Supplementary Appendix: Sustaining Collusive Outcome.

Suppose firms sustain their joint profit maximization agreement using trigger strategies taking in to account the multimarket nature of their contact. Total profits of firm A under collusion is $\pi_{1A}^{JM} + \pi_{2A}^{JM}$, while under competition this is $\pi_{1A}^{C} + \pi_{2A}^{C}$. Given that the punishment for cheating – reversion to competition – is the same whether a firm cheats in one or both markets, a cheating firm would cheat in both markets. It is possible that the firm (say, A) deviating from collusion captures fully both the markets or the entire home market but only a fraction of the foreign market. Given (τ^{JM}, t^{JM}) and $(p_{1B}^{JM}, p_{2B}^{JM})$, suppose firm A deviates to (p_{1A}^{D}, p_{2A}^{D}).

Optimal deviation the home market. The deviation of firm A from collusion could result in firm B either having a positive market share or being priced out of the market. Denote firm A's share as $s_{1A}(p_{1A}^{D}, p_{1B}^{JM})$.

■ $\mathbf{s_{1A} < 1}$. The deviating firm's problem is: $\max_p \pi_{1A}^{D} \equiv (p-c)s_{1A}(p_{1A}^{D}, p_{1B}^{JM})$, which gives:

$$\begin{aligned} p_{1A}^{D} &= \frac{v+c}{2} + \frac{\theta(5\theta+4\delta+\gamma)}{4(2\delta+3\theta)}, \\ s_{1A} &= \frac{v-c}{4\theta} + \frac{\gamma+4\delta+5\theta}{8(2\delta+3\theta)}, \\ \text{and} \qquad \pi_{1A}^{D} &= \frac{[2(v-c)(2\delta+3\theta)+\theta(\gamma+4\delta+5\theta)]^2}{32\theta(2\delta+3\theta)^2}. \end{aligned} \tag{1}$$

$s_{1A}^{D} < 1 \iff 2(v-c)(2\delta+3\theta) - \theta(19\theta+12\delta-\gamma) < 0.$

■ $\mathbf{s_{1A} = 1}$. If $2(v-c)(2\delta+3\theta) - \theta(19\theta+12\delta-\gamma) \geq 0$, then the deviating firm A captures the entire home market. It sets p_{1A}^{D} such that the consumer located at $x_1 = 1$ is indifferent between products A and B, i.e., $U_A(p_{1A}^{D}, p_{1B}^{JM}) = U_B(p_{1A}^{D}, p_{1B}^{JM})$. This implies:

$$\begin{aligned} p_{1A}^{D} &= v + \frac{\gamma-4\delta-7\theta}{2(2\delta+3\theta)}, \\ \text{and} \qquad \pi_{1A}^{D} &= (v-c) + \frac{\gamma-4\delta-7\theta}{2(2\delta+3\theta)}. \end{aligned} \tag{2}$$

Optimal deviation the foreign market. In market 2, firm A's market share is $s_{2A}(p_{2A}^{D})$. There are two possibilities.

■ $\mathbf{s_{2A} < 1}$. The deviating firm's problem is: $\max_p \pi_{2A}^{D} \equiv (p-c-\gamma-\tau_1^{JM}-t_2^{JM})s_{2A}(p_{2A}^{D}, p_{2B}^{JM})$, which gives:

$$\begin{aligned} p_{2A}^D &= \frac{v+c}{2} + \frac{\theta(9\theta+8\delta+3\gamma)}{4(2\delta+3\theta)}, \\ s_{2A} &= \frac{v-c}{4\theta} - \frac{5\gamma+8\delta+7\theta}{8(2\delta+3\theta)}, \\ \text{and} \qquad \pi_{2A}^D &= \frac{[2(v-c)(2\delta+3\theta)-\theta(5\gamma+8\delta+7\theta)]^2}{32\theta(2\delta+3\theta)^2}. \end{aligned} \tag{3}$$

$s_{1A}^D < 1 \iff 2(v-c)(2\delta+3\theta) - 3\theta(31\theta+24\delta+5\gamma) < 0$, which is a less strict condition than that for $s_{1A} < 1$ above.

■ $\mathbf{s_{2A} = 1}$. If $2(v-c)(2\delta+3\theta) - 3\theta(31\theta+24\delta+5\gamma) \geq 0$, then firm A sets p_{2A}^D such that the consumer at $x_2 = 1$ is indifferent between products A and B. This implies

$$\begin{aligned} p_{2A}^D &= v - \frac{\theta(\gamma+8\delta+11\theta)}{2(2\delta+3\theta)}, \\ \text{and} \qquad \pi_{2A}^D &= (v-c) - \frac{\theta(5\gamma+16\delta+19\theta)}{2(2\delta+3\theta)}. \end{aligned} \tag{4}$$

Note that if firm A captures the foreign market, it also captures the home market.

Supporting collusion using trigger strategies. Total profits (in each period) of firm j, $j = A, B$, under collusion is $\pi_j^{JM} = \pi_{1j}^{JM} + \pi_{2j}^{JM}$, while under competition it is $\pi_j^C = \pi_{1j}^C + \pi_{2j}^C$. When firm B colludes but firm A deviates, denote the latter's profit as π_A^D. Sustaining collusion using trigger strategies implies that once firm A deviates in one period, both firms compete in all subsequent periods. The incentive compatibility constraint for sustaining collusion as a subgame perfect equilibrium of an infinitely repeated game using trigger strategies is (where Δ is the discount factor):

$$\frac{\pi_A^{JM}}{1-\Delta} \geq \pi_A^D + \frac{\Delta}{1-\Delta}\pi_A^C \qquad \Leftrightarrow \qquad \Delta \geq \frac{\pi_A^D - \pi_A^{JM}}{\pi_A^D - \pi_A^C} \equiv \Delta^*. \tag{5}$$

It can be checked (from Section 4) that:

$$\pi_A^C = \pi_{1A}^C + \pi_{2A}^C = \frac{\theta[\gamma^2+4\delta\gamma+8\delta^2+6\gamma\theta+28\delta\theta+25\theta^2]}{4(\delta+2\theta)^2}, \tag{6}$$

$$\text{and} \qquad \pi_A^{JM} = \pi_{1A}^{JM} + \pi_{2A}^{JM} = (v-c) + \frac{\theta[\gamma^2-8\delta^2-2\theta(\gamma+12\delta)-17\theta^2]}{2(2\delta+3\theta)^2}. \tag{7}$$

Case 1: Both firms sell in both markets. In this case, $(s_{1A} < 1, s_{2A} < 1)$; $s_{1A} < 1$ requires a stricter condition: $2(v-c)(2\delta+3\theta) - \theta(19\theta+12\delta-\gamma) < 0$. Firm A's profit in the deviation period is (using eqs. (1) and (3)):

$$\begin{aligned}\pi_A^D(s_{1A}<1) &= \pi_{1A}^D(s_{1A}<1)+\pi_{2A}^D(s_{2A}<1)\\ &= \frac{[2(v-c)(2\delta+3\theta)+\theta(\gamma+4\delta+5\theta)]^2}{32\theta(2\delta+3\theta)^2}+\frac{[2(v-c)(2\delta+3\theta)-\theta(5\gamma+8\delta+7\theta)]^2}{32\theta(2\delta+3\theta)^2},\end{aligned}$$

implying $\Delta^* = \frac{\Gamma_1}{\Omega_1}$, where $\Gamma_1 \equiv (\delta+2\theta)^2[4v\theta(2\delta+3\theta)(2\gamma+10\delta+13\theta) - 4c^2(2\delta+3\theta)^2 - 4v^2(2\delta+3\theta)^2 - \theta^2(5\gamma^2+44\delta\gamma+104\delta^2+56\gamma\theta+268\delta\theta+173\theta^2) - 4c(2\delta+3\theta)(\theta(2\gamma+10\delta+13\theta)-2v(2\delta+3\theta))]$ and $\Omega_1 \equiv 4v\theta(2(\delta+\gamma)+\theta)(\delta+2\theta)^2(2\delta+3\theta) - 4c^2(\delta+2\theta)^2(2\delta+3\theta)^2 - 4v^2(\delta+2\theta)^2(2\delta+3\theta)^2 - 4c(\delta+2\theta)^2(2\delta+3\theta)(\theta^2+2(\gamma-3v+\delta)\theta-4v\delta) + \theta^2\{\delta^2(3\gamma^2+20\delta\gamma+88\delta^2)+4\delta\theta(149\delta^2+18\delta\gamma-\gamma^2)+\theta^2(1531\delta^2+96\delta\gamma-16\gamma^2)+4\theta^3(14\gamma+439\delta)+752\theta^4\}$.

Case 2: Firm A captures market 1 and both firms sell in market 2. In this case, $(s_{1A}=1, s_{2A}<1)$. Recall $s_{1A}=1 \iff 2(v-c)(2\delta+3\theta)-\theta(19\theta+12\delta-\gamma)\geq 0$ and $s_{2A}<1 \iff 2(v-c)(2\delta+3\theta)-3\theta(31\theta+24\delta+5\gamma)<0$. Firm A's profit in the deviation period is (using eqs. (2) and (3)):

$$\begin{aligned}\pi_A^D &= \pi_{1A}^D(s_{1A}=1)+\pi_{2A}^D(s_{2A}<1)\\ &= (v-c)+\frac{\gamma-4\delta-7\theta}{2(2\delta+3\theta)}+\frac{[2(v-c)(2\delta+3\theta)-\theta(5\gamma+8\delta+7\theta)]^2}{32\theta(2\delta+3\theta)^2},\end{aligned}$$

implying $\Delta^* = \frac{\Gamma_2}{\Omega_2}$, where $\Gamma_2 \equiv (\delta+2\theta)^2[4v\theta(2\delta+3\theta)(5\gamma+8\delta+7\theta) - 4c^2(2\delta+3\theta)^2 - 4v^2(2\delta+3\theta)^2 - 4c(2\delta+3\theta)(\theta(5\gamma+8\delta+7\theta)-2v(2\delta+3\theta)) - \theta\{9\gamma^2\theta+64\delta^2(3\theta-2)+16\delta\gamma(2+5\theta)+6\gamma\theta(8+17\theta)+16\delta\theta(31\theta-26)+36\theta^2(107\theta-112)\}]$ and $\Omega_2 \equiv [4v\theta(5\gamma-8\delta-17\theta)(\delta+2\theta)^2(2\delta+3\theta) - 4c^2(2\delta^2+7\delta\theta+6\theta^2)^2 - 4v^2(2\delta^2+7\delta\theta+6\theta^2)^2 + 4c(\delta+2\theta)^2(2\delta+3\theta)(v(4\delta+6\theta)+\theta(-5\gamma+8\delta+17\theta)) + \theta\{64\delta^4(2+3\theta)+16\delta^3\theta(58+81\theta)+4\theta^4(336+401\theta)+4\delta\theta^3(752+943\theta)+\delta^2\theta^2(2512+3311\theta)+\gamma^2\theta(7\delta^2-4\delta\theta-28)\theta^2)+2\gamma[8\delta^3(3\theta-2)+4\theta^3(19\theta-24)+4\delta\theta^2(33\theta-40)+\delta^2\theta(93\theta-88)]\}]$.

Case 3: Firm A captures both markets 1 and 2. In this case, $(s_{1A}=s_{2A}=1)$. Recall that $s_{2A}=1 \iff 2(v-c)(2\delta+3\theta)-3\theta(31\theta+24\delta+5\gamma)\geq 0$, implies $s_{1A}=1$. Firm A's profit in the deviation period is (using eqs. (2) and (4)):

$$\begin{aligned}\pi_A^D &= \pi_{1A}^D(s_{1A}=1)+\pi_{2A}^D(s_{2A}=1)\\ &= (v-c)+\frac{\gamma-4\delta-7\theta}{2(2\delta+3\theta)}+(v-c)-\frac{\theta(5\gamma+16\delta+19\theta)}{2(2\delta+3\theta)},\end{aligned}$$

implying $\Delta^* = \frac{\Gamma_3}{\Omega_3}$ where $\Gamma_3 \equiv 2(\delta+2\theta)^2[\gamma^2\theta+8\delta^2(3\theta+1-v+c)+2\delta\gamma(5\theta-1)+\gamma\theta(13\theta-3)+2\delta\theta(13+12c-12v+31\theta)+\theta^2(21+18c-18v+40\theta)]$ and $\Omega_3 \equiv (2\delta+3\theta)[\gamma^2\theta(2\delta+3\theta)+8\delta^3(1+$

$2c - 2v + 6\theta) + 2\delta^2\theta(23 + 44c - 44v + 123\theta) + 2\delta\theta^2(44 + 80c - 80v + 207\theta) + \theta^3(56 + 96c - 96v + 227\theta) + 2\gamma\{4\delta\theta(8\theta - 1) + \delta^2(9\theta - 1) + \theta^2(29\theta - 4)\}].$