

Seismic metasurface on an orthorhombic elastic half-space

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Abstract

The article is studying a seismic meta-surface in the case of an oscillatory system arranged on the surface of an orthorhombic elastic half-space. The approach is based on the asymptotic hyperbolic–elliptic formulation for the Rayleigh wave excited by prescribed surface loading. The latter results in hyperbolic equations for surface displacements, with the right-hand sides involving the loading components. The derived model allows a formulation for the meta-surface in the form of a periodic spring-mass system attached to the surface as a hyperbolic equation for the horizontal displacement, with smooth contact stresses emerging from averaging the effect of a regular array of oscillators. The associated dispersion relation is constructed and illustrated numerically for both cases of exponential and oscillatory decay.

Keywords

Rayleigh wave, seismic metasurface, orthorhombic, asymptotic model, dispersion

Introduction

Seismic metasurfaces present a subclass of metamaterials, involving a specialised design of the surface by means of attached systems of resonators, affecting the Rayleigh wave with the aim of diverting or suppressing its propagation. The area of seismic metasurfaces and meta-barriers has become a hot topic in the last decade.^{1–3} Designed systems of resonators may also enable tuning of phase velocities and wavelengths of propagating waves^{4–9} for ideas of natural and artificial metasurfaces, see also a more general recent review of nanophotonics and metamaterials.¹⁰

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A possible approach to analytical treatment of seismic meta-surfaces is based on explicit models for Rayleigh and Rayleigh-type waves¹¹ summarising this methodology. The discussed model for the Rayleigh wave is oriented towards extracting the contribution of the Rayleigh wave to the overall dynamic response, thus allowing a more convenient analytical tool, especially on the surface, avoiding the complications arising from the analysis of exact solutions involving more sophisticated wave phenomena.

In the case of a linearly isotropic elastic half-space, the asymptotic formulation for the Rayleigh wave induced by prescribed surface loading includes a hyperbolic equation for the longitudinal displacement potential governing wave propagation, as well as an elliptic equation characterising decay away from the surface. It is worth noting that once the longitudinal potential has been determined, the displacements are expressed through the latter, using the representation of surface wave field in terms of a single harmonic function.^{12,13} Various recent developments of explicit models for the Rayleigh and Rayleigh-type waves, accounting for the effects of anisotropy, pre-stress, inhomogeneity and nonlocality have been reported in References.^{14–17} In addition, it has been recently shown that the hyperbolic–elliptic formulation may be reduced to a hyperbolic equation at a fixed depth, with the right-hand side involving a pseudo-differential operator acting on the load, with vertical coordinate essentially being a parameter only.¹⁸

This discussed approach is applicable when the Rayleigh wave dominates the mechanical response, which makes seismic meta-surfaces an ideal application of the described approach. The methodology was implemented within the context of linear isotropic elasticity, including considerations of periodic systems of spring-mass oscillators, as well as rods and beams with various boundary conditions.^{19–21}

However, in the case of a periodic system of oscillators arranged on the surface of an anisotropic half-space, the question of constructing an explicit formulation for a seismic meta-surface relying on the asymptotic model for the Rayleigh wave remains largely open. In particular, one of the complications is due to impossibility of decomposing the displacement field via the Lamé wave potentials, hence, implementation of the methodology in the works,^{19–21} formulated in terms of the elastic potentials, becomes problematic. The present contribution aims at overcoming this challenge.

In this contribution, first, we derive the asymptotic formulation for the Rayleigh wave excited by prescribed surface loading on an orthorhombic elastic half-space. The obtained hyperbolic equations for surface displacements generalise the previously known results,²² confined within a particular case of anisotropy associated with pure exponential decay, to a more general case allowing oscillatory decay. In addition, the derived hyperbolic equations account for both normal and tangential components of the loading. Then, a seismic meta-surface comprised of a system of periodic spring-mass oscillators arranged on the surface of an orthorhombic half-space is studied. The dispersion relation is constructed and illustrated numerically for both cases of pure exponential and oscillatory decay.

Hyperbolic equations for surface displacements

In this section, the approximate formulation for the Rayleigh wave field excited by prescribed surface stresses is introduced. This formulation will be employed later in construction of a seismic meta-surface.

Consider a linearly elastic, orthorhombic half-space $x_2 \geq 0$, under plane-strain assumption, that is, with the displacements satisfying $u_3 \equiv 0$ and $u_j = u_j(x_1, x_2, t)$, where $j = 1, 2$. The equations of motion are written conventionally as

$$c_{11}u_{1,11} + c_{66}u_{1,22} + (c_{12} + c_{66})u_{2,12} = \rho u_{1,tt}, \quad (1)$$

and

$$(c_{12} + c_{66})u_{1,12} + c_{66}u_{2,11} + c_{22}u_{2,22} = \rho u_{2,tt}, \quad (2)$$

where c_{ij} are elastic moduli, ρ stands for the volume mass density, and a comma in the subscript indicates differentiation with respect to appropriate spatial or time variable.

The boundary conditions on the surface $x_2 = 0$ are taken in the form

$$\sigma_{21} = c_{66}(u_{1,2} + u_{2,1}) = f_1(x_1, t), \quad (3)$$

$$\sigma_{22} = c_{12}u_{1,1} + c_{22}u_{2,2} = f_2(x_1, t), \quad (4)$$

where σ_{21} and σ_{22} are the appropriate stresses, with f_1 and f_2 being the prescribed tangential and normal components of the loading, respectively. It is also assumed that the displacement components satisfy the decay conditions

$$\lim_{x_2 \rightarrow +\infty} (u_1, u_2) = 0.$$

Let us now derive briefly the approximate hyperbolic equations for surface displacements, accounting for the Rayleigh wave contribution to the overall dynamic response. These will extend the results obtained in the contribution²² for a particular case of anisotropy to a more general scenario allowing oscillatory decay. At the same time, below we rely on integral transforms, contrary to perturbation scheme in the aforementioned article.

On applying the Laplace and Fourier integral transform with respect to time t and the longitudinal coordinate x_1 , respectively, the boundary value problem (1)–(4) becomes

$$c_{66} \frac{d^2 u_1^{FL}}{dx_2^2} - (c_{11}s^2 + \rho p^2)u_1^{FL} + is(c_{12} + c_{66}) \frac{du_2^{FL}}{dx_2} = 0, \quad (5)$$

$$is(c_{12} + c_{66}) \frac{du_1^{FL}}{dx_2} + c_{22} \frac{d^2 u_2^{FL}}{dx_2^2} - (c_{11}s^2 + \rho p^2)u_2^{FL} = 0, \quad (6)$$

subject to

$$\left. \frac{du_1^{FL}}{dx_2} \right|_{x_2=0} + is u_2^{FL}|_{x_2=0} = \frac{f_1^{FL}}{c_{66}}, \quad (7)$$

$$is c_{12} u_1^{FL}|_{x_2=0} + c_{22} \left. \frac{du_2^{FL}}{dx_2} \right|_{x_2=0} = f_2^{FL}, \quad (8)$$

at the surface $x_2 = 0$. Here

$$w^{FL}(s, y, p) = \int_0^\infty \left(\int_{-\infty}^{+\infty} w(x, y, t) e^{-isx} dx \right) e^{-pt} dt \quad (9)$$

is the Fourier–Laplace transform of the quantity $w(x, y, t)$, with p and s denoting the parameters of the Laplace and Fourier transforms, respectively.

Then, expressing the transformed vertical displacement u_2^{FL} from (6) and substituting the result into (5), we deduce the bi-quadratic equation for u_1^{FL}

$$\begin{aligned} c_{66}c_{22}\frac{d^4u_1^{FL}}{dx_2^4} - s^2(c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{66} - (c_{22} + c_{66})\rho z)\frac{d^2u_1^{FL}}{dx_2^2} \\ + s^4(c_{11} - \rho z)(c_{66} - \rho z)u_1^{FL} \\ = 0, \end{aligned} \quad (10)$$

where

$$z = -\frac{p^2}{s^2}. \quad (11)$$

The decaying solution of the latter is given by

$$u_1^{FL}(x_2) = C_1 e^{-q_1|s|x_2} + C_2 e^{-q_2|s|x_2}, \quad (12)$$

with q_1 and q_2 satisfying

$$\begin{aligned} q_1^2 + q_2^2 = \xi(z) = \frac{c_{11}c_{22} - c_{12}^2 - 2c_{12}c_{66} - (c_{22} + c_{66})\rho z}{c_{22}c_{66}}, \\ q_1q_2 = \eta(z) = \sqrt{\frac{(c_{11} - \rho z)(c_{66} - \rho z)}{c_{22}c_{66}}}. \end{aligned} \quad (13)$$

Hence, in view of (5), we deduce

$$u_2^{FL}(x_2) = C_1 F(q_1) e^{-q_1|s|x_2} + C_2 F(q_2) e^{-q_2|s|x_2}, \quad (14)$$

where

$$F(q_j) = i \frac{c_{11} - \rho z - c_{66}q_j^2}{(c_{12} + c_{66})q_j} \operatorname{sgn}(s), \quad j = 1, 2. \quad (15)$$

Next, the boundary conditions (7)–(8) may be employed in order to determine the unknown quantities C_1 , C_2 , and, therefore, the transformed surface displacements follow from (12), (14) in the form

$$u_1^{FL}(0) = C_1 + C_2 = f_1^{FL} \frac{Q_1(z)}{|s|R(z)} + if_2^{FL} \frac{Q_2(z)}{sR(z)}, \quad (16)$$

and

$$u_2^{FL}(0) = C_1 F(q_1) + C_2 F(q_2) = if_1^{FL} \frac{P_1(z)}{sR(z)} + f_2^{FL} \frac{P_2(z)}{|s|R(z)}. \quad (17)$$

Here

$$\begin{aligned} Q_1(z) &= -c_{22}\eta(z)\zeta(z), & Q_2(z) &= -P_1(z) = c_{11} - \rho z - c_{12}\eta(z), \\ P_2(z) &= (\rho z - c_{11})\zeta(z), & \zeta(z) &= \sqrt{\xi(z) + 2\eta(z)}, \end{aligned} \quad (18)$$

and

$$R(z) = (c_{11}c_{22} - c_{12}^2 - c_{22}\rho z)\eta(z) - \rho z(c_{11} - \rho z). \quad (19)$$

The latter is readily recognised as the Rayleigh function (cf. equation (21) in Reference²²), with both of the transformed surface displacements having simple poles at $z = c_R^2$. Hence, accounting for the contribution of the poles results in the following approximate expressions for surface displacements, that is, employing the Taylor expansion

$$R\left(-\frac{p^2}{s^2}\right) \approx -R'(c_R^2)\left(\frac{p^2}{s^2} + c_R^2\right),$$

see also Section 4.3 in Reference,¹¹ we deduce

$$u_1^{FL}(0) \approx -f_1^{FL} \frac{|s|Q_1(c_R^2)}{R'(c_R^2)(p^2 + c_R^2 s^2)} - if_2^{FL} \frac{sQ_2(c_R^2)}{R'(c_R^2)(p^2 + c_R^2 s^2)}, \quad (20)$$

$$u_2^{FL}(0) \approx -if_1^{FL} \frac{sP_1(c_R^2)}{R'(c_R^2)(p^2 + c_R^2 s^2)} - f_2^{FL} \frac{|s|P_2(c_R^2)}{R'(c_R^2)(p^2 + c_R^2 s^2)}. \quad (21)$$

Applying the inverse Fourier–Laplace transforms to the above formula, we arrive at the hyperbolic equations for surface displacements in the form

$$\left(u_{1,11} - \frac{1}{c_R^2}u_{1,t}\right)\Big|_{x_2=0} = \frac{Q_1(c_R^2)}{c_R^2 R'(c_R^2)}\sqrt{-\partial_{,11}}[f_1] + \frac{Q_2(c_R^2)}{c_R^2 R'(c_R^2)}f_{2,1} \quad (22)$$

and

$$\left(u_{2,11} - \frac{1}{c_R^2}u_{2,t}\right)\Big|_{x_2=0} = \frac{P_1(c_R^2)}{c_R^2 R'(c_R^2)}f_{1,1} + \frac{P_2(c_R^2)}{c_R^2 R'(c_R^2)}\sqrt{-\partial_{,11}}[f_2]. \quad (23)$$

The hyperbolic equations (22)–(23) generalise the results obtained in Reference²² by accounting for both vertical and horizontal components of the loading, and also for a stronger degree of anisotropy allowing for oscillatory decay, whereas the results of the cited article were restricted to the case of real roots for q_j , $j = 1, 2$. Note the pseudo-differential operator $\sqrt{-\partial_{,11}}$ (for more details on the theory of such operators the readers are referred to Reference²³), acting on the appropriate part of the load, observed previously for isotropic medium, which could also be written as a Hilbert transform, see Appendix B in Reference.²⁴

Seismic metasurface

Let us now implement the asymptotic model derived in the previous section to design of a seismic meta-surface in the form of a periodic system of linear oscillators attached to the surface of an orthorhombic half-space within the framework of plane-strain assumption, extending the results of Reference¹⁹ to account for anisotropy. The analysis below is carried out in the assumption that the contribution of the bulk waves into the overall dynamic response can be neglected compared with that of the Rayleigh wave,¹¹ and references therein for greater detail. The considered periodic system of oscillators is depicted in Figure 1, where a is the distance between neighbouring oscillators.

The motion of an oscillator is governed by

$$mv_{tt} + \chi v = p, \quad (24)$$

where $v = v(x_2, t)$ is vertical displacement, and m, χ , and $p = (x_1, t)$ represent mass, stiffness of the spring, and contact vertical force, respectively, distributed per unit length. The assumed continuity of displacements and stresses on the interface $x_2 = 0$ implies

$$u_2 = v, \quad \sigma_{22} = \frac{p}{a}. \quad (25)$$

According to the hyperbolic model for the Rayleigh wave derived in the previous section, the tangential surface displacement satisfies

$$u_{1,11} - \frac{1}{c_R^2} u_{1,tt} = B_R f_{2,1}, \quad (26)$$

where the constant B_R is given by

$$B_R = \frac{Q_2(c_R^2)}{c_R^2 R'(c_R^2)}, \quad (27)$$

with quantities Q_2 and R defined in (18) and (19), respectively, and, as above, c_R denoting

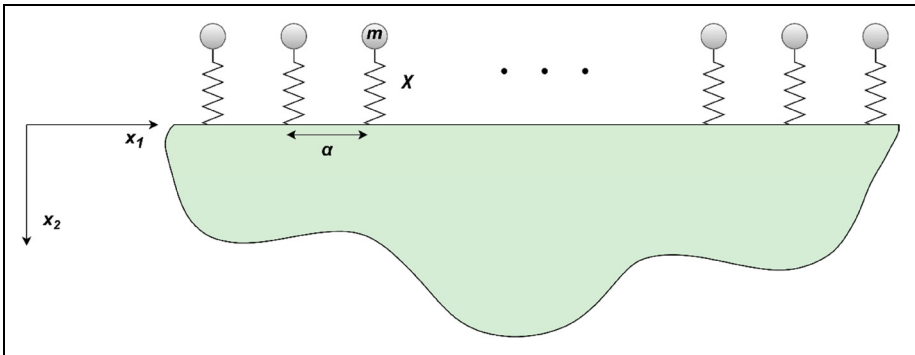


Figure 1. A periodic system of spring-mass oscillators with uniform spacing attached to the surface of a semi-infinite half-space.

the Rayleigh wave speed, being the root of $R(c_R^2) = 0$. We remark that (26) tacitly assumes zero tangential surface stress. For the considered periodic system of mass-spring oscillators, the contact surface stress may be written as¹⁹

$$f_2 = \sum_{n=-\infty}^{+\infty} p(x_1, t) \delta(x_1 + na). \quad (28)$$

As shown in the cited article, the consideration of a point contact between the single mass-spring system and the elastic media leads to an undesired spurious localised component of the solution. Therefore, the distribution of the contact stress is required, yielding

$$\sum_{n=-\infty}^{+\infty} p(x_1, t) \delta(x_1 + na) \approx \frac{1}{a} p(x_1, t). \quad (29)$$

As usual, it is assumed that the typical wavelength is much greater than the distance between the oscillators. Hence, in view of (29), the hyperbolic equation (26) may be re-cast in the form

$$u_{1,11} - \frac{1}{c_R^2} u_{1,tt} = \frac{B_R}{a} p_{,1}(x_1, t). \quad (30)$$

Note that the corresponding result within the context of linear isotropic elasticity can readily be recovered such that the surface tangential displacement is proportional to the appropriate derivative of the longitudinal Lamé potential φ (cf. equations (4.1), (4.7) in Reference¹⁹).

Now, take the distribution for the contact force in the form associated with time-harmonic waves

$$p(x_1, t) = P e^{i(kx_1 - \omega t)}, \quad (31)$$

where k is wave number and ω is angular frequency. Then, from (24), the forced solution is given by

$$v = \frac{P}{m(\omega_0^2 - \omega^2)} e^{i(kx_1 - \omega t)}, \quad (32)$$

with $\omega_0^2 = \chi / m$.

Let us now briefly derive the Rayleigh wave eigensolutions. The equations of motion are taken as (1)-(2), with the traction-free boundary conditions along the surface $x_2 = 0$ are given by (3)-(4), with $f_1 = f_2 = 0$.

Now, we search for displacement components in the form of travelling harmonic waves, that is,

$$u_j = U_j e^{ik(x_1 - ct) - kqx_2}, \quad j = 1, 2. \quad (33)$$

Then, the equations of motion imply a bi-quadratic secular equation for the attenuation

order q , given by

$$\begin{aligned} & c_{22}c_{66}q^4 + [c_{66}(\rho c^2 - c_{66}) + c_{22}(\rho c^2 - c_{11}) + (c_{12} + c_{66})^2]q^2 + (\rho c^2 - c_{11})(\rho c^2 \\ & - c_{66}) \\ & = 0. \end{aligned} \quad (34)$$

Then, by using the equations of motion, the displacements are obtained in the form

$$\begin{aligned} u_1 &= \sum_{j=1}^2 A_j e^{ik(x_1 - ct) - kq_j x_2}, \\ u_2 &= \sum_{j=1}^2 A_j f(q_j) e^{ik(x_1 - ct) - kq_j x_2}, \end{aligned} \quad (35)$$

where

$$f(q_j) = \frac{\rho c_R^2 - c_{11} + c_{66}q_j^2}{iq_j(c_{12} + c_{66})}, \quad j = 1, 2,$$

and the attenuation factors q_j ($j = 1, 2$) are the solutions of (34) satisfying $\text{Re}(q_j) > 0$ in order to ensure decay at $x_2 \rightarrow \infty$.

Finally, substitution of (35) into traction-free boundary conditions gives the relation between the unknown constants A_1, A_2 , implying the following eigensolution for the Rayleigh wave field

$$\begin{aligned} u_1 &= A \left(e^{-kq_1 x_2} - \frac{\gamma + \delta q_1^2}{\gamma + \delta q_2^2} e^{-kq_2 x_2} \right) e^{i(kx_1 - \omega t)}, \\ u_2 &= A \left(f(q_1) e^{-kq_1 x_2} - \frac{\gamma + \delta q_1^2}{\gamma + \delta q_2^2} f(q_2) e^{-kq_2 x_2} \right) e^{i(kx_1 - \omega t)}, \end{aligned} \quad (36)$$

where

$$\gamma = c_{12}(c_{12} + c_{66}) + c_{22}(\rho c_R^2 - c_{11}), \quad \delta = c_{22}c_{66}. \quad (37)$$

Now, from (30), (31), and (36), we deduce

$$A \left(1 - \frac{\gamma + \delta q_1^2}{\gamma + \delta q_2^2} \right) \left(k^2 - \frac{\omega^2}{c_R^2} \right) = -\frac{ikB_R}{a} P. \quad (38)$$

In addition, from the continuity of vertical displacements at $x_2 = 0$ we have

$$A \left(f(q_1) - \frac{\gamma + \delta q_1^2}{\gamma + \delta q_2^2} f(q_2) \right) = \frac{1}{m(\omega_0^2 - \omega^2)} P. \quad (39)$$

The dispersion relation may now be inferred as a solvability condition of the

simultaneous linear equations (38) and (39) in A and P , yielding

$$\alpha \delta \frac{q_2^2 - q_1^2}{\gamma + \delta q_2^2} \left(k^2 - \frac{\omega^2}{c_R^2} \right) = -ikB_R m (\omega_0^2 - \omega^2) \left(f(q_1) - \frac{\gamma + \delta q_1^2}{\gamma + \delta q_2^2} f(q_2) \right). \quad (40)$$

Introducing the dimensionless quantities

$$K = ka, \quad \Omega = \frac{\omega a}{c_R}, \quad s = \frac{c_R}{a\omega_0}, \quad r = i\chi B_R \frac{f(q_1)(\gamma + \delta q_2^2) - f(q_2)(\gamma + \delta q_1^2)}{\delta(q_2^2 - q_1^2)}, \quad (41)$$

the dispersion relation may be rewritten as

$$K^2 - \Omega^2 = r(s^2\Omega^2 - 1)K, \quad (42)$$

or, more explicitly, as

$$K = \frac{r(s^2\Omega^2 - 1) + \sqrt{r^2(s^2\Omega^2 - 1)^2 + 4\Omega^2}}{2}. \quad (43)$$

It may be shown that in the case of isotropic substrate the dispersion relation (43) coincides with the corresponding result in Reference.¹⁹ Also, as previously noted in the cited article, the left-hand side in (42) is zero at $\Omega = K$, which is the Rayleigh wavefront, whereas the right-hand side equals zero at $\Omega = \Omega_0 = 1/s$, which is associated with the eigenfrequency of the spring-mass oscillator. Hence, since the developed asymptotic formulation for the Rayleigh wave field is oriented towards the contribution of the Rayleigh wave, we may expect it to be valid in the vicinity of $K = K_0 = 1/s$.

Let us now illustrate the obtained results numerically. First, we consider the case of the roots q_1 and q_2 to be real, which corresponds to relatively weak anisotropy allowing operator factorisation of a bi-harmonic equation of motion for one of the displacement components into two elliptic operators, as done in Reference.²² This scenario is illustrated in Figure 2, for the unidirectionally reinforced glass-epoxy composite material, with material parameters are defined as $c_{11} = 55.15$ GPa, $c_{22} = 18.38$ GPa, $c_{12} = 4.60$ GPa, $c_{66} = 9$ GPa, $c_R = 2048.13$ m/s, $\rho = 1980$ kg/m³.²⁵ The parameters of the periodic system of oscillators are chosen as follows: $a = 0.2$ m, $\chi = 36$ GPa/m, and $m = 1000$ kg/m.

The dispersion diagram for this case is presented in Figure 2, containing the dispersion curve governed by (43) depicted by solid line, along with the Rayleigh wave front $K = \Omega$ shown by blue dashed line, the shear wave front $K = c_R / c_2 \Omega$ indicated by red dashed line, and the horizontal dotted line $\Omega = \Omega_0$ associated with the eigenfrequency of the oscillator. Clearly, in view of the decay condition, the dispersion curve (being in fact a part of a hyperbola) originates on the red line corresponding to the shear wave, with the speed of the latter given by $c_2 = \sqrt{c_{66} / \rho}$.

The next Figure 3 is showing similar results, however, for stronger anisotropy associated with oscillatory decay, when the attenuation orders q_1 and q_2 are complex. The material of the substrate is orthorhombic sulphur, for which $c_{11} = 24$ GPa, $c_{22} = 20.5$ GPa, $c_{12} = 13.3$ GPa, $c_{66} = 7.6$ GPa, $c_R = 1628$ m/s, $\rho = 2070$ kg/m³,²⁶

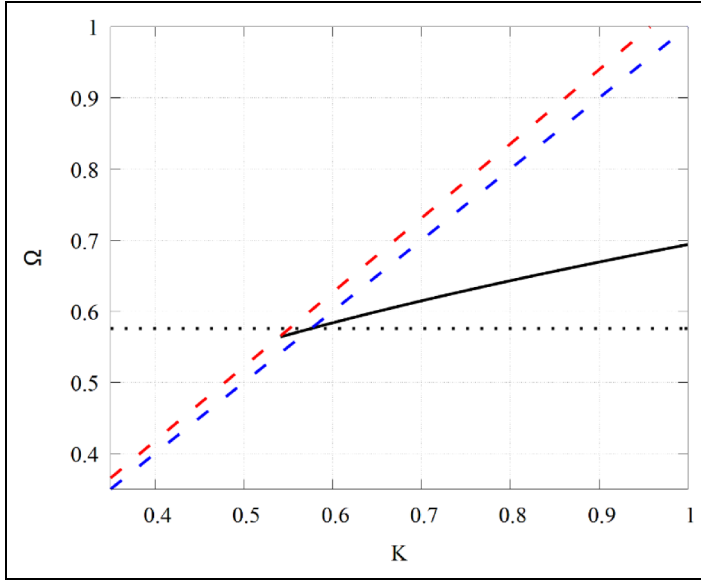


Figure 2. Dispersion diagram for the studied meta-surface, including the dispersion relation (43) (solid line), the Rayleigh wavefront $K = \Omega$ (blue dashed line), the shear wave front $K = c_R / c_2 \Omega$ (red dashed line), and the spring-mass eigenfrequency $\Omega = \Omega_0$ (dotted line). The material and geometrical parameters $c_{11} = 55.15$ GPa, $c_{22} = 18.38$ GPa, $c_{12} = 4.60$ GPa, $c_{66} = 9$ GPa, $c_R = 2048.13$ m/s, $\rho = 1980$ kg/m³, $a = 0.2$ m, $\chi = 4c_{66}$, $m = 1000$ kg/m, correspond to purely exponential decay.

with the parameters of the oscillatory system specified as $a = 0.2$ m, $\chi = 30.4$ GPa, and $m = 1000$ kg/m.

It is seen from both Figures 2 and 3 that the dispersion curve intersects the straight line $\Omega = K$ at the point $\Omega_0 = K_0 = 1/s$, with the domain of validity of the derived long-wave explicit model being in a vicinity of this point. At the same time, the angle between the two oblique straight lines corresponding to the Rayleigh and shear wavefronts seems to be wider in Figure 3, being associated with a larger relative difference between the Rayleigh wave speed and the shear wave speed.

Conclusions

A meta-surface comprised of a periodic mass-spring system attached to an orthorhombic elastic half-space has been studied. First, an explicit formulation for surface displacements oriented to the contribution of the Rayleigh wave excited by prescribed surface stresses is derived, generalising the previous results²² to a stronger type of anisotropy allowing oscillatory decay, as well as presenting the results of Reference¹⁴ in a more explicit form. The obtained hyperbolic formulation is then implemented to construct a seismic meta-surface, resulting in the associated dispersion relation, extending the results in Reference¹⁹ to account for the effects of anisotropy. Numerical illustrations

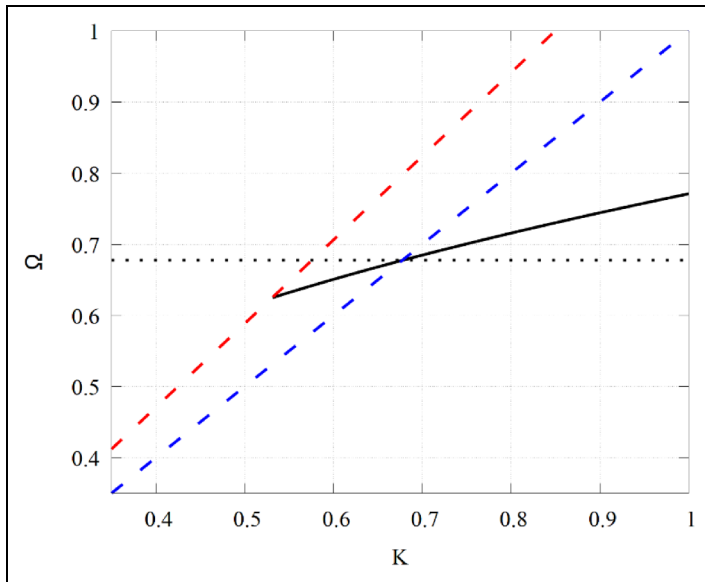


Figure 3. Dispersion diagram, including the dispersion relation (43) (solid line), the Rayleigh wave front $K = \Omega$ (blue dashed line), the shear wave front $K = c_R / c_2 \Omega$ (red dashed line), and the spring-mass eigenfrequency $\Omega = \Omega_0$ (dotted line). The material and geometrical parameters $c_{11} = 24$ GPa, $c_{22} = 20.5$ GPa, $c_{12} = 13.3$ GPa, $c_{66} = 7.6$ GPa, $c_R = 1628$ m/s, $a = 0.2$ m, $\chi = 30.4$ GPa, and $m = 1000$ kg/m correspond to oscillatory decay.

of dispersive behaviour are presented for both cases of pure exponential and oscillatory decay.

The obtained results may be extended to flexural meta-surfaces, complementing the results in Reference.²⁰ Other options are related to incorporating the effects of more general types of anisotropy,¹⁴ vertical inhomogeneity,^{16,24,27} nonlinearity,²⁸ as well as performing a refined treatment of the contact force.²⁹

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