated a problem-orientated approach. Successful courses for non-mathematicians were said to have two common features, the subject was shown to be needed in the chosen field of study and attempts to generalise concepts were left until a late stage.

A similar conclusion concerning the desirability of problem-solving was reached by Elton (1971). He categorized mathematics courses in terms of study of the subject (a) for its own sake, (b) because of relevance to other subjects and (c) as part of general education. The aims of the different courses were naturally very varied. With reference to conventional service courses, he emphasised that the needs of mathematicians and non-mathematicians may be different and for the latter, the more common approach in books of a deductive presentation was not ideal. The model-building stage was most important and should precede the analysis.

In a discussion of undergraduate difficulties in service mathematics, Tagg (1970) concluded that major problems stem from a lack of ability in algebra. He also advocated that the syllabus should include the construction of models. A great deal of practice was required to build student confidence and allow them to become familiar with the types of problem that occurred. It was suggested that an individualised instruction scheme might help to counter some of the difficulties.

Turning to the particular course under consideration, there were two important constraining features. The first was the wide range of ability and experience that the students exhibited, the second was an agreed syllabus which laid down specific topics in which some competence was required by the subject specialists and their external examiner. Consideration of the points raised in the literature led to three assumptions. Firstly, because the students were quickly exposed to the use of mathematical techniques in their main subjects, the relevance of the mathematics would be clear and motivation in studying it would be high. Secondly, not only were the subject specialists interested in basic skills but they also required students to display an ability in applying their mathematics to economics, particularly with regard to the formulation of models. Thirdly, the long established liaison between the parent department and the service lecturers would mean that few problems would be created by lack of co-ordination. Indeed the positive attitude adopted by the economics lecturers would be of considerable help in teaching the subject. The extent to which these conjectures proved to be justified is discussed in chapter 19 (section 3).

When formulating a statement of objectives, it was borne in mind that what was required was a combination of providing the necessary skills with the development of more general, problem-solving strategies. This last item was seen to involve a significant measure of confidence, the student must be willing to apply his mathematics whenever necessary. Thus, teaching mathematics to the first year undergraduates in Economics was seen to have the following purposes:

(i) to be effective in achieving a satisfactory standard of manipulative skill in a group of students displaying a wide range of ability,

(ii) to develop the students' abilities in individual and independent enquiry,

(iii) to cultivate a habit of questioning and to depart from pure exposition,

(iv) to achieve some facility in mathematical modelling where it concerned problems derived from economics,

(v) to foster motivation and confidence so that students would be led to apply mathematical skills to their study of economics.

The way in which the objectives influenced the design of the teaching scheme is shown in the following chapter.

13.5 The Research Procedure

The aims of the research were identified as follows:

(i) to establish the background to the course, including the attitudes of staff and students to mathematics in relation to the study of economics,

(ii) to design an instructional scheme which was suitable for the students involved and the course they undertook,

(iii) to carry out a formative evaluation of the instructional scheme,

(iv) to assess the effectiveness of the teaching in promoting the skills and confidence necessary to apply mathematics to the study of economics.

Thus, in addition to the theoretical study required in the design of the teaching method, the research was required to give information on three major features. It was not thought to be appropriate to adopt a procedure which allowed some form of objective comparison of the performance of students under the new instruction with those under a more conventional scheme over, say, the three preceding years. This was because no control was possible over such variables as the choice of lecturer, the ability of the groups of students or their previous experience. The possibility of using control groups over the three years of the investigation was considered to have further practical constraints. It would be necessary to provide extra resources so that experimental and control groups could be taught in parallel to minimize the effect of external factors, the parent department would be unhappy about groups being treated differently and also, the groups of students would be critical of a scheme which implied that some students had better instruction than others. In addition, a controlled experiment would make it difficult to make any changes in the instructional scheme which the formative evaluation had revealed were desirable.

The possibility of an experimental approach which envisaged some form of numerical result was rejected most strongly on the grounds of not being the appropriate way of providing the necessary data on the achievement of the research objectives. There was no satisfactory way of quantifying student confidence and motivation or their ability in problem-solving. This did however, leave the question of adequate measurement of the effectiveness of the instruction in achieving mastery of basic techniques. Should there be a programme of pre and post-tests to give numerical data on this issue? It was considered inappropriate to use specially-constructed tests for this purpose. One reason was that objective tests would convey little information about <u>why</u> a particular question was answered well or badly. Another reason was the problem of incorporating further tests in a course which already had a heavy programme of formal assessment. (Three one-hour tests plus mid-sessional and sessional examinations.) It was decided that an overall performance that was satisfactory to the parent Department in the formal assessment would be taken as an indication that this objective was achieved.

It was decided to adopt a procedure in which (as in the study of the cultural mathematics course considered previously,) the data collection would be based on student interviews, supported by the use of questionnaires. Interviews would not assess changes in students' attitudes. The objectives of the course did not reflect a concern with developing a more favourable attitude to the subject. Consequently, rather than interview the same group at regular intervals, the opportunity was taken to interview different students, so that at the end of the year, interviews would have been recorded with quite a large proportion (about one-third) of the whole student group. The interviews were to be carried out on three occasions during the year and provide the major source of data on the attitudes and opinions of the students. They would also provide some of the information required for the formative evaluation of the instructional method.

However, it was felt that significant corroborative evidence was necessary to support that obtained from the interviews. One reason for this was the fact that the interviewer was also the lecturer on the course and so there was a possibility that students might be inhibited in their comments in the interview situation. Also, in terms of the formative evaluation of the course material, it was an advantage to have information from the whole student group, so the questionnaires were to be given out and returned during tutorial periods, thus ensuring responses from all those who were present at the time. To encourage objective responses, it would be emphasised that returns were confidential and although the form had space for the respondent's name, this could be left out, if desired.

Information would also be made available by interviewing students who were well into the second year of their undergraduate studies. This was to assess their current attitude to mathematics and to discover if their subsequent experience had produced any noticeable changes in their first-year opinions.

It was felt that the attitudes of the staff in the parent Department would have a major effect on the views adopted by the students about mathematics and its role in the degree course. For this reason, it was intended to carry out informal discussions with some specialist lecturers who were closely associated with the degree, the aim being to discover what they saw as the role of mathematics, how they perceived the attitudes of the students to mathematics and how successful they judged the mathematics teaching to be.

The same format for the interviews with students was adopted in this case as for the subsidiary mathematics course, the details of which were given in 5.6. The whole discussion concerning the strengths and weaknesses of data gathering methods which was presented in chapter 5 may also be referred to in connection with this evaluation. One difference in this case was that because students were readily available, interview groups could be made up of random selections from the register list. Although the disadvantage of the investigator being the course tutor has already been mentioned, there was an advantage in the fact that by being present during the tutorial periods, there was some check on the statements of students about how these functioned and the way different groups responded.

The possibility of employing a neutral third party as interviewer was thought to present practical difficulties. It would have required someone to volunteer to commit a great deal of time and effort over a period of three years. The person concerned would require careful and detailed instructions (and possibly some practice) if the data were to be useful. If by some mischance, the interviewer had to be changed during the investigation, the problem of comparing different sets of data would be highly complicated. However, the problems for a single interviewer must not be overlooked. He may become "conditioned" by earlier responses and anticipate certain replies, with a consequent bias in the interpretation.

The following chapter discusses the design of the teaching method itself and the manner in which lectures, self-paced study and problem-solving sessions were put together. In chapters 15, 16 and 17 the results are presented which pertain to the three years of the study. This year-by-year arrangement was chosen so that each chapter contained the relevant data on the formative evaluation of the teaching scheme and any necessary changes could be identified. Chapter 18 contains the results of discussions with lecturers on the course and some second year students and chapter 19 gives the summary and conclusions.

CHAPTER 14

DESIGN OF THE INSTRUCTIONAL SCHEME

This chapter presents the background to the design and implementation of the instructional scheme. The first section looks at information concerning suitable methods of study, the second at the design of the chosen scheme itself. A third section presents the information obtained from the operation of a pilot study and a final section considers the task of organising a programme involving self-paced individual progress.

14.1 The Choice of Instructional Method

It was evident from the objectives that were formulated (see p.212) that an important feature of the teaching approach would need to be a capacity to cope with a wide range of student ability (objective (i)). It was decided that a form of self-paced study was most suitably adapted for this and that it could be designed to achieve objectives (ii) and (iii) also. There was no real evidence however, that this mode of study would be instrumental in developing problem-solving ability.

The original plan of self-paced study was devised by Keller (1968). The students studied individually, working at their own pace on course material which had been divided into "units". After studying each unit, a test was taken and progress to the next unit allowed only when a high level of mastery was demonstrated by the test performance. A scheme of revision and re-test was included for those who failed to reach the required standard. Lectures were rare and included almost as a reward for showing satisfactory progress. The large marking load generated by the tests was dealt with by "proctors" who were enlisted from the more senior student body, who were also expected to give tutorial advice in cases of difficulty.

In a discussion of individualised study methods, van der Klauw and Plomp (1974) listed some four hundred such courses in the U.S.A. (twenty-seven of which were in mathematics). They reported enthusiastic responses from teachers and students and that there was evidence of improved student performance. However, they also highlighted possible difficulties, suggesting that good material was of major importance, that in self-paced study there was always a danger of procrastination and that care was needed in assessment if mastery was to be measured satisfactorily.

B. Goldschmid and M.L. Goldschmid (1973) in a review of individualised instruction in higher education highlighted some major issues. They noted an over-riding concern with student-centred learning but were of the opinion that too much individualisation may be undesirable. Commenting on the demands on teacher and student it was noted that the teacher's role might degenerate into being a manager of learning resources, whilst the student was no longer required to be totally passive. Also a new look was often necessary at content, methods of presentation and student assessment and it might prove necessary to provide extra resources. In a later paper M.L. Goldschmid (1976) discussed recent trends and listed some remaining questions. It was asked how were individualised courses integrated within the total curriculum? Was there a contradiction between the individual pacing and the course calendar? How was creative thinking to be fostered? How were students to be prepared for various and different study methods?

A study of self-paced instruction in undergraduate science teaching was undertaken by Davies (1976 and 1977). He outlined self-study schemes which were categorized as self-paced and teacher-paced. In choosing a scheme of instruction he contended that mere training in skills was not the epitome of education. There should be concern with higher-level aims such as the solution of genuine problems. Strategies should be general and not related to specific problems. Complicated lists of behavioural objectives may not help in directing study or indicating depth. He suggested that both traditional deductive activities and more open-ended aims were required in a rounded education. He went on to advocate that the best way to achieve those aims was to adopt what he called a "hybrid" scheme in which self-pacing blocks were interleaved with over-view lectures. Self-pacing occurred only within blocks, thus the teacher retained some control over the rate of student progress.

It seemed reasonable to conclude from this that self-paced study by itself could prove rather monotonous and that certain objectives might not be so well served by this form of instruction. The idea of a mixed scheme appeared to have some advantages. It was necessary to consider how different modes should be incorporated.

In a review of the research in which lecturing and other methods of teaching were compared, Costin (1972) concluded that lecturing was not consistently worse than other modes of instruction in the acquisition of information. It compared well with discussion groups, student-centred projects and unguided reading, although programmed learning had a slight advantage. Skills such as problem-solving and the interpretation of data seemed to be aided by discussions and projects. On this evidence, he felt lecturing was not necessarily as bad as it was sometimes made out to be.

In addition to the theoretical study, it proved possible during the academic year of 1976/77 to try out a pilot scheme in which self-paced study was used to teach differential calculus and matrix algebra to a group of first year undergraduates in Business Studies. The purpose of this was to gain experience in preparing and using self-pacing material and to measure the student's enthusiasm for this mode of study. Details of this are given in 14.3.

Combining these different strands of evidence led to a decision to adopt a composite method of instruction in which different objectives would best be served by having some lectures, a major section of self-paced study and less formal class sessions in which problems involving the application of mathematics to economics were discussed and worked through. The different modes would also help those students who found one particular mode unattractive. Thus, the self-paced study would be the main vehicle for achieving objectives (i) and (ii) and some activity would be incorporated to achieve (iii). The fourth objective would be achieved by considering problems in discussion and tutorial groups and objective (v) would result if the overall programme proved to be effective.

14.2 Design of the Teaching Scheme

In the previous section, reasons were given for using an instructional method containing elements of lecturing, self-paced study and informal problem-solving sessions. The objectives of each section of the syllabus were looked at to decide which teaching method would best suit each one. Gagné and Briggs (1974) denoted five categories of learned capabilities. These were the acquisition and communication of verbal information, intellectual skills, cognitive strategies, motor skills and attitudes. A similar classification was constructed specifically with mathematics in mind by Avital and Shettleworth (1968) who classified three levels of thinking as recall, algorithmic thinking and open search. Roughly, these correspond to the combinations of the levels in Bloom's Taxonomy of Educational Objectives (1956) into knowledge, comprehension and application, analysis and synthesis.

There was a close connection between the first three categories of Gagné and Briggs and the ways of thinking of Avital and Shettleworth. It was decided that objectives in each of the three categories would best be achieved by using the modes of instruction of lecturing, self-study and tutorial work. Where a large proportion of the task was to present factual material, this was best done by lectures, whereas routine skills were developed by self-paced study and problem-solving was considered as a group or tutorial activity.

Appendix 8 gives an example of this analysis for the section of the syllabus concerned with differential calculus. Starting with the relevant section of the approved syllabus, a list of objectives was formulated and these were divided into three groups dependent on which mode of instruction was felt to be most appropriate. The programme of study was then written with these objectives in mind.

Integration of the three modes of instruction was seen as a particular problem. The lectures were basically to present facts, although it was envisaged they might provide interest by, for example, discussing interesting economic applications. However, they would not occur at precisely the right moments within the overall programme of every student, since individual rates of study would all be different. For example, some students might have progressed sufficiently to be working through a section involving the product rule for differentiation before the lecture which demonstrated this technique. The only way of avoiding this would be to give the lectures very early on in the study of each topic. This would be least effective for the slow learners, who require most support; lectures and their individual studies being so separated that one may be of little help to the other. Another important factor was the need for all students to have some knowledge of the techniques required before any attempts were made to apply them to problem-solving. Because of this, it was decided to arrange the lectures in such a way that the faster (and probably brighter) students might be ahead of the lecture presentation on the assumption that they would not find this too distracting and the self-study material was constructed with this in mind. It seemed reasonable that the proper place for the problem-solving sessions was towards the end of the study of each section, when the maximum possible amount of time had been devoted to mastery of the basic techniques. For example, the technique of finding maxima and minima is required for solving many problems in economics. This section of the differential calculus was brought forward to precede a section on the product, quotient and function of a function rules to ensure that there was the greatest possible opportunity for students to master this process. It was necessary to investigate if very slow students were to find that lack of coverage produced serious difficulty.

One other feature was developed in connection with the self-study notes. Early investigation revealed that some students saw mathematics as a process in which a set of rules and procedures was demonstrated and they were expected to go away and apply these to a collection of similar examples. It was thought to be worthwhile if a more open-ended approach was encouraged, particularly with reference to problem-solving ability. Consequently a modification of the self-paced study notes was undertaken to incorporate some sections which required students to formulate and test conjectures. This was not to suggest that the design was based on discovery method, but there were a few occasions when students were left to derive certain rules and test their assumptions on some exercises. (To give two examples of this, the notes were re-written so that the rule for writing down the derivative of x^n and the rule for multiplying two matrices were not explicitly mentioned.) The method was to present examples which illustrated the application of a certain rule. Students were expected to try and extract the rule and verify that they had done so correctly by applying it to some given exercises. (See Appendix 8.) One problem which had to be considered was the possibility of weaker students either finding the initial formulation beyond them or formulating incorrect rules. To deal with this, supplementary sheets were prepared which gave explicit statements of the rules together with some additional explanation. Students were instructed by the self-study notes to ask the tutor for a copy of the supplementary notes if they became completely stuck. Each student was given a copy of the appropriate supplementary sheet at the end of studying each topic. By adding these to their existing notes students had a convenient summary of the work for revision purposes.

14.3 A Pilot Scheme in the Development of Self-Study Material

During the academic year 1976/77, a pilot scheme was undertaken which involved the writing of self-paced study material and its trial by first year undergraduates reading for a B.A. in Business Studies. The objectives were:

(i) to gain experience in the preparation and use of self-study material,

(ii) to see how well this method was received by the students and

(iii) to determine if a satisfactory degree of learning was possible in this way.

Two topics, matrix algebra and differential calculus were chosen for this study. Together, these formed a significant part of the first year mathematics syllabus. Notes were written which contained all the material that had to be learned in the two topics. Progress tests were inserted at regular intervals throughout the work. Students were instructed to take the appropriate test when they reached a particular point in their studies. They were not allowed to proceed with their self-paced study until their performance in the progress test showed some level of mastery of the previous material. This level was a mark of between 70% and 75% depending on the individual test. Students were restrained from going ahead of the tests because the study notes for the next section were only given to a student when his progress test result was satisfactory.

At the beginning of each topic, a list of the objectives of that particular section of work was given out with the first distribution of study notes (see Appendix 7).

No time limit was set for the completion of the study of each topic. However, it was desirable that everyone should start the second topic together, so eventually a dead-line was imposed on the first topic (as far as formal classes were concerned). It became fairly clear that without some incentive to keep up, some students could get seriously behind.

The scheme was used with a group of twelve students. To measure their responses, questionnaires were administered at the end of each of the two topics. A pre-test and post-test was also used in conjunction with the study of matrix algebra, a post-test followed by a retention test with the differential calculus.

Differential calculus was studied first. In this scheme, no lectures were given and it was noticeable that the class periods were somewhat monotonous. The matrix algebra notes were supplemented by a lecture given part-way through the period of weeks devoted to this topic. This lecture was used to introduce the concept of the matrix inverse and its analogy to the reciprocal of a real number. There was no description of the rules for forming the matrix inverse, but rather a discussion of why matrix division did not exist and the relationship between matrix algebra and the solution of sets of linear equations.

Looking at the data from the questionnaire administered at the conclusion of the differential calculus, the students were unanimous in stating that self-paced study was a good way to learn and that the method should be used for other topics in the syllabus. The style, pace and layout of text and exercises was felt to be generally satisfactory. Two points which were worthy of note however, were that eight students thought more diagrams might be helpful and four students thought that it was not always clear what had to be done or learned.

This last point led to the introduction into the notes of two features. The first was to mark with a symbol †, any exercises or sections which required action by the students; the second was to introduce summaries of the objectives just covered in each section by paragraphs marked with a ■ . These were usually just before a progress test. This latter idea made the issue of lists of objectives at the beginning of the topic rather redundant and since these appeared to convey little meaning to the students, it was decided to discontinue the practice of distributing the initial lists.

The results of the post-test and a retention test showed that students had achieved a satisfactory standard in terms of performance. The post-test immediately followed the conclusion of the self-paced study, in December 1976 and the retention test was in January 1977. No warning of the tests was given beforehand. In the post-test, four students scored less than half marks, five scored between 50% and 70% and three above 85%. (The last three had all studied mathematics to A-level.) In the retention test which was a parallel test with the same format but slightly different questions from the post-test, it was interesting to note that of the eight students who sat both, five scored within one mark of their post-test score, two increased scores by 20% and 37% respectively and one dropped from 67% to 40%. The lowest mark in the post-test showed the biggest increase, from 20% to 57%, providing evidence of some continuation of studies in the interim period.

Before studying matrix algebra, a pre-test was administered to measure the extent to which the students were familiar with the subject. This required them to define a matrix by giving specific examples and asked for a demonstration of the rules for addition, multiplication and finding the inverse. The test revealed that three students had some experience of this subject and could add matrices correctly. There were no other correct attempts, all the remaining students claimed the subject was completely new.

Nine of the group attempted a post-test, three weeks after the completion of the self-paced study. There was evidence that some students had carried on studying after the end of the formal period allocated to matrix algebra, this being that five of the students gave correct solutions to questions about work in the last part of the programme and yet only one student had completed all five progress tests in the self-paced study. Counter to this, however, was the situation with the three weakest students. These had completed three of the five progress tests and did rather badly in the second part of the post-test, many questions remaining unattempted. This suggested that they did not recover very well after slipping behind in their studies.

The responses to the questionnaire which was administered after the study of matrix algebra showed that the students remained unanimous in feeling it was a good way to learn the subject. Returns were made from ten of the twelve students in the group, three rated self-paced study as very good, five as good, one as average and one was uncertain. Asked if they thought this way of learning would help them remember, nine thought it would and one was uncertain. The lecture that was included in this section of study was said by five students to be helpful and the other five were uncertain, no-one assessing it as of no value. Comparing this mode of study with a conventional programme of lectures, it was rated as good, no student was of the opinion that it was worse than lectures.

The trial had shown that administering the scheme with a group as small as twelve had presented no difficulties. There was ample time during the two one-hour periods each week to mark progress tests, hand out study notes and have time to deal with individual's problems. Responses from the students showed that eight thought adequate tutorial help was available and two felt more help was needed.

Writing the self-study notes had shown that introducing and defining terms often generated a large amount of written explanation. This gave weight to the view that if lectures were used to bring variety to the instruction, one useful function would be the dissemination of factual information.

The conclusions from the pilot scheme were summarized as follows:

(i) Self-paced study was liked by the students and was thought to be a good way to study mathematical topics.

(ii) Students showed an adequate degree of mastery of the material and were not distracted by working from photocopies of hand-written notes.

(iii) The style and pace of the study notes and the provision of progress tests had not produced any difficulties for the students.

(iv) It was reasonable to suppose that a more varied programme of study rather than pure self-pacing would make the programme more interesting.

(v) It had been possible to identify difficult sections in the self-study and to consider suitable modifications.

(vi) Learning objectives were best incorporated at points within the text, where their meaning was more evident and they were useful as a summary of the preceding section.

(vii) There could be a serious problem with procrastination in this mode of study.

14.4 The Operation of the Self-Paced Study Scheme

Bearing in mind the experience gained from the operation of the pilot scheme, the programme of study for the B.A. Economics mathematics course was prepared in the summer of 1977 and was introduced first in the academic year 1977/78. The self-paced study notes were distributed photocopies of hand-written notes. This allowed for changes to be made with relative ease and the pilot study had suggested that students were not upset by this format. Incorporated in the notes were exercises, in sections marked by the symbol †, which the students were expected to attempt as they went along. Students would assess their own answers against those given at the end of each exercise. These were clearly marked within an enclosing box, which was indicated by a large arrow in the margin. (This was the method used in the study booklets of the Continuing Mathematics

Project, Schools Council, 1976.) Students were instructed to cover up the boxes so that they did not have premature knowledge of the correct results whilst attempting the exercises. At regular intervals, the student would be asked to check his mastery of the most recent piece of work by attempting a progress test before going any further. To help him, in a paragraph marked by the symbol ■ , were listed the objectives that had just been covered. Mastery was judged by the achievement of a minimum mark of between 70% to 75%, depending on the individual test. Any student who did not reach the required standard was first asked orally if he could correct his mistakes, or he might be asked to rework a particular question (e.g. if a method was correct but there was an arithmetic blunder). In extreme cases he would be asked to go away and do some revision and to retake a test. For this reason, two editions of each progress test were always made available. Students asked the tutor for a copy of the test when they reached the appropriate point in their study. They were expected to attempt it without external aid from text or colleagues and then take their answers to the tutor for marking and comment.

The notes were divided into sections at the points where progress tests occurred. Each section of notes was to be given out only when a test was satisfactorily completed. (This procedure was later modified, see chapter 17.) The completion of a progress test was recorded by the tutor.

It was conjectured that three features would help prevent students from falling seriously behind in self-paced study. The first, (following its introduction during the first year of the study) was the distribution with each set of study notes of a timetable showing the times allocated to lectures, self-paced study and problem-solving, but most importantly the total time available for the completion of the topic. Depending on the individual topic, the student could arrange his studies within a period ranging from some four to seven weeks. There was an incentive to complete the self-paced study before the next topic was started. The second was the programme of formal assessment which involved three tests spaced throughout the year plus a mid-sessional examination. These provided reminders to the students that on the given dates, their studies would need to be up to date. The final feature, although not so compelling as the others, was the recording of the completed tests by the tutor; students would be stimulated into keeping their individual records up-to-date.

The three following chapters present the results of the investigation over three consecutive academic years; they record the attitudes expressed by the different groups of students and discuss the formative evaluation of the teaching scheme.

CHAPTER 15

THE FIRST YEAR OF THE STUDY, 1977/78

15.1 Introduction

This chapter covers the results obtained during the first year of the investigation. The mathematics syllabus can be seen in Appendix 8, and the allocation of time to the major topics is detailed in Chapter 13 (p.208). Twenty-eight students were in the first year on this occasion. We first present evidence obtained from interviews, then consider the results from questionnaires and finally look at the implications of the findings in terms of formative evaluation.

15.2 Results of Interviews

Three groups of students were interviewed, one group of four in November 1977, a second group of five in February 1978 and a final group of four in May 1978, a total of thirteen out of a class of twenty-eight. Of their mathematical backgrounds, one did not possess an O-level pass having qualified by completing a Business Studies course with no formal mathematics content; nine had gained O-level passes; one had obtained a Diploma after O-level which included some statistics; one gained a pass in Additional Mathematics and the last had an A-level pass.

Where quotations are given in the text, the source is denoted by allocating the student to one of three groups by virtue of his mathematical background. A roman numeral indicates the students as having:

(i) a weak mathematical background e.g. mature students without formal qualifications, Certificate in Secondary Education grades or entry based on courses with little or no mathematics content,

(ii) the normal entry requirement of an O-level pass grade in mathematics,

(iii) better than normal requirements, with post O-level experience of significance, e.g. Additional Mathematics at O-level, A-level pass or Certificate or Diploma with post-O-level mathematics.

This procedure is different from that adopted in the earlier study. In this case, changes in attitude were not being studied so individual students were interviewed only once. Hence it was considered more useful if students were categorized in terms of their mathematical backgrounds, since this was likely to be a strong influence on their replies during the first undergraduate year.

The first point noted in the responses was that the students were unanimous in stating that mathematics was important or very important in the study of economics. The group interviewed earliest in the year felt that some students were concerned about their ability to make satisfactory progress in mathematics. Comments on the study of mathematics were varied; some thought it to be difficult (four), others not particularly easy (one), not difficult (two) and straight-forward (one A-level entrant). One student who said he disliked the subject was also of the opinion that it was important. Three students expressed a liking or partial liking for the subject. In the first two interviews, the students were asked if they would choose to study mathematics if it were an option. Seven out of nine replied that they would, giving as a reason that it was thought to be relevant to their needs. Here were some replies when the students were asked to say how important mathematics was in the study of economics:

> "From hearing about the use made of mathematics in the second year it must be very important." (iii) (May)
>
> "Very important, because it affects your ability to pursue studies in other subjects." (iii) (May)
>
> "We see the usefulness of mathematics in studying macro and micro economics, for example graphs and so on." (i) (February)

These replies were to the question would they study mathematics as an option?

> "Yes, I think it is so important to the main subjects." (ii) (February)
>
> "I would not have chosen it as an option but I have come to realise its importance having done it as a compulsory subject." (i) (February)

One student mentioned that the numerate approach adopted by the economics lecturers helped her to a more positive attitude to mathematics.

The students were questioned about the method of instruction. There was complete agreement that it was a good way to study the subject. Some of the reasons given for this were the opportunity to go at one's own pace, the opportunity for weaker students to go over the material repeatedly and the chance to master a technique first before becoming involved in its application.

"I like this method, you do not get left behind and have to scramble to catch up." (ii) (May)
"This method is useful. I like the idea of getting knowledge of the process first (e.g. differentiation) before being concerned with applications." (i) (February)

Most were happy with the combination of lectures and self-paced study that was adopted, although two students (in May) felt that lectures could be dispensed with if the material in them was straight-forward. Problems were concerned with lack of retention, one student saying that he seemed to understand at the time but discovered difficulties later. Most students felt that this was simply a problem of forgetting. Three students were of the opinion that further revision examples might help. Examples of this kind were introduced into later sections of the self-study scheme and the students interviewed in May said that they were satisfied with the provision of exercises. However, one did comment that he felt remote from the work done at the beginning of the year. (A general revision period before the final examination was yet to come at that time.)

"(Lectures) are about the right number, they are helpful." (ii) (February)
"I like the method but I feel there are not enough exercises to emphasise the points." (ii) (February)
"The more problems you can do the better it is. The number of examples we do seems to be adequate." (ii) (May)
"I find the explanation given in the lecture before studying the notes is useful." (ii) (May)

It was asked whether falling behind in self-paced study was a problem. Three out of the five students interviewed in February acknowledged being short of time in the calculus section. Some reported knowing of cases of other students in the class falling seriously behind. It was suggested by one student that a specific number of weeks should be allocated to a topic. The group generally agreed that the issue of a timetable would be useful to help students judge their rate of progress. This scheme was adopted in all later studies. (February interview)

> "I think it (falling behind) was because I lacked basics, I had not done calculus before." (ii) (February)
>
> "I found it disturbing that people worked at different rates. I was anxious about getting behind. The time was adequate for me." (iii) (February)
>
> "It is a good idea to say how long the time is for a topic, how many pages are in the notes etc. You have no idea how fast you are going compared with your colleagues." (iii) (May)

Comments on problem-solving proved to be of interest. Examples were worked through on the black-board with the whole class, the tutor asking questions. The students were then given similar problems to work individually whilst the tutor supervised their efforts and gave advice. (In later years, this pattern was modified.)

> "It is not always clear what is required. Each question is slightly different from the previous one." (i) (February)
>
> "I can usually sort it out with a bit of thought ...

I seem to be able to spot what they want from the problem. This has improved as the year has gone on." (ii) (May)

It was clear that, particularly at the beginning of the year, some students were unhappy about their ability to solve problems. One group felt that they had not done enough of the harder examples to prepare them for the assessment tests. It was suggested that some students were reluctant to raise their individual problems in class. Asked to say why, one student put it that they may feel embarrassed if it was felt that they were wasting people's time.

"Perhaps they are reluctant to waste other people's time. Perhaps tutorials with smaller numbers might overcome this." (iii) (May)

Based on their comments about the diversity of problems, some students appeared to see problem-solving not in terms of a generalised approach but rather as a relatively small number of specific problem-related strategies. It was interesting to note that two of the four students interviewed in May said their problem-solving capabilities had improved during the year.

Certain difficulties were related to specific details of the syllabus. In the November interview, three of the four students said that the characteristic forms of linear, quadratic and cubic functions were not made clear enough. They all expressed some difficulty with natural logarithms, and the exponential function was completely new to them. Two out of the five students in February found difficulty with the function of a function rule and four of them said implicit differentiation was difficult. In the last group to be interviewed, one student was disturbed by not finding a reference in the notes to the transpose of a 2 by 2 matrix (the notes gave a 3 by 3 matrix as an example). Two students were troubled by the process of finding the turning points in a function of two variables.

In the February and May interviews, the students were asked for an assessment of their progress. Of the nine students, one replied that it was not as good as it should be, one said he did not know, the remainder were happy with their progress, three being of the opinion that it was better than they might have expected. Four students were asked to say how their progress on a more conventional course might compare with this. One replied that he thought it would be about the same, two felt they did rather better on this course and the most experienced student (A-level mathematics) thought there would be few difficulties either way but was moved to comment that the present way of studying was a good idea and he did not find it boring.

> "Not so good as it should be (present progress). Perhaps I ought to do more work in between lessons." (iii) (May)
>
> "Doing better than expected. I was dreading it when I realised how much maths was in the course, I was pleased that I could cope." (ii) (May)
>
> "When we first started I did very badly, now doing much better ... It is better the way we are doing it." (ii) (May)
>
> "I feel happy with my progress but I feel I have not done so well in the exams. The exam questions were harder." (iii) (February)

"I am making better progress than I might have hoped. I am quite pleased by this." (ii) (February)

15.3 Evidence from Questionnaires

Turning to questionnaires, information was obtained by making two distributions, one in November 1977 and one in May 1978. Twenty-four replies were received in the first instance, eleven in the second. All the students who were present at the tutorials made a response, including any interviewees. (The questionnaires may be seen in Appendix 9.) The difference in the number of responses reflects the difference in attendances on the two occasions.

The replies showed that in November all students felt mathematics to be important or very important on this course. Twenty out of twenty-four expressed some concern about their ability to make satisfactory progress in the subject. Difficulties were mainly attributed to lack of pre-requisite knowledge and also to finding that tests revealed gaps when an apparent understanding had been achieved. Most, however, thought there was enough revision in the self-paced study notes, six students expressing the opposite opinion. Exactly half in this November questionnaire thought that they were making better progress than they would on a conventional lecture course, but four felt that they were making less. Investigation revealed these four to be amongst the weaker students.

The questionnaire administered in May 1978 showed students almost equally divided between thinking the mathematics in the last half-year to have been either of average difficulty or hard. They were almost all (one exception) of the opinion that the lectures were a very useful part of the course. The time allocation to the various learning activities was said to be about right except for problem-solving, where seven thought it was adequate but four too little. When asked if more guidance was required in problem-solving, two claimed it would be an advantage, nine a big advantage. Difficulty with problem-solving was indicated by students rating its degree of difficulty as "easy" (two), "average" (four), "difficult" (three) and "very difficult" (two). (The two "easy" assessments were anonymous replies but evidence suggests that they were students with better qualifications.) When asked to compare their opinion about their mathematics with that at the beginning of the year, eight said that they were more confident, two about the same and one felt less confident.

With respect to specific items of the syllabus, the results showed that students felt that natural logarithms, change of base and use of exponential tables were difficult topics. The use of logarithms was mentioned in May by eight out of the eleven students as being difficult. In the second half of the course, the topics which emerged as most difficult involved partial differentiation and the investigation of turning points with two independent variables. It was interesting to note that although six students out of eleven in May had rated studying over the last half of the year as "hard", only two felt they had made little progress, seven felt their progress to be average and two had made good progress.

Putting together the information obtained from the two sources of interview and questionnaire, it was found that the students were very conscious of the importance of mathematics to their main area of study. This led some students to show concern about their ability to make adequate progress. The self-pacing way of studying was popular, although some less able students felt that they made little progress. The combination of lectures and self-pacing was thought to be a reasonable balance. Students saw the lectures as useful, being a good way to introduce topics. The application of mathematics to problems in economics presented some difficulties. Reasons for this were thought to stem from students finding difficulty in developing general strategies from the study of specific problems. Students suggested that more time could profitably be spent on this aspect. Nearly all students were satisfied with their progress on the course, some were of the opinion that the method of study had been of more benefit than a conventional one. There was reason to believe that the students' confidence in their mathematical ability had increased during the year. Students became less concerned with lack of prerequisite knowledge and appeared happier with applications and problem-solving as the course progressed.

15.4 Formative Aspects of the Evaluation

Students reported that they seemed to understand at the time but discovered difficulties later. It was thought that this situation might be improved by providing more practice exercises in the self-paced study. The notes were scrutinized to discover where extra exercises might be helpful. One place was the section on differential calculus, some optional extra questions were added which students could try if they felt they needed more practice before attempting the progress test.

One feature developed during the year and resulting from the early reactions, was the distribution of a time-table for the study of each section. As well as showing the allocation of time to lectures, self-paced study and problem-solving tutorials, this also gave a brief summary of the material covered in the lectures, the number of pages of self-paced study notes and the number of progress tests (see Appendix 8). If, as one student suggested, a reason for getting behind was not knowing how fast one was progressing compared with one's colleagues, it would suggest the need to set some external goals and fixing the duration for each topic would seem a useful complement to the incentive provided by regular assessment.

An interesting point emerged from a comparison of results from progress and assessment tests. The record of successfully completed progress tests showed that a minority of students completed the full programme of these tests within the time available. However, an analysis of the questions that were attempted in the assessment tests showed that many students were attempting topics which their progress test records suggested they had not studied. The number of examples of this were sufficient to suggest that students were motivated to continue their private studies after the formal class time had expired and were doing so with a fair degree of success. As an example, only four students completed the last progress test in the section on variables, functions and graphs, which was concerned with logarithmic and exponential functions. In an assessment test, half the class either were completely right or substantially correct in a graphical solution of an equation which involved the graph of $\log_e x$. A scrutiny of the scripts from interviewees suggested that this proportion was roughly the same for students with O-level backgrounds who would not have benefitted from previous knowledge of the topic.

Some amendments were made to the content of the study notes as a result of the evaluation. The section on linear and quadratic forms was expanded to give more explanation and consolidation. The work on logarithms was modified to include more exercises. Partial differentiation was given a more careful explanation, particularly the location and identification of turning points. The notes on the transpose of a matrix were enlarged to illustrate that there was a general principle involved. It was mentioned earlier that when the group of four were interviewed in May one student mentioned that he had been unable to find in the notes an example of a two-by-two transposed matrix. It was clear that the examples given on transposition had not led him to conclude that this was a general operation and independent of the size of the matrix.

This last point suggested a wider consideration be undertaken of the way in which students may be led to look for general principles. It was apparent at this stage that some students viewed mathematics in terms of the "black box" approach, an attitude which could affect their ability to solve problems. It was thought it might be helpful to encourage students to make conjectures about the procedures they were being introduced to and to test their ideas. It was for this reason that the study notes were re-designed to introduce this feature. The effectiveness of this was of major interest in the evaluation recorded in the following chapter.

Applying mathematics to problems in economics had proved rather more difficult than at first envisaged. Costin (1972) presented evidence that suggested discussion groups were an effective way of developing cognitive skills such as problem-solving. Beard (1972) advocated the value of having short lectures followed by discussions in which the students split up into groups of about ten each. Woodall (1976) used group work as a major component in teaching graph theory to undergraduates. In this last case, the students were left very much on their own in the discussion groups. It was interesting to note it was discovered that about half of the students thought there was a real benefit in working together but others preferred working on their own.

In view of these findings, it was decided that in the following year, some class sessions would be devoted to work involving small discussion groups. The class would be divided into groups of about five or six students who would discuss the solution of problems amongst themselves.

CHAPTER 16

THE SECOND YEAR OF THE STUDY 1978/79

This chapter presents the results of the second year of study of the teaching of mathematics to first-year Economics undergraduates. Results are presented from three separate interviews and supportive evidence is compiled from three questionnaires. There is a discussion of the formative evaluation of the teaching method.

16.1 Results from Interviews

Three different groups of students were interviewed, six in November 1978, four in February 1979 and two in May 1979, making a total of twelve out of a class of thirty-five. Of these twelve, three had not studied mathematics beyond O-level, four had studied either some mathematics or statistics in the sixth form but not as an examinable subject, one studied a Higher National Diploma course containing some statistics, one had passed Additional Mathematics, one obtained a Scottish Higher School Certificate including Mathematics and the remaining two held passes at A-level, one in Mathematics with Statistics, the other in Mathematics.

All those interviewed considered mathematics to be important to the study of economics. There was evidence that students were aware that mathematical techniques would be widely used in the later years of the course and there was mention that a knowledge of mathematics could be valuable to future career prospects. Nine of the interviewees said that they would study mathematics even if it were an optional subject.

"The importance to the study of micro and macro (economics) is clear. In reading (I see) the mathematical symbolism is used extensively." (ii)* (November)

"I think mathematical knowledge will be important in terms of career prospects." (ii) (November)

"I don't think it has been that apparent yet, but I can see it becoming important." (iii) (February)

Feelings about mathematics at school varied. Three students admitted to disliking it, two expressed a liking but others were mixed in their reactions. One reported being inspired by a particularly hard-working group and finding the subject reasonably interesting. Only one student was definite about enjoying the subject at school without any reservation. Even amongst some students who thought themselves reasonably successful, there was no overall feeling that it had been enjoyable.

"Depended on the topic. If I follow it at all it is very good. If I cannot do it, then it is very frustrating ... I was reasonably happy." (ii) (May)

"I liked maths but found it difficult at times, I coped with it. I quite liked it." (ii) (February)

"I never liked mathematics, always been very slow but I feel very good when I have mastered something." (iii) (November)

"I cannot say I enjoyed it but I was reasonably good at it and that gave me some satisfaction." (iii) (November)

* The roman numerals are used to denote a student whose mathematical experience was weak (i), normal for the course (ii) or strong (iii). See 15.2

Some of the students were asked if the subject was different in any way from their other subjects. All agreed that some difference was discernible, particularly its relationship to other subjects. One remarked that it was clearly the language for handling data and another mentioned the analytic rather than descriptive quality.

> "Yes, especially (different from) Social Science.* I find I can concentrate better on mathematics. It seems closer, more directly involved with the main subject." (ii) (November)

When asked to comment on mathematics being rewarding or frustrating to study, all the students except one felt that they experienced both at different times. The exception was the student who obviously enjoyed the subject and claimed to have not encountered any frustration. The students in the February and May interviews were asked if the examination posed any particular threat. One was definite that it did, one said that it posed something of a threat but not too seriously and the remaining four said that it did not. This suggested that these students were feeling relatively confident of their ability in mathematics. This was in contrast for example, to findings in the early part of the previous year, where students expressed some concern about making satisfactory progress (see 15.2 and 15.3). Typical comments on the rewards of studying mathematics were as follows:

> "It is rewarding when you can get the right answers. This happens reasonably often." (ii) (February)

* This is a reference to the other first-year subjects, Contextual Studies and The Social Framework of Britain. "Main" subjects were seen as the study of Macro- and Micro-Economics.

"When I get something I cannot understand, it is really annoying but when I understand it and have fathomed it out it is really good." (iii) (February)

and concerning the problem of passing the examination:

"I do not like the idea of an exam at the end of the year. Right up to the moment I have sat it, it is there at the back of my mind, what if I fail?" (iii) (February)

"No (problem) because I looked at many past papers and I am confident; in statistics as well." (ii) (May)

The six students interviewed in November were asked if they had experienced problems with lack of pre-requisite knowledge. The A-level student experienced no difficulty but it was interesting that he attributed this to lack of time to forget since he completed his mathematics at school. The problem of forgetting was raised by another student who gave as an example, his inability to manipulate logarithms (in dealing with growth rates). Others mentioned having to revise basic algebra and the solution of simultaneous and quadratic equations. One comment concerned a lack of experience in translating problems into mathematical expressions.

When it came to considering the method of teaching itself, the students were asked first about the lectures. All of the replies indicated that they were regarded as serving a useful purpose. The only reservation was from a better-qualified student who found them rather boring.

"They seem to supplement the self-pacing notes, very well I think. So far, most of the stuff I have done before, so it is fairly easy anyway." (iii) (February)

"Wrongly, I see them as a main part of the course. I ought to read the notes more and rely more on those. I do find it a help going through on the board." (iii) (February)

"I find the course as a whole has too many lectures. I would not be happy without any lectures at all but just one to supplement a hand-out. I find them useful to go over basic facts." (ii) (November)

"I think they could go over the difficult points in more detail ... I think the lectures could be improved if, when there is a new topic the lecturer describes what it is all about first, what use the topic is. I would not like to see lectures removed altogether." (ii) (May)

Generally the lectures were seen by the interviewees as serving two functions; one was to present basic facts, which it was felt was better than just reading about them and also to deal with the difficult points in each topic. No-one reported any difficulties with the pace and balance of the lectures, although one student did say that possibly the pace may be too brisk for weaker students and another that the lecturer tended to assume that they knew rather more than they actually did.

Some interesting replies were given when the students were asked how well the lectures combined with the self-paced study. Most students were happy with the arrangement, but two students were of the opinion that all the lectures should be given at the beginning of each topic and another felt that some form of lecture should be given each week and the students instructed which part of the study notes should then be read. This was some indication that more able students were uneasy about getting ahead of the lectures in their individual study.

"I tend to stay with the lectures and I am a bit behind, if anything, with the progress tests. I think it is all right really. I am a bit hesitant with the progress tests, I keep thinking I will do badly in this when I take it and I put it off and leave it." (iii) (February)

"I find sometimes the lectures are a bit behind. One thing is, after the lecture, you tell us how many pages you should go through the notes, then we could have the next lecture and then we could go through the notes again. The notes and the lectures would be up to date." (ii) (February)

"I cannot answer this question (balance of lectures and self-paced study) because I do not know the background of the other students. If, for example, after the lecture I go and do two hours study then you do not find any difficulty, even though you are weak at maths. It is a subject where you need practice." (ii) (May)

It was of interest to note that two of the students who were unhappy about being ahead of lectures in their self-paced study were from overseas and were probably more familiar with a carefully prescribed mode of instruction than is the case in the United Kingdom.

None of the students interviewed had experienced self-paced study previously. There was unanimous agreement that it was a good way of studying although one student was not sure if it would work as well in some other subjects.

"They (self-paced study notes) are well set out, they are very useful. Very reasonable way to study ... You could not have it in other subjects really, arts and so forth." (iii) (February)

"It is the best way, go at your own pace. You can also quickly go over what you may already know to other things. You always have some challenge in front of you." (ii) (November)

The reasons for liking this way of studying were said to be that it overcame the problem of getting lost in lectures, the notes were a constant and reliable source of reference, the capacity to study at one's own pace and the incentive of the notes always being there to study.

"I like it, a good idea, I find if I have half-an-hour to spare I sit and go through the self-pacing notes, otherwise I might watch the T.V." (iii) (February)

This last quote was interesting, not all students seemed motivated in this way. As in the previous year, comments showed that some people had difficulty keeping their work up to date. Two students reported that they had fallen behind quite noticeably at some stage, a third reported that one of her friends was in that position. Although two of the more qualified students were of the opinion that this was a problem only for the less well-motivated, one did think that there was more opportunity to fall back than with a more conventional course.

"I do not think it is (a problem) provided you are interested and want to work. If you do not want to work you have a good excuse not to do any but if you want to work you have more opportunities to catch up." (iii) (February)

"In a sense it is a disadvantage. There is more chance of this happening than with conventional lectures." (ii) (May)

The students were asked what they saw as disadvantages of a self-paced study scheme. There were no serious drawbacks mentioned apart from the one considered above. At this point one group raised the fact that in matrix algebra they preferred to be given the rules rather than find them out for themselves (February interview). In general, the students felt that the time-tables issued for each topic, showing the allocation of time to lectures, self-paced study and problem-solving were helpful in their efforts to plan their studies. However, one student expressed some doubt and when asked why, explained that in his opinion, some students would still fall behind even with a time-table.

Asked to say how effective the progress tests were as part of the self-paced study programme, everyone felt that the tests worked adequately, that the stated objectives were reflected in the tests and that they were not excessively difficult or easy. One student did suggest that summarizing the objectives before each test might be too helpful and make the tests too straight-forward.

"Yes, you are confident you can do it if you do a progress test." (iii) (November)

"If you use the tests properly they work well." (iii) (February)

This last quotation raised a point of interest. There was no formal procedure for the progress tests, students were given a copy of the appropriate test if and when they asked for it and they returned the answers to the tutor for marking. An effort was made to ensure that the purpose of the tests was understood by the student and that there was no gain, except in self-deception, by attempting to distort the results. Nevertheless, the opportunity was present and one group of students was asked to comment on differences between performances in progress tests and (formal) assessment tests. Three of the four replies suggested that there may be some students who were tempted to seek aid although no-one volunteered any knowledge of actual instances. The fourth reply pointed out that the two types of test were quite different.

> "The difference might be the tension, you take the progress tests in class but in the assessment tests you have the tension and the time difficulty, you might be careless." (ii) (February)

There were discernible reasons why results in formal and informal tests should appear different. It was also difficult to appreciate why students should want to enhance progress test performance, unless it was a case of "keeping up with the Jones". Some comment on this is given in the following chapter (see p.293).

Turning to the part of the course concerned with problem-solving, five students out of the twelve interviewed felt that there was some difficulty in applying mathematics to problems in economics. Two particularly mentioned the difficulty of formulation, another said that the main problem was understanding the question. Three students claimed no great difficulty, one saying that his main problem was avoiding silly errors.

> "I was a bit scared off at the start when I saw all the

variations in questions, but when you realise there are only a few laws really, it is all right." (iii) (February)

"I have difficulty in the mathematical formulation. Maybe it is because we are at the beginning." (iii) (November)

This year, students had been encouraged to work through problems in small groups and to discuss their own problems within the group. Five people were positive that this working in groups had helped them, one was undecided and four replied that it was of little or no help. The reasons given for being in favour were that it took the emphasis off the slow ones, people could ask questions without embarrassment, you could be prompted by other people's ideas and they were encouraged to help one another. Students who did not see any great advantage said that they preferred to ask friends for assistance.

"It is a good idea, takes the emphasis off the slow ones, they can ask other people in the group." (iii) (November)

"Yes, because sometimes you just cannot see one particular piece and somebody can see it and they can help you ... depends who you are sitting with. One group did not communicate, another group they communicated so much you could not do the question." (iii) (February)

"I do not really find it useful. I do ask people, but working in groups in class I do not think is a good idea." (ii) (February)

"I think if that was the case (you could not see the point) you would ask someone else anyway. I do not really see much point in working in groups." (iii) (February)

As can be seen, the interviewees were almost equally divided on this issue. There appeared to be two major factors which influenced students' responses; one was the personality of the individual, whether he or she communicated easily with colleagues, the second being previous educational experience. Those students more familiar with a formal atmosphere were uneasy with this approach and felt that communicating with others was perhaps being unfair by involving other people's efforts in their own work.

At each of the interviews, the students were asked about those topics of the syllabus they had recently been studying. Thus, the first group, in November was asked about variables, functions and graphs. No-one found the earlier part of the work, up to graphical solution of equations to be of any great difficulty. Five out of the six students said that logarithmic and exponential functions were completely new and three found them difficult.

The February interview was concerned with the differential calculus and the first part of the matrix algebra. Those who had studied calculus at school found it straight-forward (two). Difficulties were mentioned with remembering the formulae for product and quotient rules (twice), using the function of a function rule (once) and implicit differentiation (once). (There were four students in this group.) When asked about the matrix algebra they had studied so far, only one of the four had found it difficult although two commented on the complicated rules and cited the product of matrices as a source of error.

The two students interviewed in May gave their opinions about matrix algebra, partial differentiation and max/min problems in two independent variables. With reference to the matrix algebra, one declared that he found it easy, the main problem being to avoid arithmetic slips. His colleague declared it to be "in the middle", neither difficult nor particularly easy. Asked to comment on partial differentiation, the first thought that if you had studied differentiation well, the extension to two variables was quite straight-forward. He also felt that many students had found the section on constrained optimisation slightly easier. The other student was not so happy with this topic and confessed to getting muddled when trying to apply the rules for finding a maximum or minimum. He said that it was a question of remembering the rules and going step-by-step.

One point of interest was the students' reaction to the lecture introducing partial differentiation. It had been found that trying to write study notes involving three-dimensional diagrams to develop the idea of rates of change in different directions was rather difficult and not very convincing. It was thought that a lecture in which a solid three-dimensional model could be used was the best way of presenting this concept. Asked to say if this lecture was effective, one student said that it explained things adequately and the other thought that he had understood what was involved. The former added that he thought some students found difficulty because they did not do enough practice.

The matrix algebra notes were the first section of the work which had been redesigned to encourage students to formulate rules for themselves. Six students were asked how they viewed this experience. Three did not like the idea, two saying they preferred to be given rules, the other that he did not like the doubt that he might be wrong. Of the other three students, two were of the opinion that it had some value, the third remained neutral.

> "I find it disturbing, I do not particularly like it. I like someone to say that is it or that is wrong. (Is discovering for yourself satisfying?) Not really. It does not really help in remembering ... There is always a nagging doubt." (iii) (February)
>
> "I suppose I do find it satisfying. I can think of two instances and they seemed all right. Yes, I saw the multiplication rule." (iii) (February)

It was of interest to note that the three students who disliked this approach were the same group who, when discussing self-paced study had said that they preferred being told what to do. There was also some indication that some students who did not particularly like it, perceived the value of the experience (see p.260).

Finally, all the students were asked how they rated their progress up to the time of the interview. Their reactions were naturally varied and some had experienced more of the course than others. Overall, five students were of the opinion that they were making good progress, four were neutral or undecided and three were unhappy with their progress. In the November interview, the more mathematically qualified student viewed it as rather boring, another said that he thought at first it was all very easy and fell badly behind. One had not done as well as he anticipated, one had found a lot of it new and was finding it difficult to keep up. The two remaining had done reasonably well they thought. It was of interest that one student added that she had done better by this method than she would with a conventional approach.

In February, one student felt he had not done as well as he had hoped, one had got good marks and felt happy about his progress and the other two seemed to feel that progress was adequate without being spectacularly good. Commenting, one thought there was little he could do to improve matters, another hinted that it might be better if he worked harder. The biggest contrast was the reactions of the two students in May. One had found the work interesting and enjoyable and was disappointed only in that his results were marred by silly mistakes, the other said that he had put up with it, it was not particularly enjoyable, he had to do it. Although some students were obviously disappointed with their progress, no-one suggested that the method of instruction had been detrimental, whereas one student suggested that there had been a positive gain.

16.2 Evidence from Questionnaires

Turning to the questionnaires, these were administered in December 1978, February 1979 and May 1979, in each case within a week or two after the interviews took place. In December there were twenty-eight replies. Twenty thought a knowledge of mathematics was "very important" and the other eight rated it as "important". Twenty-four replied that they were concerned about their ability to make satisfactory progress. Although eight felt some frustration in studying this subject, fifteen were of the opinion that it was satisfying. Nearly all the students were willing to say that the discussion groups were helpful in problem-solving, three being neutral and one saying they were not helpful. Asked if there was enough revision before the progress tests, the great majority (nineteen) felt that it was about right, five that there was too little and two too much. The students' ratings of how well they had progressed compared with a more conventional course were less progress (five), about the same (nine) and more progress (twelve). It was noticed that all five students who thought they made less progress were from the least able sector of the class and more able students tended to feel they had made more progress. These results were similar to those obtained in the previous year (see Appendix 9 for summary).

Twenty-six students submitted replies to the questionnaire in February 1979. Some of the findings are given in Table 2 for easier reference. It can be seen that the study of mathematics over the previous half year was generally felt to have been average to difficult. Self-paced study was viewed by a majority as being a good method of learning. Keeping up with the work was not thought to be a source of difficulty by most students, although one did feel it was very difficult. Concern over the applications of mathematics was reflected in the seventeen students who rated this as "difficult" or "very difficult" and in the majority who thought that too little time was devoted to this aspect. This was the only activity where the majority felt the balance was not satisfactory.

The responses concerning formulating one's own rules were particularly interesting. Table 2 shows the group evenly divided about its difficulty and nine said they liked the exercise but sixteen replied that they did not. However, when they rated it as a rewarding experience, the replies were "little" (six), "some reward" (twenty) and "very rewarding" (one). This showed that many who disliked it appreciated that the experience had some real value. There are references elsewhere to occasions when similar attitudes were revealed. (See, for example p.290.)

N = 26	v.poor	poor	average	good	v.good
View of self-paced study	0	1	7	13	5
Progress in mathematics	1	5	14	5	1
	v.difficult	**difficult**	**average**	**easy**	**v.easy**
The study of mathematics	1	10	13	1	1
Keeping up in self-paced study	1	8	12	4	1
Applying maths to problems	6	11	8	1	0
Formulating own rules	1	7	10	7	1
	v.little	**little**	**about right**	**too much**	**much too much**
Time spent on problem-solving	2	15	8	1	0

Some Results from the February Questionnaire

Table 2

In May, there were twenty-seven replies to the questionnaire. Table 3 gives some of the more notable responses. It can be seen that although the study of mathematics in recent months was rated generally as average or difficult, the majority thought their progress had been average to good. Self-paced study was still looked upon favourably and keeping up with the work was not held by most students to raise any serious difficulty.

Problem solving was thought by most to be difficult, but overall not quite so difficult as previously and more students than in February were of the opinion that the time allocation was adequate. Students held the subjective view that their mathematical ability had improved during the year, only two of the weaker students expressing a contrary view. This agreed with results from the previous year and was taken as evidence that the course had been effective in raising student confidence.

16.3 Formative Aspects of the Evaluation

In this section, the reactions to the course of study are considered together with the changes in the study programme which might be contemplated as a result of student comment. The first part of the programme involving variables, functions and graphs, was considered to be not too difficult by the group interviewed in November 1978, apart from the section concerned with natural logarithms, exponential functions and change of base. That this was a source of concern was confirmed by the results of the questionnaire in the same month. Of the six interviewees, five said that these aspects were completely new to them and three of these found it difficult. The questionnaire showed that thirteen students out of twenty-eight were

N = 27	v.poor	poor	average	good	v.good
View of self-paced study	0	2	9	11	5
Progress in mathematics	0	7	9	10	1
Ability compared with earlier	0	2	9	11	5
	v.difficult	difficult	average	easy	v.easy
The study of mathematics	4	10	10	3	0
Keeping up in self-paced study	1	5	17	3	1
Applying maths to problems	4	14	7	2	0
	v.little	little	about right	too much	much too much
Time spent on problem-solving	0	10	17	0	0

Some Results from the May Questionnaire

Table 3

of the opinion that exponential functions and natural logarithms were difficult or very difficult and eleven held this opinion for changing the base of the logarithm.

Further evidence came from the scripts submitted for the first assessment test, set in November 1978. In the question requiring a change of base of a logarithm, sixteen students out of a class of thirty-five either made no attempt or made serious errors. It was concluded from this that some modification of the self-paced study notes on this particular section might be an advantage. The notes were modified to include more detailed explanation with examples and practice exercises.

The notes were also reviewed with the intention of introducing into all sections some exercises involving the formulation and testing of conjectures. It was hoped to achieve a balance so that the student was neither faced with the task of simply learning rules by rote nor left in a situation where progress was so slow or devious that he was tempted to abandon the attempt.

Another notable point emerged from an analysis of assessment test scripts. At the beginning of the year, the students had been given a diagnostic test to provide information about their pre-requisite knowledge. This was in response to comments about students finding problems associated with forgetting work done at O-level. This short test included examples of such things as solving linear equations, solving quadratic equations, laws of indices, evaluating quadratic functions for given values of the variable and sketching graphs. Not unnaturally, some gaps were disclosed, mainly due to the time lapse since previous mathematical studies. The important feature was that the student then had precise knowledge of what he required to revise and could do something about it. (In addition a tutor was provided to conduct remedial tutorials involving topics such as the solution of equations.) Despite this, ten students in the first assessment test were unable to make a successful attempt at solving a quadratic equation. Therefore, it was decided that students would require to be given more specific guidance if they were to take effective action. For example, it was insufficient to make available any remedial tutorials, it would appear an advantage to advise individual students which tutorials to go to. It was possible that the less able students had the most difficulty in organising such things for themselves.

The evidence suggested that the topic of the syllabus which gave most cause for concern was the differential calculus in one variable. Of the four students interviewed in February 1979, those two with post O-level experience were reasonably happy but the others described various topics as "not too bad", "not too happy" or "some difficulty here". When the results of the questionnaire were looked at, of the twenty-six replies, twelve rated implicit differentiation as "hard" or "very hard", whilst ten rated it as being of average difficulty. Very similar figures were recorded when students were asked to rate solving economic problems involving maxima and minima.

Generally, the data indicated that with all topics, most difficulty arose with problem-solving. Since a major objective was to provide these students with an ability to use mathematics as a tool in economics, this was an important consideration. Although the difference between the type of problem and more frequently encountered practice exercises did not appear so profound to the experienced eye, there was indication that formulating the mathematical problem and knowing what to do to get a satisfactory solution presented some difficulties for many students. (Typical problems can be seen in Appendix 8.) Of those interviewed, five students expressed difficulties with this aspect. When asked for reasons, replies included "understanding the question", "the problem of mathematical formulation", "which method to use" and "the degree of variation in the questions". There was some indication that certain students viewed the process as one of learning prescribed methods for prescribed problems. (Once, in reply to the question how could the lectures be improved, a students said "Do all possible examples in the lectures". Also see p.237.)

Another aspect of problem-solving was the student reaction to working in small groups. Throughout the year, there were occasions when the tutor would present the class with a problem, or problems (chosen from a duplicated hand-out). The students were divided into small groups and each group would attempt to produce a solution. Sometimes they would be told that at the end of the session one group would be asked to provide a spokesman to present an account of their work to the whole class. The tutor did not attempt to take an active part with any one group, but remained as an observer. Groups could ask him for advice if they thought it necessary but otherwise he did not try to influence the discussions.

Although the majority of students were of the opinion that group working was useful (the opportunity to ask questions was a typical reason for this), the questionnaire results showed that some students were firm in their opinion that it was of little benefit (in February, six and in May ten students rated it as of little or very little use, whereas fifteen and fourteen respectively held it to be useful or very useful). Several students commented that they stuck closely to their own friends and one remarked that "someone will volunteer to help only if they know you".

It was evident that there would be an advantage in devoting as much time to problem-solving as possible, even if this meant keeping students to a fairly tight schedule in the self-paced study. There was little extra time within the session that could be devoted say, to differential calculus. What could be done however, was to remind the students how important it was to keep up their study and not to fall into the trap of getting seriously behind. Group discussions would be continued. There was enough evidence to suggest that they were a useful experience for many students although some thought they were of little value. No-one suggested that they were the worse for the experience of working in groups. One positive advantage was that it fostered the idea that mathematics involved more open-ended procedures than might have been evident from the previous experiences of some students.

CHAPTER 17

THE THIRD YEAR OF THE STUDY 1979/80

This chapter follows the previously adopted pattern in presenting the results of the research for the third and last year of the investigation. The results concern the interviews, carried out on three occasions, the data from three questionnaires and finally presents the conclusions of the formative evaluation.

17.1 Results from Interviews

The three groups of interviewees were seen in November, February and May of that year and comprised four, three and four students respectively out of a total class of twenty-four. Six of these students gained a pass grade in O-level mathematics, one had a grade two in C.S.E., two had passes at A-level. Of the remaining two, one had studied abroad to a level which appeared to be between O and A level and the other had studied a foundation course in accountancy which he said had something in common with his present course. Their reactions to school experiences were mixed, with a slight majority being favourably inclined. Six students said that they enjoyed some or all of it, the others being neutral except two who said that they "did not like it really". This was notable because one of these was an A-level student.

All the students said that mathematics was important in their studies. The subject was seen very much in terms of being useful to the study of economics, in fact the majority said that they would have chosen mathematics as an option because it was so necessary to understanding various aspects of economics. There was no support for the idea that mathematics was worth studying for its own sake.

> "Very important (mathematics). You need the basic mathematics to get you through the course. We use maths in macro (economics) with all the different models. You need the maths, more so than the other subjects I think." (ii)* (November)
>
> "I agree it is coming into micro (economics) a lot, simultaneous equations and everything and it is coming into macro (economics) in the national income statistics ... Myself, just C.S.E. maths, if they had launched me straight into the economics without any maths at all it would be a bit sticky." (i) (February)
>
> "When I thought of doing economics I did not know that maths would play such an important part, but it is important." (iii) (May)

A typical reply to the question if any mathematics were worth studying for its own sake was:

> "(I) cannot think of anything off-hand. It is useful in everyday life. It is not a thing I personally would study, like Pythagoras and that sort of thing." (ii) (February)

Nearly all the students who were asked the question felt that mathematics was different from their other subjects in some way. Their reasons were based on a recognition that the way of

* The roman figures illustrate the mathematical background of the student i.e. (i) is weaker than average, (ii) is normal for the course, (iii) is a strong entrant. (See 15.2)

reasoning in mathematics was significantly different. This was mentioned as a factor in the degree of frustration that some students encountered. Of the four students who were asked to comment on the subject being particularly rewarding or frustrating, one, a better qualified student, found it rewarding since he had expected the course to be tougher and was pleased with his progress, two found a fair amount of frustration because they found the course difficult whilst the fourth experienced both elements at different times.

> "It depends whether you can do it or not. I find it quite difficult to translate the question into mathematical form. If I cannot do it, it is frustrating." (ii) (May)
>
> "It is different because with most other subjects you can perhaps get by with an awful lot of common sense but with maths you cannot, you either know how to do it or you do not. That is the main difference." (ii) (November)
>
> "It seems to me as though we are being taught down a narrow channel instead of branching off all the time. Obviously with two hours a week we could not branch out, we are being guided rather than being taught. We have to take many things for granted." (i) (February)

This last quotation was interesting in saying that the mode of instruction was guidance rather than teaching. The general tone of the answers was in keeping with the data obtained in previous years. This was true of the responses to a question about feelings concerning the final examination. Comments from the February and May interviews showed that only one student felt any real concern, the other students mostly showing a natural trepidation without it being too over-shadowing.

> "One worries about all examinations, but with maths and stats I think I worked out that I needed 35% to pass so I am not really too worried." (iii) (May)

When the four students in November were asked if they had noticed any lack of pre-requisite knowledge, one thought not and three said they had. Algebraic manipulation was cited as one problem, quadratic equations were mentioned twice and manipulation of logarithms once. This was in line with the areas of concern noted earlier and would have been expected from an analysis of the diagnostic test.

Turning to the course itself, the purpose of the lectures was generally seen by students as the introduction to the main points of each topic so that the self-paced study could follow. Nearly all of those interviewed felt that the lectures were useful and would be unhappy with no lectures at all. The one exception was a student who said that he was often late to the lectures and so could not follow them. However, he liked the notes very much and could follow them without the lectures. (This student had A-level mathematics.) Another student thought lectures were useful to go through the applications and did not think them useful for factual information and preferred more tutorial time. One interesting comment was that lectures were necessary to establish a good relationship between lecturer and students.

> "I think they help to introduce you to the subject areas. There is some duplication, but I think you have got to have the lectures so that we have some sort of relationship with the lecturer, I think that

is important." (iii) (May)

"... The lectures will give us simpler guidelines, the more complicated examples come later, I think the lectures do this. I think they fit in well, cannot complain about them really ... I think it a good idea to have the lectures spaced in between the self-study. If you were on your own you might lag behind. With the lectures, you know where you should be." (i) (February)

"The lectures ought to prepare you for dealing with the self-pacing notes, so that you can go away and work by yourself on the notes. I have found this has helped me because my maths is sticky and I have been able to go home and spend a fair amount of time by myself. From my point of view, yes, the lectures are doing this." (iii) (November)

"Given the study notes that we have got, the lectures are not necessary. Usually I am a bit behind." (ii) (May)

Generally, the number of lectures was thought to be about right. Weaker students tended to favour rather more lectures, stronger students less. No-one was of the opinion that the course should be given entirely by lectures.

"With self-pacing you are using the lecture material and using it yourself. Sometimes in lectures you do not take it in half the time, just forget about it. With the notes you use it." (ii) (November)

The majority of the students who were questioned rated self-paced study as a good way of learning the subject.

Two students who were guarded in their approval pointed specifically to the problem of falling behind. Again, these were amongst the weaker members of the group. In the November interviews it was interesting to note that one person was uneasy about going at a pace that was faster than the lectures although he appeared to cope easily with the work. In the final interviews, in May, the students were specifically asked to name any disadvantages of this mode of study. They could not think of any apart from one student who said that early on in the course he had fallen behind but an assessment test had given him a shock and he promptly caught up.

> "Not really (any disadvantage), I think it is probably the best mode of study. It gives me more structure, you know where you are going, you know where you are supposed to have come from so if you do fall behind you can catch up." (iii) (May)
>
> "It is a good way of studying, it is a new way for me. I have to refer to other books as well as the handouts you give .... It should be good (way of learning). I find it difficult to keep up. It has been so many years since I studied (maths). I find it difficult with new topics." (ii) (February)
>
> "Very well, I find I learn a lot more doing it this method than I did by any other method. A useful aspect is I can work on the self-pacing notes and keep up with the rest of the class because I do tend to do a lot of maths at the weekend so that I know I am keeping up with everybody. That is the beauty of this method. I do this and it helps me understand it better if I go over it by myself." (ii) (November)

"It is rather slow. As I said, it is better that we have complete lectures on the subject first and then do our own self-study. (Do you fall behind?) No, I mean our progress is slow because of this way of study. The notes are easy to read, very comprehensive. The method is new to me." (iii) (November)

In the early planning, it was thought that a possible drawback might be that the self-paced approach was not too much help to the slow learners in preparation for dealing with applications. (p.224.) Judging from the above and similar results obtained in previous years (p.237 and p.252) there did not appear to be any general concern about this. It was apparent that some weaker students would not be so well-equipped in basic techniques when faced with applications, but the experience of the tutorials suggested that this was no more serious than on a more lecture-based course.

With reference to falling behind, it was worth looking at the students' reactions to the time-table issued at the beginning of each topic showing how the time was to be allocated to the various modes of study. Of the seven students asked, three said it was useful and four said that they made no real use of it. The replies may have been influenced by their ability in keeping up with the work-load.

"Yes (it is a help) but if it was a pure lecture course they would not have the misfortune to fall behind unless you missed the lectures. With self-pacing you are more likely to lose track." (ii) (February)

This last quotation was notable in that it displayed a belief that experiencing the lectures was in itself all that was required. This was in contrast to a remark of a colleague quoted earlier who thought the lectures were not so necessary (p.272). Compared with the previous year, the interviewees were not so enthusiastic about the value of time-tables, but evidence from the February questionnaires suggested that a greater proportion than previously viewed them as very useful, a substantial majority seeing them as being of "some use" or "very useful".

When it came to assessing the value of trying to formulate their own rules and test their conjectures, all the students saw some benefit in this although some also said that they found it difficult. No-one was of the opinion that it was better left out. It was noteworthy that several students were of the opinion that they would remember better as a result of this exercise. Comparing these results with those obtained in the previous year, this group were better disposed to this type of approach. There were no cases, as previously, of people who found the experience disturbing.

> "It is very useful. Then again ... it is helpful. After we have done that, you give us the supplementary notes so that we can check." (ii) (November)
>
> "Oh yes, helps us to build up our creativity. Hypotheses can be tested and proved to be correct. Useful, not only in mathematics. It helps us to understand and interpret information in other subjects as well." (iii) (November)
>
> "It obviously makes you think, it is going to give you a greater understanding rather than, as I said, just learning parrot-fashion. Sometimes it is a bit of a struggle but I can see the relationships in the end.

> ... yes, I think if the answers to those kind of questions were on the sheet, then I personally would be tempted to see what the answer was. If you have to go and get another sheet you would have another think about it." (ii) (February)

Two groups were asked specifically if they thought the progress tests had a useful function. Of the seven students questioned, all were unanimous that the progress tests were helpful for demonstrating mastery of the objectives or pinpointing the gaps. One, however, did mention that some students used their notes for reference in the tests and so defeated their objectives. (See p.293.)

> "Yes, I would make one qualification to that, I do not think you should be allowed to refer to your notes while doing a progress test." (ii) (November)
>
> "If you do look you know yourself you are not doing it right. You know it will be a false mark at the end. But if you are totally stuck and there is nobody around, you think I should not have done, but you do. Sometimes, in a way, it helps because it sinks in." (ii) (November)
>
> "It is a guide to our personal progress so it is up to the individual whether he wants to use it as a guide or not. Overall I think it is working reasonably well." (iii) (November)
>
> "An excellent idea because they expose all your gaps." (ii) (February)

Turning to the usefulness or otherwise of working in groups, the students expressed opinions very similar to those of previous years, six students found it of positive help, one was neutral whilst four found it of no real benefit. Again, the response seemed to depend very much on the individual's preference for working on his own. Students from overseas in particular were more willing to say that they preferred to work on their own and that any problems they would discuss with their personal friends. Rather weak students still gave the impression that they were uneasy about raising problems, as in the first quotation of the following examples.

> "I find it (solving problems) difficult. I do not think working in groups helps. That is just for myself, because I think I tend to be lazy. Perhaps it is uncertainty. It can be useful if everybody contributes." (i) (February)
>
> "I do not think I can agree with that (groups are helpful). It has its advantages but it also had disadvantages. You can see other people's point of view but then, when you are in groups, dependent on the group you can get people who let someone else do all the work for them. It could lead you astray, listening to other people's methods. I would not say it is <u>very</u> useful but it has its advantages." (ii) (November)
>
> "Definitely useful, not only a good way of arriving at the answer but also to learn to be critical. It is stimulating, much better than working out examples on the blackboard." (iii) (November)

The replies to questions during the interviews about recent topics in the syllabus also followed an expected pattern. Because of lack of time, the November group were not asked specific questions about topics in the first part of the course.

However, their responses to more general questions showed that they were all satisfied with their progress at that point. The one drawback they were unanimous in stating was that they felt too little time had been spent on the applications to economic problems.

> "Applications to economics, I do not think we spend enough time ... By comparison with all the methods by which I have learned maths I would say this one is the best ... I am satisfied with my progress. I do not really consider I would have done as well by any other method." (ii) (November)

During the February interview, the students were asked questions about various aspects of the differential calculus part of the course. All three students were happy with the development of the rule for finding the rate of change, although one mentioned problems with the algebraic manipulation. The product and quotient rules posed no problems apart from remembering the formulae but one student said that he could not cope with the function of a function technique. Finding maxima and minima was also seen as relatively straightforward but two of the three students found difficulty with implicit differentiation.

In May, the four students were asked about that section of the course involving partial differentiation. Three of the four found the process quite straightforward and the other said that he was badly behind on this one and was busy catching up. When it came to applying the technique to finding unconstrained optima in two variables, three students were happy and one said that he found the rules confusing. It was interesting to note that this was not the same student who said he was slow in mastering the basic differentiation rules. Constrained optimisation (using the Lagrange multiplier method) was too new for two people to offer any comment. Of the other two, one thought it was easier than trying to follow the unconstrained case, the other was reasonably happy but said he was making too many errors when solving the equations. There was a general feeling that if students had mastered the techniques in one variable, it made this part of the work relatively easy.

> "In the last assessment (test) it was the only thing I could do. Basically it goes back, it was in two variables but very similar to what we did in one variable. I was able to apply the same knowledge and get more or less the right answer." (iii) (May)

When the students were asked about their progress on the course, the majority were happy about it, of the eleven students interviewed, only three students were not satisfied. It was considered important that students should feel that they were making progress, whatever may be the outcome of the assessment, since one of the objectives was to try and produce some measure of confidence in their ability to think mathematically and apply their skills. In the November group, all the four students expressed satisfaction with their progress although one girl thought that her progress may not be so good when compared to others. Two of the three interviewed in February felt that their progress was not satisfactory. One had a poor mathematical background and although he thought he was improving he knew there was still a long way to go. Finally in the May group, three of the four were satisfied with their progress. It was noteworthy that no-one held the opinion that their achievement could have been any better by any other method of instruction.

> "Not getting on very well at all. With my general mathematical background I would have done the same I think, whatever type of instruction. I seem to be understanding things slightly quicker (now) but there are still great gaps in my knowledge from before."
> (i) (February)
> "Yes I am happy (with my progress). No, I would not prefer another teaching method, I am happy enough as it is." (ii) (May)

17.2 Evidence from Questionnaires

Three questionnaires were administered during the year, in November, February and May. (See Appendix 9.)

In November, eighteen replies were obtained. The results showed that thirteen students thought a sound knowledge of mathematics very important with respect to an economics course, four of some importance and one of little importance. Fifteen were concerned about their ability to progress satisfactorily, one had no particular feeling and one was not concerned. Replies about their feelings concerning the study of mathematics showed that three were neutral, nine found it satisfying or very satisfying whilst six generally found it frustrating or very frustrating. The great majority of the students reported that the amount of revision before the progress tests was about right and that the paragraphs setting out and summarizing the objectives were useful, only three being of the opinion that this was not the case and one was undecided. A mixed reaction was evident concerning the value of group discussions in solving problems. Two replies said they were "very unhelpful", one "unhelpful", four were "neutral", six said they were "helpful" and five "very helpful". The ratings given to the various topics in the part of the course studied so far showed that, apart from logarithmic and exponential functions, this part of the syllabus was, on the whole, considered to be fairly straightforward. Asked to say how they rated their progress compared with a more conventional course of lectures, they replied that it was thought to be "worse" (two), "about the same" (five) and "better" (eleven). The replies in the questionnaire given by students who had been interviewed earlier were compared with the responses on that occasion. They were seen to be consistent in all four cases, except for one reply. In the interview, this student said that group discussions had some good points, although he did not find them particularly useful; in the questionnaire he rated them as "very unhelpful".

Nineteen replies were collected to the February questionnaire. A summary of some of the major points is given in Table 4. The students were almost equally divided in rating the difficulty of studying mathematics, but it was also noted that sixteen of the group thought the lectures were useful or very useful and thirteen thought the lectures and self-paced study combination was about right. Self-paced study was generally viewed favourably and keeping up with the work was not rated as difficult. Compared with November, there was less enthusiasm for working in groups and the class divided almost equally in rating the discovery of their own rules as difficult or average. However, it was noted that nine said they liked the idea, one was neutral and six did not like it, but when

N = 19	v.poor	poor	average	good	v.good
View of self-paced study	2	0	7	8	2
Progress in mathematics	1	2	11	4	1
	v.difficult	difficult	average	easy	v.easy
The study of mathematics	1	4	10	3	1
Keeping up in self-paced study	1	1	9	5	3
Applying maths to problems	2	10	5	1	0
Formulating own rules	0	8	8	3	0
	v.little	little	about right	too much	much too much
Time spent on problem-solving	3	10	6	0	0

Some Results from the February Questionnaire

Table 4

asked if it was rewarding, the replies were "little reward" (one), "some reward" (ten) and "very rewarding" (six).

The difficulties encountered with problem-solving are again reflected in the returns, the response to the time allocation being contrary to the replies for lectures and self-paced study, where a considerable majority thought it to be about right.

Of the seventeen replies to the May questionnaire, Table 5 shows that a majority found the mathematics to be of average difficulty. On the whole, students also felt their progress to be average, but compared with earlier, a substantial number felt that their mathematical ability had improved. Again the balance of lectures and self-paced study was thought to be about right. When the students were asked to rate the usefulness of group discussions in problem-solving, the replies were "little use" (eight), "average" (four) and "useful" (five).

The results as previously, show that, on the whole, students were confident that their ability had improved during the year, even if they viewed their progress as being no more than average. The time-tables issued with each set of study notes were thought to be of benefit by all but three of the seventeen students. This distribution of opinion was very similar to that expressed in February and was rather more favourable than that expressed in the previous year, when in May, eight out of twenty-seven rated them as of "little use".

Comparing these results with those obtained in the two previous years showed that the attitudes and opinions

N = 17	v.poor	poor	average	good	v.good
View of self-paced study	0	0	3	8	6
Progress in mathematics	0	3	10	4	0
Ability compared with earlier	0	0	4	10	2
	v.difficult	difficult	average	easy	v.easy
The study of mathematics	1	3	9	4	0
Keeping up in self-paced study	0	0	10	5	1
Applying mathematics to problems	3	9	2	3	0
	v.little	little	about right	too much	much too much
Time allocated to problem-solving	1	8	7	0	1

Some Results from the May Questionnaire

Table 5

expressed by the students, both in interviews and questionnaires were quite consistent. The same sources of difficulty were mentioned by the students and the features which were thought to be of benefit were generally the same throughout the three years. There was no doubt that students saw mathematics as important to any study of economics and were concerned to master the techniques involved. There was overall approval of the mode of instruction, involving a significant amount of self-paced study. One area where responses were mixed was in applying mathematical techniques to the solution of problems couched in economic terms and particularly the role in this of discussion groups. This source of student concern is considered again in the following section.

17.3 Formative Aspects of the Evaluation

The form of the course was substantially the same this year as in the previous year, the one exception was some modification of the notes to involve more devising of rules. For this reason, the opportunity is taken to compare the students' reactions in the two years on four issues. These were the responses to the diagnostic test, the degree of difficulty experienced with individual topics, the reactions to forming and testing rules and the degree of difficulty experienced with problem-solving.

One point revealed during the previous year was that having used a short diagnostic test to pin-point areas where some students needed revision, the evidence of subsequent assessment suggested that a number had not made good use of the available means of sorting out their individual difficulties.

It was noted that in the November questionnaire, when asked to choose of seven different sources of difficulty, the one ticked by the largest number of students was a "lack of pre-requisite knowledge". This year, a series of tutorials was arranged to cover the basic pre-requisite knowledge, and based on their results in the diagnostic test, students were advised individually which tutorials would be of benefit. This produced a far better response than when students were left to their own initiative. (The diagnostic test can be seen in Appendix 8.)

The questionnaires gave the general picture of the students' ratings of how difficult various topics were thought to be. In Table 6, below, there is a list of those topics which were rated as most difficult, other topics being rated as, at most, average difficulty by the majority. In comparing the figures year-by-year, it must be remembered that with such sample sizes, large percentage difference may only represent a slight change of opinion by one or two individuals.

In both years, the early part of the curriculum gave rise to difficulties concerned with natural logarithms and their manipulation. In both years, about half the students felt that these topics were of greater than average difficulty. This result suggested that the topic held some intrinsic difficulty, changes in the study notes had not noticeably improved student ratings.

The results from the questionnaires were corroboration of a view formed from the interviews and the experience of student questioning in the tutorials that differential calculus was the subject giving rise to most difficulty for the students. The ratings show rather more variation than with

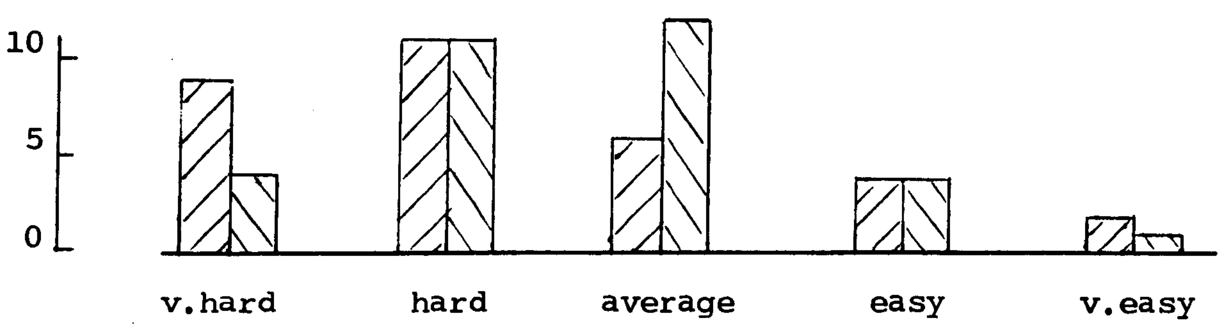
Topic Rating:	v.hard	hard	average	easy	v.easy	sample size
Naperian logarithms	0.1, 0.1	0.4, 0.4	0.4, 0.2	0.1, 0.3	0.05, 0	27, 18
Change of base of logs	0.1, 0.2	0.3, 0.2	0.4, 0.2	0.2, 0.3	0, 0.1	26, 18
Implicit differentiation	0.3, 0.2	0.15, 0.4	0.4, 0.05	0.15, 0.3	0, 0.05	27, 19
Problems involving max/min	0.05, 0.2	0.4, 0.4	0.4, 0.2	0.15, 0.1	0, 0.1	26, 19
Problems involving matrices	0.05, 0	0.3, 0.5	0.5, 0.3	0.1, 0.2	0.1, 0	27, 15
Finding max/min in two variables	0 0.05	0.4, 0.3	0.5, 0.5	0.15, 0.05	0, 0.05	26, 15
Constrained optimisation	0.15, 0.05	0.4, 0.4	0.3, 0.5	0.1, 0	0.05, 0.05	27, 13

(78/79 figure given first, 79/80 figure given second)

Student Ratings Expressed as Proportions of Sample Size

Table 6

other topics, and there is a tendency for opinion to polarise away from the average. This could be a result of the wide variety of school experience with respect to calculus; obviously A-level entrants would view the subject very differently from someone with no previous experience. This is difficult to quantify precisely, because of the number of anonymous replies which could not be classified. However, figure one shows that for those results that were identified, the non-A-level entrants found the two topics of more than average difficulty, whereas the only two known A-level returns rated them as average and easy (Implicit differentiation) and both easy (Problems in max/min).



Implicit Differentiation ///

Problems in max/min \\\

Combined ratings of topics in Calculus, 78/79 and 79/80

<u>Fig. 1</u>

This factor also makes direct comparison of figures in the different years rather difficult. Referring back to the comments made during the interviews, two factors seemed to be of major importance in formulating opinions about calculus. One was the algebraic manipulation involved and the other was applying the basic techniques to problem-solving.

Anxiety with problem-solving was evident when students rated the various aspects of matrix algebra. This topic was generally reported to be well understood and the source of few worries apart from the applications, which is rated as a difficult topic in Table 6. In both years, the centre of opinion lies between "average" and "hard", the opinions tending to show less variation than with other topics. The questionnaire responses were similar in the two groups apart from that involving Cramer's rule for solving simultaneous equations. In 1979, 29% rated it as "hard" or "very hard" but in 1980, 11% rated it as "hard", no-one as "very hard". It was not obvious why this was so. However, since this topic was the subject of a lecture, one possible conclusion was that this should be looked at carefully to ensure that the presentation was as clear as possible.

The last section of the syllabus, involving partial differentiation, was again shown to present no problems in terms of basic technique but likely to be rated as difficult when applications were involved. One possible influence in this case was the relatively short interval between the introduction to this topic and the administration of the questionnaire. Some students were still involved in working through the self-study notes and mastering the basic ideas, a point which may have resulted in an exaggeration of the difficulty. Another factor was mentioned during an interview; one student thought that the time interval between the two sections on calculus (filled by a study of matrix algebra) had allowed the ideas to "sink in", to the advantage of his later studies. Again, the ratings from the two years were reasonably consistent, the sizeable majority in both cases being in the average to hard region. It was noticeable that no topics which were ranked difficult by one group were thought to be easy by the other or vice-versa.

The overall impression was to emphasise that student concern increased when the applications of the basic techniques were introduced.

The 1979/80 group were the first to use a set of notes in which all the sections contained some exercises where the students were required to formulate and test a rule (e.g. the multiplication rule for matrices). In the previous year, students had split more or less equally in rating the difficulty they experienced in doing this and this pattern occurred in the current year. However, replies to the questionnaire suggested that they differed in their support for it as part of the self-paced study. Previously one-third had said that they liked it, whereas the second group produced nine replies in favour out of fifteen. This was reflected in replies during the interviews, no-one finding the experience disturbing, whilst previously, three students out of six had said that they were uneasy about it. Looking at the replies given when they were asked if it was rewarding showed that students in the latter year felt it had more merit. Six rated it as "very rewarding", ten as giving "some reward" and one "little", compared with figures of one, twenty and six respectively a year earlier. Two points were noted. Firstly, it was interesting that some students who thought that the formulation of rules was difficult nevertheless acknowledged that it was a worthwhile experience. Secondly, the second class had experience of this right throughout the course and may have benefitted from the extra practice. However, the four interviewees in November had been unanimously in favour when their experience was at its most limited. Another factor that may have been influential was that the second group were better qualified, five more students entering with A-level mathematics.

It was encouraging to note that amongst the interviewees, there were several who thought that formulating their own hypotheses and testing them was a valuable part of an undergraduate course, whatever the subject. Some students were not too happy about the experience because they found it disturbing. As long as the exercise was not too confusing or disturbing, leading the student into novel situations was considered a worthwhile undertaking in an undergraduate course.

In all the years covered by the investigation, the questionnaire responses showed that a majority of students held the view that applying their mathematics to solving problems was of more than average difficulty. (The ratings for 78/79 and 79/80 have been summarized in Table 7, which also shows the results for the usefulness of working in groups.) This feeling was also evident in figures from the same two years which showed that students were about equally divided between thinking that the time allocated to this aspect was either too little or about right. In contrast, the overwhelming majority in each year thought that the times given for lectures and self-paced study were about right. In both years, when asked to pick the sources of difficulty from a given list, the majority indicated the translation of the problem into mathematical form. One difference in the two classes was that in 78/79, ten students found difficulty "remembering the mathematical method required" but in 79/80, only two reported this as a problem. Reference back to the interview data suggests that the open-ended nature of problem-solving was

		v.difficult	difficult	average	easy	v.easy	sample size
Difficulty in	Feb	0.2, 0.1	0.4, 0.5	0.3, 0.3	0.1, 0.05	0, 0	26, 18
Problem-solving	May	0.15, 0.2	0.5, 0.5	0.3, 0.1	0.05, 0.2	0, 0	27, 17
		v.little	little	average	useful	v.useful	
Usefulness of	Feb	0.1, 0.1	0.1, 0.4	0.2, 0.1	0.5, 0.4	0.1, 0	26, 17
Group-working	May	0, 0	0.4, 0.5	0.1, 0.2	0.45, 0.3	0.05, 0	27, 17

(78/79 figure is given first, 79/80 is given second)

Student Ratings as Proportion of Sample Size

Table 7

relatively new to many students who had been more accustomed to an "operational" approach to mathematics.

Rating the value of working in groups when trying to solve problems produced a strong division of opinion in both years. The figures in table 7 suggest that the class of 79/80 were more likely to feel that there was less merit in this experience, but the most noticeable feature is the small proportion who remained fairly neutral. One encouraging feature was that, in both years, between February and May, two or three individuals foresook the view that they were very unhelpful. This, of itself, was not firm evidence of an overall change in attitude, but it does help to remind one that these students were at the beginning of their degree course and this mode of study may be more attractive with greater experience.

Finally some thought is given to the value of progress tests in a self-paced study scheme. (Observations on these tests have been made earlier; see pages 243, 253 and 276.) In some implementations, incentives were provided for students to undertake progress tests. Thus, sections of the study notes have been issued only after successful completion of tests (Keller, 1968) or the results have been given some weight in course assessment. (van der Klauw and Plomp, 1974.)

In this scheme, the process of giving out small sections of notes after each test was dispensed with because of the time and effort required in distributing and recording the status of each individual student. To incorporate marks in the formal assessment grades would have required the facilities and supervision necessary for such results to comply with

Polytechnic regulations. This was quite impractical.

The method adopted, involving the distribution of study notes covering between four and six weeks of work, with students attempting the progress tests informally and bringing the results for marking, raised two possible questions. Would students either be tempted to ignore tests and press on regardless or would they be tempted to seek external assistance when attempting the tests? The following points were considered to have some bearing.

In a scheme which allowed students to progress at their own pace, it was not unnatural to discover that not all students completed a full diet of progress tests in each topic. This did not imply necessarily that parts of the syllabus were left out. The results of the assessment tests suggested that either students were already familiar with certain topics or continued their individual study elsewhere. There were a few examples of progress tests being asked for some time after formal study of the topic had ceased.

It was thought that the likelihood of a few students misusing tests was not sufficiently great as to warrant major changes in the scheme. A strong reason for retaining the current procedure was thought to lie in the worthwhile experience it gave to students in being involved with responsibility for their own progress. This was considered to outweigh the disadvantage that, on any course, there will be a small minority who will be tempted to abuse the system. Reference to the comments made by interviewees provided evidence that the students were aware of the benefits of these tests and the opportunities they presented to monitor their own success.

The following chapter presents two further views of the course, that of the specialist economics lecturer and that of the second year student looking back at his first-year studies. This precedes the summary of the main outcomes of the investigation as a whole.

CHAPTER 18

OTHER VIEWPOINTS OF THE COURSE:- LECTURING STAFF AND SECOND YEAR STUDENTS

18.1 Introduction

This chapter presents the data obtained from interviews and discussions with members of the lecturing staff from the Department of Economics and Social Studies and also with students in the second year of the three year course who had experienced the mathematics course under investigation. In the first case, three members of staff were consulted, all of whom were involved in lecturing to the students, mainly in relation to macro and micro-economics, one was also involved in teaching mathematically-based subjects of Economic Statistics and Econometrics. Their experiences ranged over all three years of the degree course. The other evidence was gathered by interviewing two groups of second year students who were, by then, over half-way through their second year. The purpose of this part of the investigation was to establish the attitudes adopted by two factions that were known to have a significant influence on the overall tenor of the course.

18.2 Views of Lecturing Staff

The following details were the outcome of discussions during the 79/80 academic session. The interviews were carried out with individuals and were recorded on tape. Thus each record is uninfluenced by comments from any other person and represented, as nearly as possible, the views of that particular individual. Since the lecturer transmits his own individual approach and view of the subject to the students, this was seen as the most appropriate mode in which to gather the information. In the interviews themselves, information was sought on what was seen as the role of mathematics in economics generally, and also on the way in which the course under review actually functioned.

When asked to comment on the importance of mathematics to the study of economics, all of the lecturers pointed out that it depended on the course. Some could be very descriptive but all were of the opinion that on a degree course of this nature, a sound knowledge of some mathematics would be necessary.

> "It varies with the course. If we are talking about a specialist course in economics then I think it is necessary. I do not think it is possible to obtain an in-depth understanding of economics without a knowledge of mathematics. Outside specialist degrees it is possible to give an alternative presentation by description or diagrams but after a time that runs out of force." (b)*

All three lecturers acknowledged making extensive use of mathematical development in the way they taught economics, although one pointed out that in the first year, one had to be careful not to jump ahead of their mathematical development and another said that where it was possible he tried to use both approaches (descriptive and mathematical) for those students who found the mathematics difficult to follow. When asked to comment on the amount of mathematical studies the student undertakes,

* For the purpose of identification, the lecturers will be denoted as (a), (b) and (c).

they were unanimous in thinking that the total was probably right but there were problems associated with it all being in the first year. There seemed to be reservations about some students' ability to make satisfactory progress in these circumstances.

> "I do not think the total is wrong, there definitely is a gap in their knowledge though. I am not sure how that could easily be rectified." (a)
>
> "My feeling over the years is that we have not been terribly successful in persuading economics students to involve themselves in the maths ... If they were persuaded to - not do more than they do now - do it preferably over three years, I feel we might be more successful in persuading them of its desirability." (c)

There was also agreement when asked if a reasonable understanding was necessary in order to make progress on the course. All replied that progress would be possible but not very satisfactory. There was agreement that certainly to make good progress and become a good economist usually demanded a sound mathematical understanding.

> "I think it would be difficult to make progress (without reasonable mathematics). It would certainly be difficult for them to be good students of economics, though not impossible." (c)
>
> "If you mean passing the course, then I do not think they need to have such a fantastic ability in mathematics ... Those who obtain better quality degrees however, invariably are fairly competent on the quantitative side. I would not have thought there were many exceptions." (b)

Before turning to questions concerning the course itself, the lecturers were asked if any other objectives were desirable other than to provide a basic collection of mathematical techniques. It was interesting to note the unanimous view that there was rather more to it than that. One remarked that the logical argument in mathematics was important, equally the ability to see the connection between a problem and its mathematical expression. Two commented on the need for mathematics in following published papers and articles. There were also two who mentioned the general value of an education in mathematics, although wondering how much one may pursue wider issues on a course of this nature.

> "Arguably, certainly as a mathematician, I would say the subject has an integrity of its own and it can be an educational experience on its own ... The question is, could you involve the characteristic economics student in this aspect? It is doubtful really, so you are compelled in a sense to do it as an applied subject." (c)

When the issue of mathematical formulation and the students' ability to cope with this was raised, all the views were that the formulation of models was an important tool to the economist and an ability to do this often characterized a difference between good and not-so-good students. There was some concern expressed that for some reason not discernible, there was often a reluctance on the part of students to employ a mathematical argument.

> "It is important as far as the economics is concerned, a lot of them have difficulty in doing this, I suspect this is why they tend to steer clear. I am not sure

why they find it so difficult. It is strange because even those who are quantitatively competent also will not use it in their answers." (b)

The replies were unanimous in expressing a conscious desire to encourage students to make good use of their mathematical skills and a concern that some of the less mathematically able students did not do so.

"I think they use it less well and in a less desirable way than they ought to, certainly given the amount of time that we devote to it in year one ... There are the exceptions who take the quantitative options." (c)

In trying to discern why some students displayed these attitudes, the lecturers were asked about the effect of student confidence. It was acknowledged that this was a significant feature and it was hoped that the emphasis given to the subject in the first year made a positive contribution. One contrary factor suggested in one reply was that the methods used to teach economics at A-level had developed habits that were difficult to change. Another lecturer mentioned the possible influence of popular belief that only certain people could do mathematics well. It was also pointed out that the way economists dealt with functions was, in fact, quite a sophisticated process.

"I think that is partly it, partly that things are difficult to remember. When I was doing maths, you struggled to learn it, then you did not see it again for six months or a year. There was nothing to make it stick in your mind ... It is a vicious circle, because our students find it difficult, we tend not to push them too much, then you find they do not use the skills enough and they forget them." (a)

All the lecturers who were interviewed saw as a serious objective the development of students' abilities to think for themselves and this was seen as a general objective of any undergraduate course and not specific to mathematics. Two people mentioned that it was a far easier process to reproduce what had been given and one went on to illustrate this by pointing out that in preparing lectures it was always tempting to rely on what was printed in a text-book. It was agreed that students tended to perceive the experience as one of learning specific techniques.

> "Inevitably it has got to be a situation where they have got to be taught a set of techniques. It is perhaps expecting too much of them to be involved beyond that kind of capability in the subject. Subsequently we are not going to get involved in the philosophical foundations of mathematics, we just make use of the techniques involved. But often, when I teach economics with the use of maths, the biggest problems I encounter in teaching are not explaining the economics but explaining the mathematics." (c)

The members of staff were asked for their opinions about why students might not display a satisfactory mathematical ability. One thought that it did not appear as a lack of ability so much as a reluctance to use the ability that was there. One responded that over the years he had become aware of a kind of threshold beyond which it was difficult to push without the student encountering some problems. Although there was unanimous agreement that they were always concerned about this, there was no clear identification of where the difficulty lay and what could be done about it.

> "I think that is a difficult one. There is obviously a deficiency, as I said, but I find it difficult to put my finger on where it lies. Whether it is lack of ability, lack of technique or simply lack of practice. It could be any of those things and I am not sure which. Given that they have passed their first-year exams, presumably they have learned the technique at some point, so it must be either lack of practice or lack of ability." (a)

The discussions revealed that many students commented to the lecturers about their mathematical progress. This was particularly true of the two lecturers who had acted in the capacity of year tutors, where part of their function was to monitor the students' progress. Talking about specific sources of difficulty, it was suggested that fundamental topics, like algebraic manipulation were real problems. One lecturer quoted the method of putting two algebraic fractions over a common denominator as one which had been the subject of student enquiries. Another suggested that this uncertainty concerning what was considered to be pre-requisite knowledge may have been a source of student lack of confidence and that early problems sometimes seemed to be unresolved. However, it had to be remembered that there was a wide range of ability displayed by the students.

> "The only common factor is that it is the weaker ones

who find it difficult, right from the start. Perhaps it is in the early stages that they find it most difficult, even things you would imagine they had done before ... When they try and pick up the steps in that area, by and large this does work, for when you come back to them later they say,'yes we have come on to calculus or matrix algebra, I find that O.K., I find that quite easy! Very often they still find the earlier bit difficult ... It might be an advantage if we could build their confidence early on in that a number of them seem to lose their confidence at an early stage and this panics them terribly." (b)

One lecturer commented that it should not be concluded that all students were weak, some did choose the more mathematical options later on in the course. These were the ones who complained when you tried to explain things in a different way after you had put it down mathematically on the blackboard. Generally however, it was considered that mathematics was the subject most likely to give rise to student concern.

There was a strong feeling that it would be inappropriate to try and teach the mathematical techniques as "black boxes". The danger was that the students would associate specific techniques with specific problems. There was a feeling expressed in all three discussions that the strategies of problem-solving, involving formulation, manipulation and interpretation would not be properly developed by this approach. One lecturer did suggest that one possible alternative approach was to present a specific problem and then develop the know-how for its solution. Although seeing this as a possibility, he personally preferred the idea of developing the technique independently of the problem so that one was not seen as specific to the other. Another lecturer, commenting on the process of expressing problems in a mathematical form, made the important point that this was not a problem peculiar to economics, it was more general and he had noticed similar responses with other courses in the Polytechnic.

> "No I do not (think mathematics should be presented as a black box). If the worst came to the worst it would be better to do that than leave out the maths altogether, otherwise they would become so bogged down. At least, in the second year, they could follow some part of the reasoning, but if they only ever did that they would never get to the stage, as you said earlier, of being able to think for themselves or to do very well in economics. For that they need to understand more about the concepts, the logical concepts, than the techniques." (a)
>
> "... I would prefer from an economics (standpoint) to do the techniques independently and then for them to be able to draw on these techniques for a problem. Otherwise they tend to miss out on the ability to use a bundle of techniques in relation to any problem. They see the technique as specific to a problem ... I still feel that if they have a good grounding in the techniques they should be able to apply these to the problems they are given." (b)
>
> "As I have said before, my feeling would be that perhaps one should start from a slightly lower base when they come in year one and make sure that they have a better understanding of basic algebra so that their

manipulative ability later on is better. I think they need more time to assimilate the sort of maths that they have to do ... The biggest problem is not the economics but sorting their way through the required calculus (in micro-economics)." (c)

A point worth noting was the measure of consistency between the views expressed by the three lecturers about the role of mathematics. That this was an expression of a general attitude was given some weight by comments made by other members of staff during informal conversations. These demonstrated a similar involvement with the use of mathematical symbolism and methods of analysis. It was note-worthy that the consensus was not to see the mathematics course simply as a device for imparting a finite number of skills but the opportunity to develop more general, problem-solving strategies which would play a significant part within the overall degree. It was apparent that their attitude was strongly conveyed to the students as evidenced by the comments which were recorded in the previous chapters.

What was encouraging from the mathematician's point of view was that the overall attitude to the subject was shown to be positive, that it was seen as an important and basic part of the course and there was a genuine desire to integrate it with the other sections of the curriculum. Without exception, the lecturers commented that the economics was often more succinctly expressed in terms of mathematics than otherwise. There was no major concern about the way in which the mathematics was being taught although there was a continuing awareness that some students made slow progress and were reluctant to adopt a numerate approach.

18.3 Views of Second-Year Students

The results presented in this section are based on interviews with two groups of second-year students, the first in April 1980 and the second in March 1981. The objectives of the interviews were (i) to establish the attitude of these students to the mathematics course, (ii) to ascertain how they perceived the course as a preparation for later study and (iii) to compare their opinions on the role of mathematics with those expressed by the staff.

The students were a cross-section of the classes with respect to mathematical ability, ranging from a student who had to resit the examination to one whose mark was first class. There were four students in the first group and five in the second. Of the nine, three had chosen options in the more mathematically-based areas (Econometrics and Applied Statistics).

Judging by their choice of options it was likely that there was a division of opinion about liking for mathematics and indeed, their replies showed four did not, three were neutral and three did like mathematics. All the students questioned acknowledged that mathematics was playing an important part in their economics studies. Not only were basic techniques mentioned (calculus, matrix algebra) but there were many references to the problem of expressing economic concepts in terms of mathematical expressions. One student was of the opinion that expressing things in mathematical terms made it easier, but the general opinion was not in support of this. Two people in particular, whilst agreeing with the conclusion that mathematics was important in their studies, were nevertheless of the opinion that the whole philosophy was questionable and felt that a less numerate approach should be adopted. The following quotations were representative:

> "Economists do see mathematics as important, but I do not. I try and avoid it ... I think mathematics tries to say that economics is precise, but it is not, I do not think it is. There is a place for mathematics but only in the theory. When you come to practice in the real world not a lot of mathematics can be useful."
> "You have to qualify that a little (importance of mathematics to an economist). We do an awful lot of mathematical analysis in micro-economics. I find a problem is you rarely see a real figure on the board. This tends to make it a little bit abstract, difficult to relate at times. Obviously, mathematics is going to be important, otherwise you just cannot follow it."
> "You need mathematics to quantify whatever you are working on ... This is evident in the second year work, very much so ... I do not know if it is central but it is certainly there to a large extent. If not at the centre of things as a very important back-up."

When asked to say if the lecturers made any use of mathematics in their economics teaching, the interviewees were unanimous in saying that they were aware of a substantial usage. As mentioned earlier, there was some feeling that the emphasis was too mathematical. One group was obviously influenced by the fact that in micro-economics they were currently studying a topic in which the mathematics was rather difficult and each student in turn mentioned this subject specifically. However, the replies from the other group indicated that all the interviewees were conscious of the crucial role of mathematics.

Students were asked to say if they felt their attitude to mathematics had changed in any way since their first year. The replies suggested some difference between the two groups. In one, the students were all of the opinion that their attitude was unchanged. Their attitudes were expressed in terms of seeing the subject as helpful or relevant. One made an interesting point that you may see the subject as useful without necessarily liking it.

> "No, not at all (changed my attitude). Mathematics at school tended to be rather irrelevant to everything else, whereas brought in with the economics, it is very relevant."
>
> "I think there is a difference whether someone likes it. You may like some things but because you cannot do it you will not choose it. If someone asks me if it is useful, I would say yes, but I do not have to like it."

In the other year, for the reason mentioned above, the students were influenced by the fact that they were finding the mathematics difficult and this was reflected in the responses that they were all, in some degree, less confident than they were previously.

> "I used to enjoy it last year, but not this year (more difficult than you hoped?). It is not difficult - well it is difficult but it would be a lot better if we were taught in a different manner ... This year you are thrown in at the deep end, you know it or you don't."

When it came to identifying the reasons for their concern, there was a general response that suggested that it was the actual applications that were difficult. Three students were of the opinion that more about applications in the first year would have been better, another thought that extra economics lectures would have been better, with the mathematics integrated into these. The last student saw it more as a problem with the economics than the mathematics and suggested that there was a significant difference in the approach adopted in the two years.

> "I think the trouble was, it was not a problem of what was lacking in the maths so much as what was lacking in the micro and macro. Although we did applications in the first year, they were very general, at the basic level. We did not actually do any economic analysis involving the use of mathematics which we are totally concerned with in micro-economics in the second year."

The mode of instruction adopted for the mathematics course, involving a large measure of self-paced study was considered by all the students interviewed to have been a good way of studying the subject. The reasons given for this were the opportunity to go at one's own pace, the opportunity to monitor one's progress and the study notes which set out all the steps and were useful when it came to revision. The one reservation was from a student who said that he lagged behind. Asked if he could suggest how that problem could be overcome he was not sure what to suggest except something "more strict". Turning to the idea of having to formulate your own rules, five of the nine students said that at that moment of time they viewed it as useful experience, two were neutral and two thought it had been unhelpful. Of the two who were not happy about it, one said he simply avoided doing that sort of exercise and the other preferred being given rules and did not like the doubt left in one's mind that the rule might be wrong. Two replies noted that there were problems if the rule was too difficult to devise, there was a tendency not to bother with it.

> "I remember doing that. It would be O.K. for somebody who has a working idea of the rule. I think it could be useful if well used."
>
> "I have to say yes and no. It was useful if you could latch on to the solution relatively quickly ... If you could spot what the rule was it was good because you remembered it better, but if you could not, it worked in the opposite direction."

There was much comment suggesting the real difficulties were not encountered with the acquisition of basic skills but the application of these to solving problems given in economic terms. This seemed to be the major concern of the students who were interviewed, how to successfully integrate their mathematical and economic techniques. As one student put it, last year they were separate, this year it is all combined. In all cases except one, students reported that the process of mathematical formulation was a difficult step.

> "Yes, it is very difficult. Solving is not difficult, the problem is to understand the question and to extract the mathematics."
>
> "Yes it is (difficult). You have it in economics, to translate the economics to mathematics, sometimes it is difficult. That is why I think instead of having this self-pacing thing, say, they give you a set of problems to solve at home. Then you can come back."

This last quotation presents a view that more problem-solving should be included at the expense of time given over to learning basic techniques. This was the implication when this same student was asked how the mathematics course might be improved and he replied that greater emphasis should be given to doing problems. One of his companions wondered if some basic rules could be given for problem solving.

> "... If you could be given basic rules:- if you have a problem like this do so-and-so, or try so-and-so, then it might be considerably easier."

Other students, speculating on improvements, thought that the mathematics syllabus might be extended further than it was. This produced responses from colleagues who already considered the mathematical emphasis too great, who felt this would only make things worse. One reply was that it really needed less emphasis on mathematics in the <u>second</u> year. The responses were mixed when they were asked how confident they felt about applying their mathematics. Nearly all the students had experienced some difficulties at some time in the second year. There was a definite feeling that they had been more confident at the end of their first year and that this had waned somewhat in the light of experience. Overall, four students felt satisfied or reasonably confident about their mathematics and the remaining five expressed some reservations. The following replies are representative of the opinions that were expressed:

> "I felt at the end of the first year a lot more confident about mathematics than when I started. I do not find it a great deal of use in the second year because of the approach we have to use."
>
> "Well it was a help to a certain extent since I did not really know anything about mathematics. I could follow things on the board but I would not use it."

"I could have done with more knowledge but I think the background I had in the first year would help me if I want to concentrate ... Looking back at things I could see that it was easy if you concentrated."
"Quite satisfied. I said I took A-level, in actual fact I only got an O-level pass. I think if I had had teaching of the style and type I had in the first year of the economics course I would have passed without any doubt."

It was note-worthy that students commented that their first-year experience did not always seem a reasonable background to the second-year studies; remarks seemed to suggest they felt some difficulty existed in relating their mathematics to subsequent studies of economics. One example showed their mathematical technique had been stretched considerably. Perhaps this suggested a close look at the teaching approach to the second-year topics. Might it be necessary to "round out" the student's mathematical knowledge for certain applications or possibly delay some topics until a greater experience of formulation was gained?

Comparing the views outlined above with those in the first section of this chapter, it was clear that the significant place given to mathematics in this degree course by the lecturers was firmly conveyed to the students. The emphasis given to a numerate approach in so many of the economics topics was evident from the examples quoted by the students. Some of them were inclined to feel that the emphasis was too mathematical. The replies showed agreement between lecturers and students in the area of applying mathematics to problem-solving. The lecturers perceived that students encountered difficulties with this and comments from the students confirmed this. The most oft-quoted source of difficulty was the process of mathematical formulation. The lecturers felt that students would sometimes avoid using mathematical formulations if it were possible and some of the interviewees also suggested this was so. The lecturers saw no particular reason for this, but two replies by different students gave some hints why it was so:

> "Once or twice when there has been a graphical and a mathematical approach I have read them both but I have made extra effort on the graphical because I think for an exam it is easier to remember a picture."
>
> "As I have said already, if it involves transforming from economic information to mathematical form, sometimes I try to avoid it because I am not sure if the equation I am forming is the right one. It involves a big risk you see."

The first quotation brings into focus the difficulty that most students of this kind have in manipulating mathematical ideas. (This student was one of the most able.) He obviously felt much safer in dealing with pictures. The element of safety is nicely illustrated in the second quotation. Students do feel vulnerable when examinations have to be passed and grades accumulated. Perhaps it is too easy to forget that for some weaker students, the objective is viewed as one of survival and there was no gain in taking unnecessary risks.

The mathematics content in the B.A. Economics degree has now been looked at from three different points of view. Earlier chapters also contained details of the formative evaluation of the method of instruction that was developed.

In the next chapter a summary of this information and certain conclusions are presented and indications are looked for that the teaching method achieved the stated objectives.

CHAPTER 19

SUMMARY AND CONCLUSIONS - MATHEMATICS AND THE B.A. ECONOMICS COURSE

The previous four chapters have presented the details of the information gathered during the investigation of the mathematics course in the first year of a B.A. degree in Economics. A summary is presented here together with certain conclusions. Basically, two views of the investigation are presented, one identifies the factors that contributed to the student's attitude to the course and how he responded to it and the other considers the development and effectiveness of the teaching method.

19.1 The Student: Opinions, Background and Responses

It was evident that the students either arrived at the Polytechnic with, or quickly developed, an opinion that mathematics was going to be very important in their study of economics. Since many of them could be said to have only a fair ability, represented by a middle-grade pass at O-level achieved some two years previously, there was a general feeling of concern about their ability to progress in mathematics. The students were most likely, in their past experience, not to have experienced a complete dislike or liking for the subject but to have enjoyed some parts (those they could cope with) and been frustrated by others. It was noticeable to them that the role of mathematics was quite different from that, say, of the social sciences; they were aware that it had a mode of thought and symbolism which was quite different.

The method of instruction, involving a strong emphasis on self-paced study was viewed as a good way of learning the subject. In addition, the use of some lectures to present basic facts and to discuss difficult points in the topics was held to be useful and the balance of lectures and self-paced study was considered to be about right. The opportunity to go at one's own pace was almost unanimously welcomed, the exceptions were generally amongst the weaker students who tended to fall behind. One problem was that students were far more responsible for organising their studies than was the case with conventional lectures.

The independent initiative that was required in self-paced study, particularly in those, albeit very limited, sections requiring some formulation and testing of rules, was likely to be the cause of some unease in some students, not necessarily the less able. Possibly because of their limited experience of undergraduate study, a certain number displayed some preference for a course which was much more expository in nature. Students who held this opinion were likely to be at a disadvantage when it came to problem-solving, which demanded a more open-ended approach.

Almost without exception, the students encountered some elements of difficulty with solving problems which were expressed in economic terms and required translation into a mathematical form. Even problems which to the experienced eye were rather thinly disguised were likely to give rise to concern. For some students, a reappraisal of their attitude was necessary, for they considered that it was very much a question of rote-learning techniques to deal with the different kinds of questions they were likely to encounter. It was possible that one or two students never quite lost this rather mechanistic approach to the subject, although the experience of the first year produced in the great majority of the students at least an appreciation that the application of mathematics involved rather more than learning a few basic skills.

In this respect, the experience of working in small groups on specific problems was felt to be of value to many students. The advantages of this approach were seen to be the opportunity in a small group of raising questions without embarrassment and the possibility of having one's ideas stimulated by other people's remarks. Another factor which gave some encouragement was the realisation that the individual was not alone in his difficulties. However, not all students accepted there was a real advantage, they seemed to be influenced by such factors as a preference for working alone and previous experience which fostered the view that study was an individual activity.

The majority of the students were of the opinion that their ability in mathematics had improved during the year. The favourable opinion concerning self-paced study was retained through to the end of the course. That the students felt they had improved during the year was taken as an indication that confidence in their ability to use mathematics would be higher than previously.

19.2 Views of Members of Staff and Other Evidence

One factor which was seen as a major influence in forming the students' attitudes was the position adopted by the academic staff who were involved in the economic subjects, both in the first and subsequent years. It became clear from the results of both formal and informal discussions with the lecturers that the important role attributed to mathematics by the students was a reflection of the opinions held by the staff and put into practice in the way the subject was used in the teaching of economics. They all agreed that extensive use was made of mathematical symbolism and the formulation of mathematical models. There was general agreement with the view that mathematics had an integrity of its own, was not simply a collection of techniques but it was also felt that the students were likely to be motivated by its relevance to economics.

Despite the commitment to a mathematical approach, it was also the generally accepted view that some students did not always welcome the mathematical analysis. The reason for this was not clearly discerned, although there was a definite feeling that it was not always associated with lack of ability. This meant that students often adopted a descriptive approach as an alternative, sometimes to the disappointment of the staff involved. It was widely felt that where the students were sufficiently confident to adopt a mathematical approach and use their skills, the result was very often a real benefit in their understanding and progress in the study of economics. Lecturers found it difficult to make constructive comments on how to improve the willingness of some students to involve mathematics. However, from the point of view of this investigation, it was noted that no comments were recorded which were critical of specific aspects of the existing first-year mathematics teaching.

When considering the views expressed in the interviews with second-year students, it was of interest to compare them with those expressed by first-year students and lecturers. The over-riding impression was given that the approach adopted by the lecturers in economics involved far more mathematics than they may have imagined in the first year. This opinion appeared to be fairly generally held judging by the comments made during the interviews. Some remarks were evidence of a concern amongst a number of students about their ability to cope with the mathematics. They noted the role of mathematical formulation in the second year studies. Lecturers had commented on the need to employ mathematical models in the development of economic theory. Bearing in mind the opinions of first-year students concerning formulation and problem-solving, it was not surprising to discover that these aspects were thought to be far from straightforward at the second year stage.

Because of this, some students were prompted to feel that the first year mathematics course had not been wholly successful as a preparation for later studies. However, some did not view this as attributable to the mathematics course alone, the teaching approach in the economics subjects and the problems some students had in integrating the mathematics and economics were also mentioned as influential factors.

Both the lecturers and the interviewees commented on the tendency for some students to choose a descriptive approach rather than a numerate alternative. The lecturers suggested that this was not directly related to mathematical ability. Two comments by different students gave a hint of why this might be the case, one mentioned the relative ease of remembering a pictorial representation, the other pointed to the risks that students feel they take when using mathematics. Overall, the difficulties cited by the students seemed to be centred around the transfer of training and the general strategies of problem-solving. It may be worth noting that these are educational issues of wide concern and not specific to an economics course. It has been noted elsewhere that transfer of training may exist in terms of structure rather than content. (Bruner, 1966.) One less encouraging feature that came to light in interviews with the second year students was that, at that moment in the course, they were generally less confident mathematically than at the end of their first year.

19.3 Conclusions

In making a summary of the conclusions, consideration is given to the general and specific points detailed in 13.3. Firstly, there were certain conjectures made concerning attitudes of staff and students. The investigation confirmed (i) that students were amply aware, from the beginning, that mathematics would play a significant part in their study of economics. The results of discussions with staff confirmed (ii) that the specialist economics lecturers were definite that students were required to display manipulative skills but they also required them to understand and cope with an extensive use of the mathematical formulation of economic models. The last conjecture (iii) that co-ordination would present no serious problems, was justified on the evidence of the considerable willingness of specialist staff to discuss the teaching of mathematics with the investigator and the way in which the course operated over the period of the study, involving a close liaison between the two Departments involved.

With reference to the specific purposes of the mathematics teaching, the five points listed in 13.3 may be considered separately.

(i) The effectiveness of reaching a standard considered to be satisfactory by the parent Department was measured in terms of the number of students who had to leave the course because of a specific lack of achievement in quantitative methods (mathematics and statistics combined) and the opinions expressed by economics lecturers. During the period of investigation, eight students failed in quantitative methods while passing in economic subjects, five in 77/78, three in 78/79 and none in 79/80. In the same period, six students required resit examinations before passing, a total of eighty-six students sat the examinations. The opinions expressed by lecturers were indicative that they were satisfied with the service provided in mathematics and they did not mention any specific points which were thought to require attention. It was concluded that the course gave to nearly all students the basic mathematical skills appropriate for the study of economics. As the development of basic skills was seen as predominantly a function of the self-paced study, it was considered that this method was a satisfactory way of studying, a conclusion reinforced by the evidence that it met with the approval of a great majority of the students.

(ii) Self-paced study was also seen as providing valuable experience to undergraduates and to make a strong contribution to the development of their ability in individual study. Group discussion and more problem-orientated activities were also seen as making a contribution to the establishment of more open-ended study methods.

(iii) The reduction of lecture time and the introduction of a scheme involving student-centred activities made demands on the students to play an active part in their own learning. Their reactions were generally favourable. The students' reactions to the formulation and testing of their own rules tended to be mixed. Although not always welcomed by some students, the conclusion was that it was a fruitful experience and made a real contribution to the development of a desirable attitude to learning.

(iv) The application of mathematics to problems in economics was found to be more difficult for the student than first envisaged. The difficulties encountered with mathematical formulation were under-estimated. Some students tended to cling to the notion that problem-solving was a matter of doing enough examples to cover all eventualities. Evidence pointed to the fact that the development of more general problem-solving strategies were difficult for nearly all students. The adoption of small group discussions was found to be helpful to rather more than half the students, although no-one reported them to be detrimental. Because of the difference in the responses of the students to the teaching methods it was felt that it was appropriate to provide a variety of experiences. This meant that in problem-solving for example, the student encountered work in groups, problems worked out on the blackboard by the tutor and individual study (e.g. homework).

(v) From the student responses it was concluded that by the end of the year, the course had been successful in making most students more confident about their ability in mathematics. However, it was disappointing to discover some evidence from lecturers and second year students that this confidence may have waned and was not necessarily reflected in later performance. Reasons were proposed which might have accounted for this but no definite conclusion was drawn.

Turning to the study scheme itself, it had proved possible to develop a programme of lectures, self-paced study and problem-solving activity which was compatible with the achievement of the identified course aims and which was generally approved of by the student body. The involvement of several different learning modes was a worthwhile feature in view of the fact that different learning objectives were involved. It was also of benefit for some students who were not well inclined to self-paced study.

The balance of the lectures, individual study and problem-solving was thought to be about right. Students reported satisfaction with the mix of lectures and self-paced study, the lectures were thought to be helpful and were welcomed by most students. Some of them would have preferred more time on problems, but this could only be achieved by spending less time on acquiring more basic skills. Clearly, there is always likely to be a conflict between these two demands and the correct balance will remain a matter of debate.

The notes and exercises which formed the basics of the self-paced study were developed over the three years of the study. The final form of the notes in which sections were marked to indicate the need for action (†), to show that objectives were summarized (■) and involving the use of progress tests were found to be satisfactory to the students in form and content. Although the informal way of dealing with progress testing was open to misuse, it was thought this possibility was outweighed by the advantage which was gained by the stress given to the responsibility the individual must accept for his own discipline and the value of this in developing a worthwhile attitude.

In certain ways, this method of teaching made greater demands on the student than a more conventional programme. He was required to sustain his individual effort over quite long periods and to organise his own work schedule and be responsible for monitoring his own progress. There was some indication that less able students found this difficult and so viewed self-paced study with less enthusiasm. Whether they would learn more effectively by a more expository method is not easily established. Ideally, such students should receive more attention on a self-study scheme because the tutor is not involved with the majority of students who can make progress on their own (hopefully). However, it was noted that the weaker students were sometimes amongst those whose attendance was erratic and who were reluctant to seek help with remedial work. It was found necessary to keep rather careful records of the students' progress and try and give positive direction whenever necessary if some of the benefits of this method of study were to be realised. Generally, students were satisfied with self-paced study as preparation for the application of mathematics to problems; the difficulties experienced by weaker students did not appear noticeably different from those associated with conventional lectures.

Although it has been concluded that student confidence in their ability to apply mathematics had been increased as a result of the mathematics course, it was clear that the general field of applications, involving modelling and problem-solving gave considerable concern to many students. As a final comment, an economics lecturer, after reading these chapters, was moved to add that too much must not be expected of first-year students. Mathematical formulation and problem-solving were central to the whole undergraduate course; it was unreasonable to expect every student to display a highly-developed facility by the end of the first year.

This chapter concludes the details of the investigation into the teaching of mathematics to undergraduate students of economics.

CHAPTER 20

FINAL SUMMARY AND CONCLUSIONS

A comparison of the two courses under review is presented and differences are noted. The resulting changes in research procedure are discussed and the outcomes of each study are considered in a wider context. A retrospective view is taken of the research objectives and possible future points of interest are considered.

20.1 A Comparison of the Two Courses Involved in the Investigation

Both courses came under the heading of non-specialist undergraduate mathematical education yet each presented rather different characteristics. Both played a significant role in the degree courses of which they were part and both had to be passed for the respective student to satisfy the requirements for progress. They were designed for entrants who were qualified at O-level in the subject, although one allowed entry with higher qualifications. There were other features however, which ensured that each had its own particular flavour. These had their origin in the fact that whereas one was a study of the subject from the point of view of its cultural significance, the other was a traditional service course within a degree with a distinct vocational slant. In addition, each course was offered by different institutions, each with different histories and traditions and having different student perceptions of their respective philosophies and aspirations.

The difference in concept was seen in the aims that each course was designed to achieve. The cultural course did not set out to bring about a precise level of mastery of specific skills; the aims were couched in terms of significant experience and contact with genuine mathematical activity. Outcomes were seen as the development of favourable attitudes, a broadening of experience and an appreciation of a different way of thinking, the values of which may be long-term. Nevertheless, the students were required to employ the symbolism and precise ways of thinking that were in the very nature of the subject.

On the other hand, the service course, whilst having a similar general aim of developing independent enquiry and initiative, was centred on a list of identified skills which were considered necessary by the parent Department or similar controlling body. Additionally, it was possible in the investigation described here, to identify a need for students to apply their mathematical ability to problems in economics. The ability to do this was seen partly as a function of confidence in using their skills and partly of less tangible competencies in problem-solving.

Other differences were discernible in the students' reactions to the aims of the two courses. There was evidence to show that all the students accepted in some measure that mathematics was a worthwhile subject to study but that its value was firmly grounded in its usefulness. In the case of the cultural course, this made students more ready to question the existing aims and to speculate if their efforts might be better rewarded in other spheres. In the case of the service course, the students were likely to accept the syllabus as relevant to their later studies of economics and to feel that the time spent on the mathematics was necessary even if not particularly welcome. However, this did lead to a source of concern because some students apparently viewed the subject as what was described earlier as a "cook-book" of techniques which had to be memorized to achieve success.

The final differences may be termed operational. The two courses operated with large student numbers in one case, relatively small numbers in the other. In the former, this imposed constraints on the teaching methods that could be adopted. For example, the study scheme developed for the service course, which was in part a response to the wide range of ability within the group, involved individualised instruction and a system of progress tests. The students perceived an over-riding aim of achieving a competence which they concluded was necessary from the manner in which mathematics was introduced in their main subjects. In the other course, larger student numbers would have required a considerable administrative commitment to operate a self-paced study scheme. In addition, the aims of the course were not thought to be achieved most effectively in this way. It was felt that this form of study was most useful in teaching specific mathematical techniques rather than in conveying a wider appreciation of mathematical ideas. There was evidence that in some cases the aims were only partially understood. This may have been influenced by the fact that the purposes of studying the various topics were expressed in broader and therefore less precise terms.

20.2 Comparison of the Research Objectives

Because the aims, characteristics and developments were so different for the two courses, there was a corresponding diversity of objectives of the research. In the case of the subsidiary mathematics for arts undergraduates, the problem was more fundamental in that there was no agreed consensus concerning the nature and content of a non-vocational course. One of the primary objectives of the investigation was defined as the establishment of a rationale and the identification of the essential nature of a cultural mathematics course.

Another identified objective was the task of carrying out a detailed survey of the existing course, to identify those elements which contributed to its particular character and to determine the way in which it operated; the model for this type of study being an "illuminative" approach to evaluation which seeks basically to provide an understanding of activities and values. (Parlett and Hamilton, 1972; Stake, 1976.) It was evident that a review of syllabus content would also involve gaining knowledge of the way in which the topics were taught and what were thought to be the major teaching objectives. Once the objectives were identified, appropriate research procedures were considered. The subsequent programme was based on a major use of interviews, this being seen as the most appropriate way of collecting the kind of data that was required. Other sources of evidence were used to provide supportive evidence and to give some measure of "triangulation". Some practical experience was gained from the preparation of two study booklets whose effectiveness as supplementary learning material was investigated.

With the service course for economists, it was also considered necessary to investigate the climate in which the course operated, but the motivation of students, the course aims and the range of student ability were different. The overall course aim was to develop specific mathematical skills that could be applied in formulating and solving problems in economics. The distinctive aspect of this part of the programme of research was the design, implementation and formative evaluation of a teaching package integrating three modes of instruction. Since the aims of the course included concern with developing ability to solve problems, it was considered inappropriate to base conclusions only on the attainment of test scores. For reasons that were detailed in the text, it was decided to use similar data gathering methods to those in the former study. However, certain features combined to make the information rather different in kind. One was the difference in class size (and hence sample ratio). A second factor was that the initial assumptions did not require the same students to be interviewed more than once, hence a greater number of individuals were involved as interviewees. Since in this case the investigator was also the course lecturer, it was considered necessary to make rather more use of questionnaire responses as a check against the interviewees not wishing to appear too critical. This was also valuable in the formative evaluation because the questionnaires were employed to gain information from the entire group. Corroborative evidence was produced in this study by carrying out discussions with staff who were specialist economists and from interviews with groups of second year students.

20.3 The Outcomes in a Wider Context

The two studies are looked at here to see how each related to the background of the general considerations which preceded the formulation of their aims. The course for arts students is considered first.

It was of interest to look at the conclusions in the light of arguments for a broad-based education (Phenix, 1964 et al). These seek to widen undergraduate experience to include an appreciation of several disciplines. It is not uncommon to discover examples of scientists and technologists being required to consider the social and economic implications of their work. As in the case of subsidiary mathematics, these may be included as statutory components of the course with corresponding pressures on the students. Despite the uniqueness of individual courses, it is possible therefore, to consider if the findings in Chapter 11 have wider significance.

Firstly, it was perceived that students' attitudes to subsidiary studies and their perceptions of the aims and objectives were of considerable influence in the way the course operated. Students may not be convinced of other people's arguments that such things are worthwhile. Pressures of study on the individual are high and affect the approach to a subsidiary course. Students may be moved to discover a "hidden curriculum", that is to eliminate all that appears unnecessary and to maximise the return in formal credit that results from their efforts. Also, a subsidiary course almost invariably involves a different and unfamiliar department with different traditions and habits. The subject itself involves unfamiliar ways of thought and has its own criteria for establishing truth and falsity. The student will have a much lower level of competence compared with a main subject. In these circumstances, it is not always easy to appreciate the problems that students have to face. In the case of mathematics, it is almost universally acknowledged by the non-specialist as being a difficult subject and lecturers may discover attitudes which were formed as a result of less than enjoyable experiences at school. Attention has been drawn to the large number of adults who display a marked lack of confidence in the use of mathematics. (Cockcroft, 1982, pp 5-7.) It was discovered, for example, that some arts graduates deliberately restricted their choice of career in order to avoid the subject (para. 21).

Considering the content of a non-vocational course, it was far easier to identify general aims than to find agreement on how these should be translated into classroom objectives. From the comments of various authors a trend was perceived towards a study of the processes and away from the content of mathematics (e.g. Douglis, 1970; Jacobs, 1970). Students appeared likely to adopt different criteria of worth from those of the course designers; to take a view which was based on an assessment of "practicality" and to be unsympathetic to courses which were judged to be irrelevant to their main studies. This divergence of views may have a strong influence on the conduct of the course, a feature which has been noted in other cases. It may result in some frustration of the achievement of intended aims (Sheldrake and Berry, 1975, ch. 3), or it may produce results that were not entirely expected. For example, a course in physics for arts students at Lancaster University had general objectives which included providing a base for independent reading and the stimulation of further study of physics. Students' responses to a questionnaire suggested that it did not motivate them to carry on a study of physics and a main interest was seen by them as an ability to communicate with physicists. Additionally, students were so unimpressed by an historical approach to the subject that such sections were removed from the syllabus (Heywood and Montagu-Pollock, 1977).

Careful thought needs to be given to communicating to students what the real aims of the course are because they may not possess a sufficiently sound background to comprehend what is being attempted by the lecturers. Perhaps it is a difficulty in communication that leads to a fairly common notion that "service" departments are out of touch. (Macdonald-Ross and Rees, 1972.) It is also worth remembering the point made earlier that subsidiary, broad-based courses may be the last opportunity for the specialist to arouse some interest in those who some day may wield considerable influence. (Kline, 1977.)

In the study of the service course for economists, no stress was considered necessary on establishing the content of such a course. In this case however, there was increased interest in the development of an appropriate teaching scheme. The theoretical discussions that provided the background were those which pointed to a change in educational aims. Thus, the student is no longer seen as the recipient of prescribed information but as an individual who will be required to respond to changes in opportunity (Tawney ed., 1976, ch. 3). There is a corresponding demand in mathematical education mentioned earlier for an emphasis on the processes rather than the content as being more appropriate. Also, there is a concern for the development of individual enquiry and those higher educational objectives which make effective problem-solvers. (See, for example, Cockcroft, 1981, para. 243, p 71.) In this particular case, there was evidence that the objectives were best met by a mixed instructional scheme which allowed students freedom to plan their own studies and yet provide a variety of learning experiences. (Davies, 1976 and 1977.) It was interesting to compare the difference between motivating factors in this and the subsidiary mathematics course. Although fostering student interest was not seen to be as important as in the previous study (students were amply aware of the role of mathematics), nevertheless some consideration of the specific objectives of the instruction was very necessary. For example, it was essential for the mathematician to consider the balance between presenting a fairly rigorous mathematical development and merely providing the specialist with a "box of tools". How much does an economist need to know about the limiting process to make proper use of the derivative of a function?

The conclusions about the efficacy of the teaching scheme may be viewed in the light of educational ideas that wish to develop greater individualisation and to see the lecturer become more a manager of learning resources. There is also the climate created by the changing aims of education, in this case the view that achievement may not be expressed in test scores on routine mathematical techniques but in those abilities concerned with formulating and manipulating mathematical models.

The teaching method that was developed proved popular with the students and was thought to provide rewarding experience for them. However, its use involved relatively modest student numbers. In the current situation, where economic factors may dictate increases in class size with no increase in staff availability, it may prove necessary to consider teaching methods involving less individual student attention than formerly.

Finally, in view of comments that have been made about problems caused by lack of communication between departments, this work serves to illustrate what can be achieved when a good relationship exists between the two groups of staff involved.

20.4 Research Objectives: a Retrospective View

Following the previous discussion, it is of interest to look back at the research procedure and speculate if alternative approaches which were considered and rejected may, with hindsight, have proved worthy of further thought.

Concern which is sometimes voiced about the "case-study" approach to educational research centres on the validity of the results and comparison is made with those from more experimentally-based programmes. (However, some authors highlight problems with the results of "objective" methods, see for example Tawney (ed), 1976, Ch 1.) It was explained why it was considered reasonable to look for results that were not strictly quantifiable, nevertheless it was clear that good, systematic interviewing method together with careful sifting of the data were required to obtain evidence of the desired quality. One over-riding consideration was the problem of avoiding bias in the results, and reasons were put forward to suggest that this was achieved to a reasonable degree when the information about the course for arts students was analysed. However, greater confidence would accrue if the results could be shown to be derived from a random sample and on other occasions, given similar circumstances, this would be attempted. The objective could still be frustrated by less-interested students failing to attend. To counteract this, interviews could be arranged at more congenial times and places and more direct contact achieved between investigator and the interviewees, so that they could be given greater encouragement to attend.

Another feature was the diffuse nature of the material obtained from the interviews; some interesting comments would arise quite unexpectedly. The subsequent analysis was that much more time-consuming than with some other forms of data and the extraction of the relevant points increased the opportunity for the evaluator to introduce unwitting bias. It is probably fair to say that later, when issues became more precisely defined and the technique of interviewing more polished, the quality of the evidence was improved. This experience highlighted the importance of the early identification of major issues in conserving effort and increasing the value of the information. If this could have been achieved with greater effect in this case, it is possible that some of the earlier investigations would have been more fruitful, particularly in providing more precise information about students' attitudes and opinions. This however, is not to say that secondary issues, if they arose would not be pursued; this was found to be a valuable feature of the interview programme.

One final feature concerned the course for arts students. There was some evidence that the reactions of a number of students were influenced by the differences between the real aims of the course and what they perceived them to be. It would have been of interest to investigate any effect of a different approach to the dissemination of information about the nature of the course. This would have required the co-operation of the department concerned and some additional administrative effort, but it might have been possible to study the effectiveness of a different method and to discover if this had any effect on the attitudes of students.

The service course for economists also offered opportunity to develop alternative research objectives. It might be said, in retrospect, that preoccupation with writing and developing the teaching scheme distracted somewhat from interest in the applications side. However, it was not envisaged, a priori, that given a sound basis of necessary skills, the applications would prove quite as difficult as they turned out to be. It might have been possible therefore to include in the research the objective of identifying and fostering the precise attributes which are required by the problem-solver. However, this is not a straightforward question and useful information on this issue is not plentiful, although more recent publications reflect an increasing interest. (See, for example, the International Journal of Mathematical Modelling (Avula and Rodin, eds.) and "The Importance of Mathematical Modelling for University Education in Mathematics"; Gross, 1981.) Certainly, much work remains to be done in this field and this is discussed further in the next section.

Another interesting feature that emerged was a certain lack of enthusiasm amongst very weak students for individualised study methods. This was somewhat unexpected and seemed to arise from the difficulty they encountered in organising their studies; sometimes for example they showed a reluctance to attend remedial tutorials. An experiment to try and compare their progress under different modes of instruction would have been impossible in the given circumstances but it might have been possible in the overall study to have focussed more attention on this group to try and identify problems and perhaps modify the remedial tutorial system. However, taking note of the few students who over the years were required to leave the course through unsatisfactory progress in mathematics alone, it may be that at least some of these students were unlikely to profit from an undergraduate course and were wise to seek alternatives.

20.5 Further Investigation

Before leaving the subject, it is worthwhile to consider if lines of future study have been suggested by these investigations. A serious discussion concerning cultural mathematics courses for arts-based students will continue to centre on the way in which the syllabus is interpreted and translated into teaching objectives. (Cultural mathematics courses for mathematical specialists would also be worthwhile but the problems here would be quite different.) The syllabus content itself is problematical. There is a vast amount of literature containing material of "liberal" or "cultural" value and the course designer will inevitably be faced with decisions about what to leave out. Further useful research might concern the development of relevant and detailed teaching objectives and the preparation of corresponding teaching material.

There is little disagreement with aims which seek to present something of mathematical significance and to involve genuine mathematical activity. The problem is one of top-down analysis; expanding the aims until they emerge as the substance of classroom study and seminar discussion in a form that will stir the interest of the students. Because content has traditionally been the centre of so much concern, there is perhaps a suspicion that some topics have remained in the literature because of what they are rather than because of what they can offer the non-specialist. More effort could well be expended on investigating how worthwhile activity can be generated within the context of a formal syllabus. There is no reason why the specialist and non-specialist should study the same topic in a similar way or for the same reasons. It might be rewarding to move away from content entirely and instead of considering the question, "Which topics should we be teaching?" consider the alternative "What is this teaching trying to achieve?"

The research that was carried out has provided an insight into the factors that combined to make the course what it was and determine how it functioned. There is now an opportunity to explore how the lecturer's task might be made more rewarding and the students' experience more fruitful.

In the case of the service course for economists, there will be undoubted rewards in giving more attention to the development of problem-solving ability. More needs to be known about what constitutes a good problem-solver. In the sphere of formulation for example, there is informal evidence that this is not necessarily related to overall ability or to a particular level of mastery of manipulative skills. Some authors point to the advantages of experiencing open-ended questions in developing the correct strategies (Avital and Shettleworth, 1968). Are there questions which are more appropriate to economists say, than are others? What are the implications in terms of teaching method? If it can be assumed that teaching techniques are now available that deal effectively with the development of basic technique, the main interest in service-type courses in the future is likely to be in the way these deal with the applications of mathematics in the various disciplines.

20.6 Conclusion

This brings to a conclusion the documentation of the research that has arisen from an interest in the problems associated with teaching mathematics to non-specialists at undergraduate level. It was fortunate in many ways that the author was able to study two courses under this same heading which displayed such varied characteristics. The combinations of institutional differences, course aims, student attitudes, staff expectations resulted in some fascinating contrasts. The type of research that was required has led a mathematician some way into the realms of social science and has demanded the mastery of new skills and the exploration of new situations. The author is of the opinion that a function of educational research is to produce worthwhile changes in the experiences of the student in the tutorial and lecture room. It is hoped that the work detailed here has helped to achieve this. Some questions arose that were only partially answered, but some of the issues involved when teaching mathematics to the non-specialist are a little clearer. At least, when situations are encountered that are similar to those discussed in this thesis, it will be possible to identify that much more precisely what the major features are, what the aims and objectives should be and how best to set about achieving them.